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A ROBUST STUDY OF SPECTRAL EFFICIENCY FOR SPACE CONSTRAINED MASSIVE MIMO SYSTEM

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ABSTRACT

Spectral efficiency (SE) of massive multiple-input multiple output (MIMO) systems with a very large number of antennas at the base station (BS) considering for space constraint antennas are investigated in this paper. In literature, fixed spacing inter-element were considered, the number of antennas increases with a fixed total distance in a practical topology an inversely proportional to inter antennas spacing decreases. In this paper, we attain exact and approximate values of lower and upper limits for the SE of massive MIMO system with linear receiver such as MRC, ZF and MMSE. Simulation results show that the SE increasing as the number of antennas increases at BS using ZF and MMSE where as MRC is sub optimal for space limit in massive MIMO. Numerical results conclude that the effect of the large number of antennas, the number of users and the total space of the antenna array on the sum SE performance.

KEYWORDS: Spectral Efficiency, Massive MIMO, MRC, MMSE, ZF

Massive MIMO is becoming mature for wireless communication and has been incorporated into wireless access networks standards like LTE, LTE-Advanced and fifth generation (5G) systems [Larsson, et.al., 2014 & Zheng et.al., 2015], where several mobile/users simultaneously communication with BS with equipped with very large number antennas(e.g., hundreds or thousands) that are operated fully coherently and adaptively. Additional antennas help us focusing the transmission and reception of signal energy into smaller regions of space.

An interpretative issue involve to practical massive MIMO systems is the deployment of with limited spacing between a large numbers of antennas. In generally channels are uncorrelated if the spacing of inter- antenna is more than half wavelength. Due to space constrained antennas will arrange less than half wavelength in practical massive MIMO system more likely. Different Channel vectors for each UE will not be asymptotically orthogonal under these conditions. Therefore, increased spatial correlation between inter-elements limitation in massive MIMO system, this impact need to be analyzed and quantified rigorously.

Large amount literature works have investigated the performance of conventional MIMO system spatial correlation effect has been existed with relatively small number BS antennas. The exact and approximate achievable sum SE upper and lower limits of MIMO system with ZF and MMSE receivers over correlated Rayleigh and Rician fading channel has been studied in [Ngo et.al., 2013 & McKay et.al., 2010]. The approximated performance of massive MIMO with two linear precoding techniques derived over spatial correlation at the transmitter [Masouros et.al., 2013]. When the physical space is constrained for the favorable propagation in massive MIMO is violated, only maximum ratio-transmission (MRT) precoding was considered recently [Masouros and Matthaiou, 2015]. The achievable lower limit SE performance of uplink transmission with MRC at BS, in addition the effect of space is constrained on the performance of subspace estimation techniques were derived in [Ngo et.al., 2013 & Teeti et.al., 2015].

Therefore based on literature, there is no numerical and theoretical results on the SE of space constrained massive MIMO system (MMS) with MRC, ZF and MMSE receivers. Hence, we attain analytical work for the achievable SE of space constrained MMS with linear receivers.

In this paper, we defined the following contributions

- The achievable sum SE of MMS with MRC approximately defined firstly. Space constrained antennas will cause a saturation of the achievable sum SE as increases number of antennas for MMS with MRC receivers.
- Upper and lower bounds on the achievable SE of MMS with ZF are derived. We show that the achievable SE increases with number of antennas increases at BS antennas M along with number of UEs K increase the sum SE of ZF receivers when only $M \gg K$.
- Exact closed form for achievable SE of MMS with MMSE at BS derived finally and its performance is similar to ZF receiver. The sum SE of MMSE receiver

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also increases by arranged more antennas at BS with space limited MMS.

SYSTEM MODEL

The uplink (UL) of massive MIMO system (MMS), where the BS with equipped M antennas has shared simultaneously with K single antenna UEs, at the BS received signal vector $y \in \mathbb{C}^{M\times 1}$ is given as

$$y = \sqrt{p_u}\, Gx + n \tag{1}$$

Where the average power of each UE is p_u, transmitted symbol is x and n is denoted as AWGN with zero mean and unit variance $\sigma^2 = N_o B$. The channel matrix can be represented as $G = AHD^{1/2}$ where $H \in \mathbb{C}^{P\times K}$ is the propagation channel fading such as small scale fading, $D \in \mathbb{C}^{K\times K}$ denotes a diagonal matrix, ς_k represented as the large-scale fading of the k[th] UE (assumed as constant) and $A \in \mathbb{C}^{M\times K}$ is transmit steering matrix.

