Indian J.Sci.Res. 13 (1): 39-45, 2017 ISSN: 0976-2876 (Print)
ISSN: 2250-0138 (Online)

A STUDY ON THE TIME DEPENDENCE OF BRANS-DICKE PARAMETER (ω)

SUDIPTO ROY[1]

Department of Physics, St. Xavier's College, 30 Mother Teresa Sarani (Park Street), Kolkata, West Bengal, India

ABSTRACT

Brans-Dicke (BD) field equations, for a space of zero curvature, has been used to derive an expression of BD parameter (ω), in terms of time and the equation of state (EOS) parameter. An empirical expression of the BD parameter, as a function of the scalar field (φ), has been chosen for this purpose. Several cosmological parameters have been expressed as functions of the BD parameter and its time derivative. The time variation of BD parameter has been shown graphically for different cosmological era, characterized by the values of the EOS parameter. Dependence of the density parameters, for matter and dark energy, upon the BD parameter, has been shown graphically.

KEYWORDS: Brans-Dicke Theory, Scalar Field, Cosmic Expansion, Gravitational Constant, Density Parameters for Dark Energy and Matter, Cosmology.

Brans-Dicke (BD) theory of gravitation is known to be characterized by a dimensionless coupling constant (ω), known as BD parameter and a scalar field φ which is the reciprocal of gravitational constant (Brans and Dicke, 1961). The dimensionless coupling parameter, which was initially regarded as a constant in BD theory, is now regarded as a function of time in the generalized Brans-Dicke theory (Nordtvedt, 1970). This time dependence may also be expressed by assuming ω to be explicitly dependent upon the scalar field φ (Banerjee and Ganguly, 2009). There are several important reasons for which generalised BD theory has gained so much of important in explaining and analyzing cosmological phenomena. In Kaluza-Klein theories, super gravity theory and in all the known effective string actions, this theory has a natural appearance. It is regarded as the most natural extension of the General Theory of Relativity (GTR), which may justify its applicability in fundamental theories (Will, 1993). In the generalized version of Brans-Dicke theory, which is also known as graviton-dilaton theory, ω has been shown to be a function of the scalar field φ (dilaton). Thus, there can be several models depending upon the functional form of the BD parameter. This theory generates the results obtained from GTR, for a constant scalar field and an infinite ω (Alimi and Serna, 1996). Using a constant value of ω, BD theory was found to account for almost all important cosmological observations regarding the evolution of the universe. BD theory is capable of explaining the features like inflation, early and late time behaviour of the universe, cosmic acceleration and structure formation (Bertolami and Martins, 2000), quintessence and the coincidence problem (Benerjee and Pavon, 2001), self -interacting potential and cosmic acceleration (Sen and Seshadri, 2003). For a small negative value of ω it correctly explains cosmic acceleration, structure formation and the coincidence problem and, for a large value of ω, BD theory gives the correct amount of inflation and early and late time behaviour. The time dependence of ω in Brans-Dicke theory has many interesting features. From string and Kaluza-Klein theories it gets a strong corroboration, and in several studies, the dynamics of the universe has been analyzed within its framework. Through these attempts, the phenomena like the evolution of the universe, its accelerated expansion and quintessence, have been explained in a qualitative way without deriving any explicit time dependence of the BD parameter (Benerjee and Pavon, 2001). Some recent studies have shown that several models can be formulated based on the concept of a time-varying ω (Alimi and Serna, 1996). Therefore it would be quite natural for the researchers to find an analytical expression of ω as a function of time, using the field equations of the Brans-Dicke theory. Some attempts were made to derive the time dependence of the BD parameter using very simple empirical forms of scale factor and the scalar field (Sahoo and Singh , 2002; Jamil and Momeni, 2011). Contrary to all experimental observations, the chosen scale factors in these cases produced time independent deceleration parameters.

