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INTEGRARAL FORMULAE ON $\overline{H}$ - FUNCTION INVOLVING MUTLIVARIABLE POLYNONIALS

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ABSTRACT

The object of this paper is to study and develop integral formulae involving $\overline{H}$ - function of one variable and multivariable polynomials in the literature of special functions. The results are obtain in the compact form containing the multivariable polynomials. Some special cases have also been discussed.

KEYWORDS: $\overline{H}$ Function and Multivariable Polynomials

Recently, for modeling of relevant systems in various fields of sciences and engineering, such as electromagnetic, fluid mechanics, signals processing, stochastic dynamical system, plasma physics, earth sciences, astrophysics nonlinear biological system, relaxation and diffusion processes in complex systems , propagation of seismic waves , anomalous diffusion and turbulence, etc. see, Glöckle and Nonnenmacher (1993), Mainardi et al. (2001), Metzler and Klafter (2000) and others. The $\overline{H}$ - function of one variable defined by Buschman and Srivastva(1990) and we will represent here the following manner:

$$(1.1)\qquad \overline{H}^{M,N}_{P,Q}[z] = \overline{H}^{M,N}_{P,Q}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M};(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\overline{\phi}(\xi)Z^{\xi}\,d\xi\ ,$$

where $i=\sqrt{(-1)}, z(\neq 0)$ is a complex variable and in (1.2) $z^{\xi} = \exp\left[\xi\left\{\log|z|+i\ \arg z\right\}\right]$.In which log |z| represent the natural logarithm of |z| and arg |z| is not necessarily the principle value. Also,

$$(1.3)\qquad \overline{\phi}(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)},$$

here, and throughout the paper $a_j,(j=1,\ldots,p)$ and $b_j,(j=1,\ldots,Q)$ are complex parameter $\alpha_j\geq 0,(j=1,\ldots,P),\beta_j\geq 0,(j=1,\ldots,Q)$ and exponents $A_j,(j=1,\ldots,N)$ and $B_j,(j=N+1,\ldots,Q)$ are non- negative integer values . Integral is convergent, where

$$(1.4)\qquad \Omega=\sum_{j=1}^{M}\left|\beta_j\right|-\sum_{j=n+1}^{N}\left|A_j\alpha_j\right|-\sum_{j=M+1}^{Q}\left|\beta_jB_j\right|-\sum_{j=N+1}^{P}\left|\alpha_j\right|\geq 0\ .$$

The general class of multivariable polynomials is defined by Srivastava and Garg (1987) .

$$(1.5)\qquad S_{\mathrm{L}}^{h_1,\ldots,h_r}\left[x_1,\ldots,x_r\right]=\sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\leq \mathrm{L}}(-\mathrm{L})_{h_1k_1+\ldots+h_rk_r}A(\mathrm{L};k_1,\ldots,k_r)\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_r^{k_r}}{k_r!}\ ,$$

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Where h_1 ...hr are positive integers and the co-efficient A(L; $k_{1,}$.....,k_r), (L; $h_1 \in$ N; i = 1,....,r) are arbitrary constant, real or complex.

Evidently the case $r=1$ of the polynomials (1.5). Would correspond the polynomials given by Srivastava (1972).

(1.6) $$S_{\mathrm{L}}^{h}[x]=\sum_{k=0}^{[\mathrm{L}/h]}\frac{(-\mathrm{L})_{hk}}{k!}A_{\mathrm{L},k}\,x^{k}\left\{\mathrm{L}\in N=(0,1,2,\ldots)\right\},$$

where h is arbitrary positive integers and the co-efficient $A_{\mathrm{L},k}\,(\mathrm{L},k\geq 0)$ are arbitrary constant, real or complex.