We assume that all UEs are the same set of directions with cardinality P for simplicity analysis. Let us consider uniform linear array [Ngo et.al., 2013 & Zi et.al., 2014] A can be written as

$$A = \left[a\left(\theta_1\right), a\left(\theta_2\right), \ldots\ldots\ldots, a\left(\theta_P\right) \right] \tag{2}$$

Where $a\left(\theta_i\right)$ normalized steering vector with length-M, for i = 1, 2................, P

$$a\left(\theta_i\right) = \frac{1}{\sqrt{P}} \left[1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_i}, \ldots\ldots, e^{-j\frac{2\pi d}{\lambda}(M-1)\sin\theta_i} \right]^T \tag{3}$$

Where d is the antenna spacing, λ denotes the carrier wavelength, and θ_i represents the direction of arrival (DOA). The normalized total antenna array space d_0 at the BS can be expressed as $d_o = \frac{dM}{\lambda}$, the factor $\frac{1}{\sqrt{P}}$ to normalize the power of steering vector. MMS is the simple linear precoding techniques because it's near optimal and implementation complexity is very low level [Zheng et.al., 2015]. Therefore the performance of space constrained MMS with linear receiver were considered here. The perfect CSI is available at the BS further we assumed [5]. $T \in \mathbb{C}^{M\times K}$ is the linear receiver matrix which used to separate the signal into K streams by

$$r = T^H y = \sqrt{p_u}\, T^H Gx + T^H n \tag{4}$$

Then, the detected signal k[th] elements of UE is given by

$$r_k = \sqrt{p_u}\, t_k^H g_k x_k + \sqrt{p_u} \sum_{l\neq k}^{K} t_k^H g_l x_l + t_k^H n \tag{5}$$

The achievable UL SE of the of the k[th] UE is given by [Ngo et.al., 2013]

$$R_k = E\left\{ \log_2 \left(1 + \frac{p_u \left| t_k^H g_k \right|^2}{p_u \sum_{l\neq k}^{K} \left| t_k^H g_l \right|^2 + \left\| t_k \right\|^2} \right) \right\} \tag{6}$$

The sum SE of uplink MMS can be defined as

$$R = \sum_{k=1}^{K} R_k \tag{7}$$

In the next sections, we analytical defined the achievable sum SE of space-constrained MMS with MRC, ZF, and MMSE respectively.

MRC Receiver

We assumed **T** = **G** for MRC receivers [Zhang et.al., 2016]. The uplink SE for the k[th] UE is given by

$$R^{MRC}_k = E\left\{ \log_2 \left(1 + \frac{p_u \left\| g_k \right\|^4}{p_u \sum_{l\neq k}^{K} \left| t_k^H g_l \right|^2 + \left\| g_k \right\|^2} \right) \right\} \tag{8}$$

Where

$$g_k = \sqrt{\varsigma_k}\, Ah_k \tag{9}$$

Next we present an approximate analysis for the achievable sum SE of MRC receivers.

Proposition 1: The approximated sum achievable SE for space constrained MMS with MRC is given by

$$R^{MRC} \approx \sum_{k=1}^{K} \log_2 \left(1 + \frac{p_u \left(M^2 + \sum_{i=1}^{P} \beta_i^2 \right) \varsigma_k}{p_u \sum_{l \neq k}^{K} \varsigma_l \sum_{i=1}^{P} \beta_i^2 + M \varsigma_k} \right) \quad (10)$$

Where β_i is the i[th] eigen value of the matrix $A^H A$.

ZF Receiver

We now define the ZF receivers, which is forced to eliminate inter user interference in MMS system. Let us consider concept of ZF is submitted in equation (1) obtain the ZF matrix is given as $T = G\left(G^H G\right)^{-1}$. Therefore the sum SE of MMS with ZF receiver can be expressed as

$$R^{ZF} = \sum_{k=1}^{K} E \left\{ \log_2 \left(1 + \frac{p_u}{\left[\left(G^H G \right)^{-1} \right]_{kk}} \right) \right\} \quad (11)$$

Next, we define the lower and upper bounds on the achievable sum SE of MMS with ZF receiver (11).