The present theoretical study has been carried out to determine the time dependence of the BD parameter, in different cosmological era which are characterized by the values of the EOS parameter of the cosmic fluid. An expression of the ω, in terms of the scalar field, has been assumed for this purpose and its constant parameter has been determined from the field equations. An empirical expression of the scalar field, in terms of the scale factor, has been used. A time-dependent empirical expression of the scale factor has been so chosen that it ensures a signature flip of the deceleration parameter with time. We have also used the experimental results regarding the time variation of the gravitational constant (G), for the present theoretical

[1]**Corresponding author**

formulation. The scale factor (a), Hubble parameter (H), deceleration parameter (q) and matter density (ρ), gravitational constant, density parameters for matter and dark energy (Ω_m, Ω_d) have been expressed as functions of the BD parameter (ω) and its time derivative. These expressions show mathematically, the role played by the dimensionless parameter (ω) in cosmic expansion. Time dependence of the BD parameter and its role in controlling the density parameters have been shown graphically.

THEORETICAL MODEL

The gravitational field equations, based on the generalized Brans-Dicke theory, for a universe filled with a perfect fluid and described by Friedmann-Robertson-Walker space-time with scale factor $a(t)$ and spatial curvature k, are given by,

$$3\frac{\dot{a}^2+k}{a^2}+3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}-\frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2}=\frac{\rho}{\varphi} \quad (01)$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2+k}{a^2}+\frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2}+2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}+\frac{\ddot{\varphi}}{\varphi}=-\frac{P}{\varphi} \quad (02)$$

The wave equation for the scalar field (φ), in Brans-Dicke theory of gravity, where ω is a time dependent parameter, is expressed as,

$$\ddot{\varphi}+3\frac{\dot{a}\dot{\varphi}}{a}=\frac{\rho-3P}{2\omega+3}-\frac{\dot{\omega}\dot{\varphi}}{2\omega+3} \quad (03)$$

The energy conservation for the cosmic fluid is given by (Sahoo and Singh , 2002; Jamil and Momeni, 2011),

$$\dot{\rho}+3\frac{\dot{a}}{a}(\rho+P)=0 \quad (04)$$

The equation of state of the fluid is expressed as,

$$P=\gamma\rho \quad (05)$$

Here, the values of γ are -1 (vacuum energy dominated era), 0 (matter dominated era), $1/3$ (radiation dominated era), 1 (massless scalar field dominated era).

The solution of equation (4), using equation (5), is obtained as,

$$\rho=\rho_0 a^{-3(1+\gamma)} \quad (06)$$

For the present study, based on the equations (1), (2) and (3), we assume the following empirical relations regarding scale factor (a), scalar field (φ) and the BD parameter (ω).

$$a=a_0\ (t/t_0)^{\varepsilon}\ Exp[\mu(t-t_0)] \quad (07)$$

$$\varphi=\varphi_0(a/a_0)^n \quad (08)$$

$$\omega=\omega_0\left(\frac{\varphi}{\varphi_0}\right)^m \quad (09)$$

The scale factor (in eq. 7) has been chosen to ensure the change of sign of the deceleration parameter with time, in accordance with recent observations regarding the phenomenon of late time acceleration of the expansion process of the universe (Banerjee and Ganguly, 2009; Roy, 2016]. Here $\varepsilon, \mu>0$ to ensure increase of scale factor with time. The Hubble parameter (H) and the deceleration parameter (q), calculated from this scale factor, are written below.

$$H=\mu+\frac{\varepsilon}{t} \quad (10)$$

$$q=-1+\frac{\varepsilon}{(\varepsilon+\mu t)^2} \quad (11)$$

For $0<\varepsilon<1$, we get $q>0$ at $t=0$ and, for $t\to\infty$, we have $q\to -1$.

Using the boundary conditions that, at $t=t_0$, $H=H_0$ and $q=q_0$ we have obtained the following values of the constants in the scale factor.

$$\varepsilon=(H_0t_0)^2\ (q_0+1) \quad (12)$$

$$\mu=H_0-{H_0}^2t_0(q_0+1) \quad (13)$$

The empirical form of the scalar field (in eq. 8) has been chosen on the basis of some previous studies on Brans-Dicke theory (Banerjee and Ganguly, 2009). The value of n, in terms of relevant cosmological parameters, has been determined from the field equations.

The empirical expression of the BD parameter (in eq. 9) has been chosen in conformity with the fact that, in the generalized Brans-Dicke theory, ω is regarded as a function of the scalar field (φ) (Banerjee and Ganguly, 2009). The values of ω_0 and m have been determined, in terms of relevant cosmological parameters, from the field equations.