MAIN RESULTS

Let x stands for $\left(ax+\dfrac{b}{x}\right)^{2}+c\cdot$

First Integral Formula

(2.1) $$\int_0^{\infty} x^{\rho-1}S_L^{h_1,\ldots,h_r}\left[c_1x^{\delta_1},\ldots,c_rx^{\delta_r}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^{\eta}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}\ (4ab+c)^{\rho-\frac{1}{2}}}{2a}\sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\leq L}(-L)_{h_1k_1+\ldots+h_rk_r}\,A(L;k_1,\ldots,k_r)\frac{c_1^{\,k_1}(4ab+c)^{k_1\delta_1}}{k_1!},\ldots,\frac{c_r^{\,k_r}(4ab+c)^{k_r\delta_r}}{k_r!}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right)\\\left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

This is convergent under the following conditions: (i) $a>0,b\geq 0;c+4ab>0$ and $\eta\geq 0,\delta\geq 0,\rho>0$; (ii) $\rho+\sigma\min_{1\leq j\leq m}\mathrm{Re}\left(\dfrac{b_j}{\beta_j}\right)+\eta\min_{1\leq j\leq m}\mathrm{Re}\left(\dfrac{b_j}{f_j}\right)<\dfrac{1}{2};$ (iii) $|\arg z|<\frac{1}{2}B\Omega$.

Second Integral Formula

(2.2) $$\int_0^{\infty}\frac{1}{x^2}x^{\rho-1}S_L^{h_1,\ldots,h_r}\left[c_1x^{\delta_1},\ldots,c_rx^{\delta_r}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^{\eta}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M,}(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}\ (4ab+c)^{\rho-\frac{1}{2}}}{2b}\sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\leq L}(-L)_{h_1k_1+\ldots+h_rk_r}\,A(L;k_1,\ldots,k_r)\frac{c_1^{\,k_1}(4ab+c)^{k_1\delta_1}}{k_1!},\ldots,\frac{c_r^{\,k_r}(4ab+c)^{k_r\delta_r}}{k_r!}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right)\\\left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M};(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

This is convergent under the following conditions: (i) $a\geq 0,b>0;c+4ab>0$ and $\rho>0,\delta\geq 0,\eta>0$;

(ii) $\rho+\sigma \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right)+\eta \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{f_j}\right)<\frac{1}{2}$; (iii) $|\arg z|<\frac{1}{2}B\Omega$.

Third Integra Formula

(2.3) $$\int_0^\infty \left(a+\frac{b}{x^2}\right)x^{\rho-1}S_L^{h_1,\dots,h_r}\left[c_1x^{\delta_1},\dots,c_rx^{\delta_r}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$=\sqrt{\pi}\ (4ab+c)^{\rho-\frac{1}{2}}\sum_{k_1,\dots,k_r=0}^{h_1k_1+\dots+h_rk_r\le L}(-L)_{h_1k_1+\dots+h_rk_r}\ A(L;k_1,\dots,k_r)\frac{c_1^{\ k_1}(4ab+c)^{k_1\delta_1}}{k_1!},\dots,\frac{c_r^{\ k_r}(4ab+c)^{k_r\delta_r}}{k_r!}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right)\\\left(\frac{1}{2}-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M};(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

This is convergent under the following conditions: (i) $a>0,b>0;c+4ab>0$ and $\rho>0,\delta\ge 0,\eta\ge 0$;

(ii) $\rho+\sigma \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right)+\eta \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{f_j}\right)<\frac{1}{2}$; (iii) $|\arg z|<\frac{1}{2}B\Omega$.

Proof

Proof of Integral first , second and third by definition of $\overline{H}$ -Function , using equation (1.5) and the formulae Quareshi et al. (2011) equations (3.1), (3.2)and (3.3) , Interchanging the order of integration , we can find the results (2.1) ,(2.2) and (2.3) respectively .

SPECIAL CASES

If we put $r=1$ in the general class of multivariable polynomials given by Srivastava and Garg (1987) reduces to the polynomials given by Srivastava (1972) the equations (2.1), (2.2) and (2.3) take the following form :

(3.1) $$\int_0^\infty x^{\rho-1}S_L^{h_1}\left[c_1x^{\delta_1}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2a}(4ab+c)^{\rho-\frac{1}{2}}\sum_{k_1=0}^{[L/h_1]}(-L)_{h_1k_1}\ A(L;k_1)\frac{c_1^{\ k_1}(4ab+c)^{k_1\delta_1}}{k_1!}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right)\\\left(\frac{1}{2}-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