Lower Bound

Proposition 2: The achievable lower bound sum SE for space constrained MMS with ZF receivers is given by [Gradshteyn and Ryzhik, 2007 & Krishna et.al., 2015]

$$R^{ZF} \geq R_L^{ZF} = \sum_{k=1}^{K} \log_2 \left(1 + p_u \varsigma_k \xi(.) \right)$$

$$\xi(.) = \exp \left(\sum_{n \neq k}^{K} \varsigma_n \left(\psi(K) + \frac{\left| Y_{P-K+1} \right|}{\prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} \right) - \left(\psi(n) + \frac{\left| Y_{P-K+1} \right|}{\prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} \right) \right) \quad (12)$$

Where $\psi(.)$ is digamma function and Y_n representing P×P matrix whose entries are

$$\left[Y_n \right]_{p,q} = \begin{cases} \beta_p^{q-1}, & q \neq n \\ \beta_p^{q-1} \ln \beta_p, & q = n \end{cases} \quad (13)$$

Upper Bound

Proposition 3: The achievable upper bound sum SE for space constrained MMS with ZF receiver is given by

$$R_U^{ZF} \leq R_U^{ZF} = K \log_2 \left(\frac{\left| \Delta_2 \right|}{\prod_{i=1}^{K-1} \Gamma(K-i) \prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} + p_u \frac{\left| \Delta_1 \right|}{\prod_{i=1}^{K-1} \Gamma(K-i+1) \prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} \right)$$

$$- \frac{K}{\ln 2} \left(\sum_{n=1}^{K-1} \psi(n) + \frac{\sum_{n=P-K+2}^{P} \left| Y_n \right|}{\prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} \right) \quad (14)$$

Where $\Gamma(.)$ represented as the Gamma function.

MMSE Receiver

The receiver matrix T for MMSE receiver is given by [Jin et.al., 2010 & Shin et.al., 2006]

$$T^H = \left(G^H G + \frac{1}{p_u} I_K \right)^{-1} G^H = G^H \left(G G^H + \frac{1}{p_u} I_M \right)^{-1}$$

The achievable sum SE of MMS with MMSE receiver can be written as

$$R^{MMSE} = \sum_{k=1}^{K} E \left\{ \log_2 \left(\frac{1}{\left[\left(I_K + p_u G^H G \right)^{-1} \right]_{kk}} \right) \right\} \quad (15)$$

$$= KE \left\{ \log_2 \left(\left| I_K + p_u G^H G \right| \right) \right\}$$

$$- \sum_{k=1}^{K} E \left\{ \log_2 \left(\left| I_{K-1} + p_u G^H G \right| \right) \right\} \quad (16)$$

Here we derived equation (16) from equation (15) which is an important matrix property as

$$\left[\left(G^H G \right)^{-1} \right]_{kk} = \frac{\left| G_k^H G_k \right|}{\left| G^H G \right|} \quad (17)$$

Proposition 4: The exact achievable sum SE for the space constrained MMS with MMSE receiver is given by

$$R^{MMSE} = \frac{K \log_2 e}{\prod_{i<j}^{P} \left(\beta_j - \beta_i \right)} \sum_{l=1}^{P} \sum_{n=P-K+1}^{P} \beta_l^{n-1} e^{1/\beta_l p_u} \quad (18)$$

SIMULATION RESULTS

Let us considered that the all UEs are uniformly distributed at random in small hexagonal cell with a radius of 1000 meters, the smallest distance between the UE and BS is r_{min} = 100 meters. The path loss is represented as r_k^{-u}, where the distance between the UE and BS is r_k and path loss exponent can be defined as u=3.8 respectively. A random variable s_k with standard deviation is 8 d B is used for shadowing. Therefore large scale fading can be obtained by combining these factor, which has given by $\varsigma_k = s_k \left({r_k}/{r_{min}} \right)^{-u}$ further also assumed θ_i are uniformly distributed within the interval $\left[-{\pi}/{2}, {\pi}/{2} \right]$.

Figure 1 shows that the simulation and analytical approximation of achievable sum SE for space constrained MMS with MRC receiver. It is easily observed that the sum SE saturates with increases the BS antennas for different total antenna array space d_0. Therefore we conclude that MRC suffers substantial performance degradation when small antenna array space if spatial correlation is high. If the same number of BS antennas used, constant increases in the sum SE is obtained as total antennas array spacing is larger. Also observe that the gap between curve decreases as antenna array spacing is increases which imply that the effect of space constrained become less pronounced.

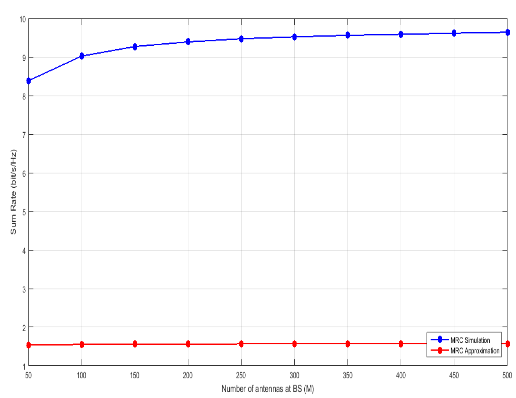

Figure 1: simulation and analytical approximation values of sum SE for space constrained MMS with MRC receiver (P = 12 and K = 6)

In Figure 2 shows that simulated achievable upper and lower bounds sum SE against the number of BS antennas and total array space. Clearly obtained all lower bounds can predict the exact sum SE for space constrained MMS with MMSE, which validate their tightness. Other hand the upper bounds are relatively looser due to large variance of the random variables. Hence we conclude that by adding more antennas at BS significantly improve the sum SE of MMS by reducing thermal noise.