Considering ω as a function of φ, equation (3) can be written as (Banerjee and Ganguly, 2009),

$$\ddot{\varphi}+3\frac{\dot{a}\dot{\varphi}}{a}=\frac{\rho-3P}{2\omega+3}-\frac{\dot{\varphi}^2}{2\omega+3}\frac{d\omega}{d\varphi} \quad (14)$$

Subtracting equation (1) from (2) and applying equation (8) we get,

$$\omega=\frac{1}{n^2}\left[2+2q+2n-n^2+nq-\frac{\rho+P}{\varphi H^2}\right] \quad (15)$$

Taking $P=0$ (for the present matter dominated era) and writing all parameter values for the present time, $t=t_0$, one obtains the following expression of ω_0 from equation (15).

$$\omega_0 = \frac{1}{n^2}\left[2 + 2q_0 + 2n - n^2 + nq_0 - \frac{\rho_0}{\varphi_0 {H_0}^2}\right] \qquad (16)$$

Combining the equations (5), (8) and (9) with equation (14), one obtains,

$$\omega = \frac{\rho(1-3\gamma) - 3\varphi n H^2(2+n-q)}{m\varphi n^2 H^2 + 2\varphi n H^2(2+n-q)} \qquad (17)$$

Using equation (17), one obtains the following expression of m by taking $\gamma = 0$ (for the present matter dominated era) and writing all parameter values for the present time, $t = t_0$.

$$m = \frac{1}{n^2\omega_0}\left[\frac{\rho_0}{{H_0}^2\,\varphi_0} - (2\omega_0 + 3)(n^2 + 2n - nq_0)\right] \quad (18)$$

The value of ω_0, in the above expression of m, is given by equation (16). This value of m (in eqn. 18) is required for the expression of ω (eqn. 17).

Equation (17) shows the time dependence of ω through the time dependence of the parameters ρ, φ, H, q whose time variation can be easily obtained from the equations (6), (7), (8), (10) and (11). Equation (17) also shows the dependence of ω upon the *equation of state* parameter γ whose values characterize the different cosmological era of the expanding universe. One needs to determine the value of the parameter n, upon which ω, ω_0 and m are explicitly dependent.

Eliminating ω from the equations (1) and (2), taking $k = 0$ and $P = \gamma\rho$ one obtains,

$$2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} + 5\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} + \frac{\ddot{\varphi}}{\varphi} = \frac{\rho}{\varphi}(1-\gamma) \qquad (19)$$

Using equations (8) and (19) for $\gamma = 0$ and taking all parameter values at $t = t_0$ one gets,

$$n^2 + (4 - q_0)n + \left(4 - 2q_0 - \frac{\rho_0}{\varphi_0 {H_0}^2}\right) = 0 \qquad (20)$$

Equation (20) is quadratic in n. Its two roots are given by,

$$n_{\pm} = \frac{1}{2}\left[q_0 - 4 \pm \left({q_0}^2 + \frac{4\rho_0}{\varphi_0 {H_0}^2}\right)^{1/2}\right] \qquad (21)$$

The values of different cosmological parameters used in this article are:

$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18}\, sec^{-1}$, $\quad q_0 = -0.55$, $\rho_0 = 2.83 \times 10^{-27} Kg\, m^{-3}$

$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kg^2\, m^{-2} N^{-1}$, $t_0 = 4.36 \times 10^{17}\, s$

Using these values in equation (21) we get,

$$n_+ = -1.94 \text{ and } n_- = -2.61 \qquad (22)$$

Since $\varphi = \varphi_0 a^n$ was chosen empirically (and not obtained as a solution of the field equations) the parameter n can also take values other than the two values shown by equation (22). The important fact about these two values is that both of them are negative, indicating a decrease of φ and an increase of $G(\equiv \frac{1}{\varphi})$ with time.

According to a study by Banerjee and Pavon (Benerjee and Pavon, 2001), $-3/2 < \omega_0 < 0$.

Using equation (16), the ranges of n values, satisfying this requirement, are found to be,

$$n < -2.061, \quad -0.838 < n < -0.455 \text{ and } n > 1.905 \qquad (23)$$

The sign of n determines whether φ increases or decreases with time. The larger the value of $|n|$, greater would be its rate of change with time. Using equation (8), the fractional rate of change of the gravitational constant is given by,

$$\left(\frac{\dot{G}}{G}\right)_{t=t_0} = \left[\frac{1}{1/\varphi}\frac{d}{dt}\left(\frac{1}{\varphi}\right)\right]_{t=t_0} = -\left(\frac{\dot{\varphi}}{\varphi}\right)_{t=t_0} = -nH_0$$

$$\text{or, } n = -\frac{1}{H_0}\left(\frac{\dot{G}}{G}\right)_{t=t_0} \qquad (24)$$

Using equation (24), the values of n can be more reliably determined from the experimental findings of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$. Its sign is found to be both positive and negative experimentally (Ray et al., 2007).

Using the equations (6), (8) and (9) one may write the following expressions of different cosmological quantities in terms of ω and its derivatives.

$$a = \left(\frac{\omega}{\omega_0}\right)^{\frac{1}{mn}} \qquad (25)$$

$$H = \frac{\dot{a}}{a} = \frac{1}{mn}\frac{\dot{\omega}}{\omega} \qquad (26)$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + mn\left(1 - \frac{\ddot{\omega}\omega}{\dot{\omega}^2}\right) \qquad (27)$$

$$\rho = \rho_0\left[a_0(\omega/\omega_0)^{\frac{1}{mn}}\right]^{-3(1+\gamma)} \qquad (28)$$

$$\frac{\dot{G}}{G} = -\frac{1}{m}\frac{\dot{\omega}}{\omega} \qquad (29)$$

In the above expressions (25-29), the parameters ω_0 and m are functions of the parameter n (eqns. 16, 18), which controls the change of scalar field with time.

Combining the equations (7), (8) and (9), one gets,

$$\omega = \omega_0 \ (t/t_0)^{\varepsilon mn} \ Exp[\mu mn(t - t_0)] \qquad (30)$$

Equation (30) shows the time dependence of ω which may be substituted into the equations (25-29). Here, ε, μ, ω_0 and m are given by equations (12), (13), (16) and (18) respectively.

It is quite evident from equation (30) that, if n and m have the same sign, ω increases with time if ω_0 is positive, since ε and μ are both positive quantities. When they have opposite signs, ω decreases with time if ω_0 is positive. For these two cases, the reverse happens if ω_0 is negative.

Using equation (28), the density parameter for all matter (dark + baryonic) can be written as,

$$\Omega_m = \frac{\rho}{\rho_c} = \frac{\rho_0}{\rho_c}\left[a_0(\omega/\omega_0)^{\frac{1}{mn}}\right]^{-3(1+\gamma)} \qquad (31)$$

Here, ρ_c is the critical density of matter-energy of the universe. $\rho_c \cong 10^{-26} Kg\ m^{-3}$

The density parameter for dark energy is given by,

$$\Omega_d = 1 - \Omega_m = 1 - \frac{\rho_0}{\rho_c}\left[a_0(\omega/\omega_0)^{\frac{1}{mn}}\right]^{-3(1+\gamma)} \qquad (32)$$

Equations (31) and (32) show the dependence of density parameters, of matter and dark energy respectively, upon the Brans-Dicke parameter.

RESULTS

We have graphically shown the time variation of ω and its effect on density parameters, using the expressions derived here. Figure 1 is based on equation (17), showing the time variation of ω for four values of the EOS parameter (γ), characterizing four different cosmological era. Here we have taken $n = -2.61$ which is consistent with the values of n in the equations (22) and (23). These four graphs would be valid for four different ranges of time, corresponding to different cosmological era of the expanding universe. There are fluctuations, above and below the value of $\omega = -3/2$ over short periods of time. Figure 2 shows the time variation of ω for four negative values of n, for the present matter dominated universe which is a pressureless dust with $\gamma = 0$. The present and future values of BD parameter are greater than $-3/2$, as predicted in earlier studies (Benerjee and Pavon, 2001). Figure 3 shows the time dependence of ω for the present matter dominated universe for four positive values of n. The present and future values of BD parameter are greater than $-3/2$, as predicted in earlier studies (Benerjee and Pavon, 2001). For each value of n, ω is found to decrease monotonically with time. Figure 4 shows the variation of the density parameters, for matter and dark energy, as a function of ω/ω_0 for the matter dominated era ($\gamma = 0$). As the ratio ω/ω_0 increases, these curves show an increase and decrease of the density parameters, for dark energy and matter respectively.