Figure 3 shows that the simulation and analytical approximation of achievable sum SE for space constrained MMS with ZF, MMSE and MRC receivers. Clearly obtained sum SE for space constrained massive MIMO system (MMS), SE increases as number of BS antennas increasing with ZF and MMSE receivers, sum SE of Massive MIMO with ZF and MMSE receiver almost same expect that MRC receiver as compared with analytical approximations.

Figure 4 shows that the simulation and analytical average SE versus number of receiving antennas at BS of space constrained MMS systems, which compare with first average, second, third and final average SE of Massive MIMO systems.

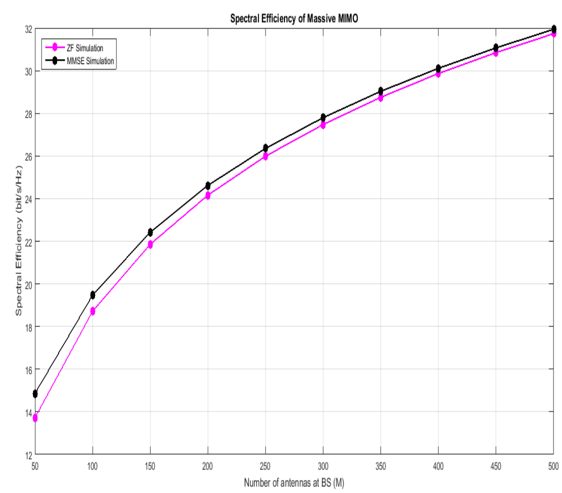

Figure 2: simulation and analytical approximation values of sum SE for space constrained MMS with MMSE receivers (P = 12 and d_0 = 4).

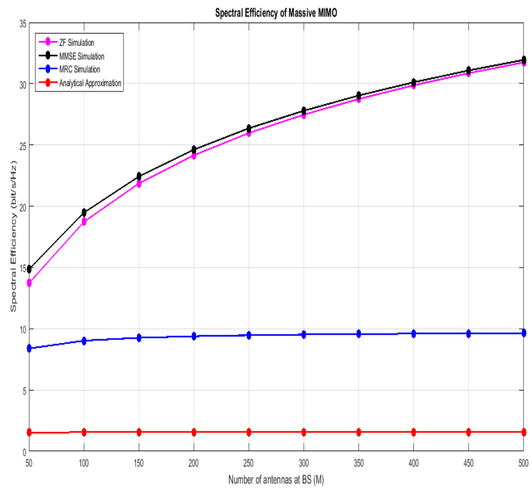

Figure 3: simulation and analytical approximation values of sum SE for space constrained MMS with ZF, MMSE and MRC receiver comparisons (P = 12 and K = 6)

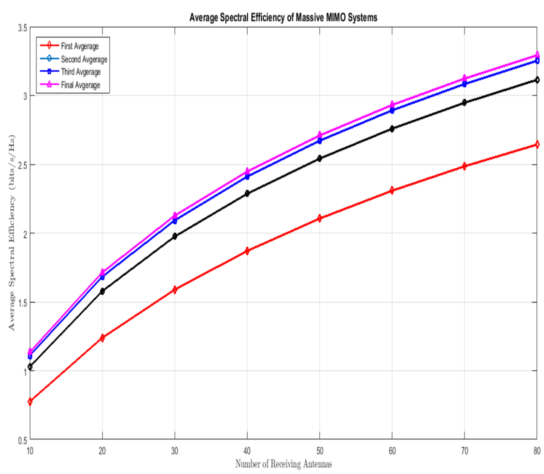

Figure 4: Average Spectral Efficiency versus number of receiving antennas

CONCLUSION

In this paper, we investigated the performance of space constrained massive MIMO system (MMS) with linear receivers, where total antennas array spacing at BS has limited. As the achievable sum SE increase with increasing number of BS antennas along with increasing spatial correlation. Though analytical and simulation results confirmed that saturation of the achievable lower and upper sum SE, which increases for a larger number of UES as long as M ≫ K. Moreover, lower bounds are tighter than the upper bounds for massive MIMO with MMSE receivers, exact express for the sum SE is derived and validated by simulation results. This is due to that SINRs of ZF and MMSR receivers increase with number of BS antennas while MRC receivers can only well suited at low SINRs.

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