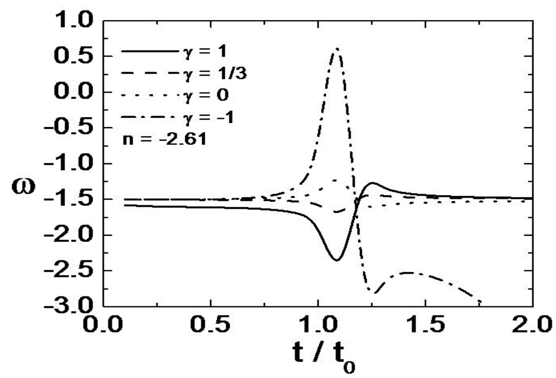

Figure 1: Time variation of ω for four values of EOS parameter (γ), for four different cosmological era.

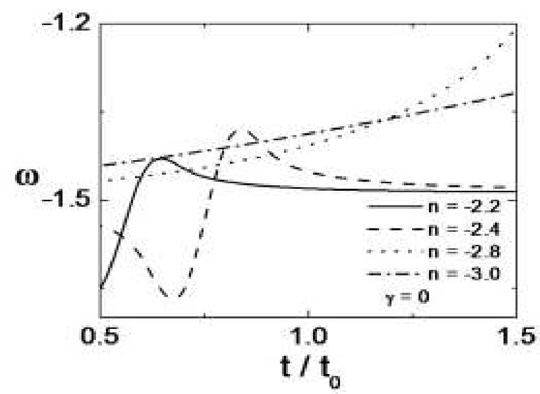

Figure 2: Time variation of ω for the present matter dominated universe for four negative values of n.

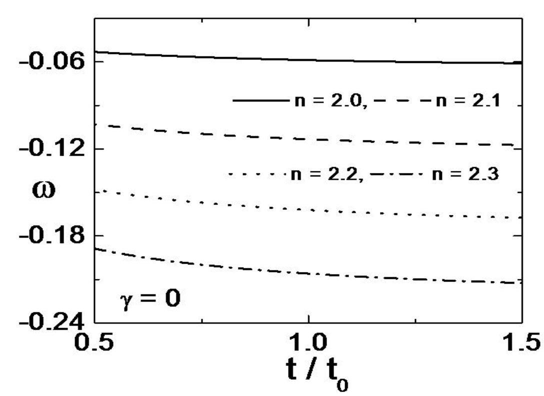

Figure 3: Time variation of ω for the present matter dominated universe for four positive values of n.

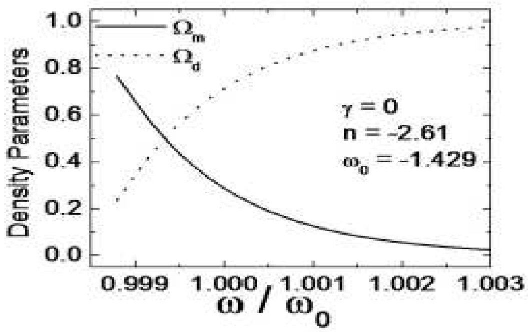

Figure 4: Variation of density parameters for matter and dark energy as a function of ω for the matter dominated era.

In Table-1, we have listed the values of n, m and ω_0 for different values of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$. Here, the range of variation of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ has been mostly chosen on the basis of experimental findings in this regard (Ray et al., 2007; Roy, 2016). The values of n has been determined by equation (24) and the values of ω_0 and m have been calculated from the equations (16) and (18) respectively. It is evident from these values that very large values of $|\omega|$ causes an extremely slow rate of change of the gravitational constant. It was predicted in some earlier studies that ω_0 must have a small negative value (Benerjee and Pavon, 2001; Sahoo and Singh, 2002). It is found from the data in Table-1 that $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ can be both positive and negative, for ω_0 to have a small negative value greater than $-3/2$. Although the results of equation (22) are in favour of an increase of gravitational constant with time, experimentally one finds both positive and negative values of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ (Ray et al., 2007). There were theoretical studies where n was shown to have only negative values, predicting an increase of gravitational constant with time (Banerjee and Ganguly, 2009). There were studies where the scalar field ($\varphi \equiv 1/G$) has been shown to decrease with time (Sahoo and Singh, 2002).

Table 1: Values of n, m and ω_0 for different values of $\left(\frac{\dot{G}}{G}\right)_{t=t_0}$

$\left(\frac{\dot{G}}{G}\right)_{t=t_0}$ (Yr^{-1})	n	m	ω_0
-1E-9	13.60935278	1.63290E+00	-8.88784E-01
-7.5E-10	10.20701459	1.91300E+00	-8.49636E-01
-5.0E-10	6.804676391	2.61804E+00	-7.68226E-01
-2.5E-10	3.402338196	7.01121E+00	-4.99082E-01
-1E-10	1.360935278	-2.18999E+01	5.32569E-01
-7.5E-11	1.020701459	-1.53588E+01	1.25104E+00
-5.0E-11	0.680467639	-1.42182E+01	2.99939E+00
-2.5E-11	0.34023382	-1.93355E+01	1.07358E+01
-1E-11	0.136093528	-4.04913E+01	5.63670E+01
-7.5E-12	0.102070146	-5.27408E+01	9.62505E+01
-5.0E-12	0.068046764	-7.74693E+01	2.07159E+02
-2.5E-12	0.034023382	-1.52148E+02	7.89019E+02
-1E-12	0.013609353	-3.76821E+02	4.77680E+03
-7.5E-13	0.010207015	-5.01706E+02	8.44551E+03
-5.0E-13	0.006804676	-7.51504E+02	1.88971E+04
-2.5E-13	0.003402338	-1.50096E+03	7.51653E+04
-1E-13	0.001360935	-3.74939E+03	4.68190E+05
1E-13	-0.001360935	3.74548E+03	4.66059E+05
2.5E-13	-0.003402338	1.49704E+03	7.43129E+04
5.0E-13	-0.006804676	7.47586E+02	1.84709E+04
7.5E-13	-0.010207015	4.97789E+02	8.16139E+03
1E-12	-0.013609353	3.72906E+02	4.56371E+03
2.5E-12	-0.034023382	1.48255E+02	7.03783E+02
5.0E-12	-0.068046764	7.36592E+01	1.64541E+02
7.5E-12	-0.102070146	4.90755E+01	6.78386E+01
1E-11	-0.136093528	3.70457E+01	3.50581E+01
2.5E-11	-0.34023382	2.19333E+01	2.21223E+00
5.0E-11	-0.680467639	-1.09384E+00	-1.26239E+00
7.5E-11	-1.020701459	1.48861E-01	-1.59015E+00
1E-10	-1.360935278	9.57293E-02	-1.59832E+00

2.5E-10	-3.402338196	5.28525E-02	-1.35144E+00
5.0E-10	-6.804676391	3.19323E-01	-1.19440E+00
7.5E-10	-10.20701459	4.84372E-01	-1.13375E+00
1E-9	-13.60935278	5.87065E-01	-1.10187E+00

CONCLUSION

The dependence of the Brans-Dicke parameter (ω) upon time and also upon the scalar field (φ) has been explored in this article, by a very simple mathematical model based on an empirical power-law dependence of the BD parameter upon the scalar field and also upon an empirical power-law dependence of the scalar field upon the scale factor. Thus we have been able to formulate an expression of the scale factor (eqn. 25) in terms of the BD parameter. The constants involved in this expression (m, n) have been determined from the field equations. Using this expression of the scale factor we have derived expressions, in terms of ω and its time derivative, of different cosmological parameters that play a very important role in understanding and characterizing the accelerated expansion of the universe. Each of these expressions has an explicit dependence on the BD parameter, enabling us to explore the role of the BD parameter in governing the cosmic expansion. Explicit dependence of the BD parameter upon time has been determined and its relation with the EOS parameter (γ) has been shown. The signs of the parameters n, m and ω_0 determine whether the BD parameter increases or decreases with time. Actually, both ω_0 and m are functions of n (eqns 16, 18), which determines the time dependence of the gravitational constant (eqn. 24). According to a study by Banerjee and Pavon, the value of ω_0 is negative and it must be greater than $-3/2$ (Benerjee and Pavon, 2001). We have determined the range of n values for which this requirement would be satisfied. The values of n can be estimated from the experimental findings regarding the time dependence of the gravitational constant (Ray et al., 2007). For a wide range of values of this quantity, we have determined the values of the parameters n, m and ω_0 and listed them in Table-1. This table shows that for extremely low values of $\left|\left(\frac{\dot{G}}{G}\right)_{t=t_0}\right|$, one gets very large values of $|\omega_0|$. It indicates the fact that large values of ω causes a smaller rate of change of the gravitational constant ($G = 1/\varphi$). Time dependence of BD parameter has been graphically shown for four different values of γ, characterizing different cosmological era. Time variation of ω has been graphically depicted for both positive and negative values of n. Functional forms of density parameters, for matter and dark energy, in terms of ω, have been derived (eqns. 31, 32). Their dependence on ω have been shown graphically. Equal importance have been given to the differential equations (1) and (2), in formulating the expression of ω_0 (eqn. 16). Instead of doing this, one may calculate the value of ω_0 separately from the equations (1) and (2) and determine their weighted average with unequal weights assigned to these two values. A new parameter, for this purpose, can be introduced to represent their relative importance, enabling us to determine the value of ω_0 correctly, making it more consistent with more advanced studies in this regard. Based upon the same set of empirical relations (eqns. 8 and 9) for the scalar field and the BD parameter, we have determined two expressions of $\omega(t)$, represented by the equations (17) and (30), showing clearly its dependence upon time and the equation of state parameter (γ). Using one of them, one may determine the time evolution of ω for different values of the equation of state parameter (γ), corresponding to different cosmological era of the expanding universe. A shortcoming of the present study is that, it is based upon empirical forms of the scale factor $a(t)$ and the scalar field (φ), which are not the solutions of the Brans-Dicke field equations. One may go for a better way of theoretical exploration by assuming a suitable ansatz regarding one of them and obtaining the other by solving the field equations. This would be our method of study for a future project in this regard.

ACKNOWLEDGEMENT

The author of this article is thankful to his students and colleagues for whatever he has gained from the fruitful discussions he had with them and also for their encouragement.

REFERENCES

Alimi J.M. and Serna A., 1996. Scalar-tensor Cosmological Models. Phys. Rev. D, **53**:3074-3087.

Banerjee N. and Ganguly K., 2009. Generalised Scalar-Tensor theory and the cosmic acceleration. Int. J. Mod. Phys. D, **18** (3): 445-451.

Benerjee N. and Pavon D., 2001. Cosmic acceleration without quintessence. Phys. Rev. D, **63**: 043504-043510.

Bertolami O. and Martins P.J., 2000. Nonminimal coupling and quintessence. Phys. Rev. D., **61**: 064007-064012.

Brans C. and Dicke R.H., 1961. Mach's Principle and a Relativistic Theory of Gravitation. Phys. Rev., **124**: 925-935.

Jamil M. and Momeni D., 2011. Evolution of the Brans-Dicke Parameter in Generalized Chameleon Cosmology. Chin. Phys. Lett., **28** (9):099801, 1-4.

Nordtvedt K. Jr., 1970. Post-Newtonian metric for a general class of scalar-Tensor gravitational theories and observational quintessence. Astrophys. J., **161**: 1059-1067.

Ray S., Mukhopadhyay U. and Dutta Choudhury S.B., 2007. Dark energy models with time-dependent gravitational constant. Int. J. Mod. Phys. D, **16**: 1791 - 1802.

Roy S., 2016. Time Evolution of Various Cosmological Parameters and Their Inter-Dependence in the Framework of Brans-Dicke Theory. IOSR Journal of Mathematics, **12** (3): Ver. VII, 27-35.

Sahoo B.K. and Singh L.P., 2002. Time Dependence of Brans-Dicke Parameter ω for an Expanding Universe. Mod. Phys. Lett. A, **17**(36):2409-2415.

Sen S. and Seshadri T.R., 2003. Self interacting Brans Dicke Cosmology and Quintessence. Int.J.Mod.Phys. D, **12:** 445-460.

Will C.M., 1993. Theory and Experiment in Gravitational Physics, Cambridge University Press, Chapter-**5**: 123-126.