(3.2) $$\int_0^\infty \frac{1}{x^2}x^{\rho-1}S_L^{h_1}\left[c_1x^{\delta_1}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$= \frac{\sqrt{\pi}}{2b}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{[L/h_1]} (-L)_{h_1 k_1} A(L;k_1) \frac{c_1^{k_1} (4ab+c)^{k_1\delta_1}}{k_1!}.$$

$$\overline{H}^{M+1,N}_{P+1,Q+1}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P}, \left(1-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right), (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

(3.3) $$\int_0^{\infty} \left(a+\frac{b}{x^2}\right) x^{\rho-1} S_L^{h_1}\left[c_1 x^{\delta_1}\right] \overline{H}^{M,N}_{P,Q}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \sqrt{\pi}\,(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{[L/h_1]} (-L)_{h_1 k_1} A(L;k_1) \frac{c_1^{k_1} (4ab+c)^{k_1\delta_1}}{k_1!}.$$

$$\overline{H}^{M+1,N}_{P+1,Q+1}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P}, \left(1-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right), (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

The condition of convergent of equations (3.1), (3.2) and (3.3) as equations (2.1), (2.2) and (2.3) under the condition (i), (ii) and (iii) respectively.

By applying the our results given in equation (3.1), (3.2) and (3.3) to the case of Hermite polynomials Gupta and Soni (2006) by setting $S_n^2[x] = x^{\frac{n}{2}} H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $L = n, h_1 = 2, A_{L,k} = (-1)^{k_1}$, we have following interesting results.

(3.4) $$\int_0^{\infty} x^{\rho-1} (c_1 x^{\delta_1})^{\frac{n}{2}} H_n\left[\frac{1}{2\sqrt{c_1 x_1^{\delta_1}}}\right] \overline{H}^{M,N}_{P,Q}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{2a}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{\left[\frac{n}{2}\right]} \frac{(-n)_{2k} (-1)^k c_1^{k_1} (4ab+c)^{k_1\delta_1}}{k_1!}.$$

$$\overline{H}^{M+1,N}_{P+1,Q+1}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P}, \left(1-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r} k_i\delta_i, \eta\right), (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

(3.5) $$\int_0^{\infty} \frac{1}{x^2} x^{\rho-1} (c_1 x^{\delta_1})^{\frac{n}{2}} H_n\left[\frac{1}{2\sqrt{c_1 x_1^{\delta_1}}}\right] \overline{H}^{M,N}_{P,Q}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{2b}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{\left[\frac{n}{2}\right]} \frac{(-n)_{2k}(-1)^k c_1^{k_1}}{k_1!(4ab+c)^{k_1\delta_1}}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

(3.6) $$\int_0^\infty \left(a+\frac{b}{x^2}\right) x^{\rho-1}(c_1x^{\delta_1})^{\frac{n}{2}} H_n\left[\frac{1}{2\sqrt{c_1x_1^{\delta_1}}}\right] \overline{H}_{P,Q}^{M,N}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \sqrt{\pi}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{\left[\frac{n}{2}\right]} \frac{(-n)_{2k}(-1)^k c_1^{k_1}(4ab+c)^{k_1\delta_1}}{k_1!}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

By applying the our results given in equations (3.1), (3.2) and (3.3) to the case of Lagurre polynomials Gupta and Soni (2006) by setting $S_n^1[x] \to L_n^{(\alpha)}[x]$ in which , $L=n, h_1=1, A_{L,k_1}=\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}}$ we have following interesting results

(3.7) $$\int_0^\infty x^{\rho-1} L_n^{(\alpha)}\left[c_1x^{\delta_1}\right] \overline{H}_{P,Q}^{M,N}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{2a}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{[n]} \frac{(-n)_{k_1}}{k_1!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}} c_1^{k_1}(4ab+c)^{k_1\delta_1}$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

(3.8) $$\int_0^\infty \frac{1}{x^2} x^{\eta-1} L_n^{(\alpha)}\left[c_1x^{\delta_1}\right] \overline{H}_{P,Q}^{M,N}\left[zx^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{2b}(4ab+c)^{\rho-\frac{1}{2}} \sum_{k_1=0}^{[n]} \frac{(-n)_{k_1}}{k_1!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}} c_1^{k_1}(4ab+c)^{k_1\delta_1}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^{\eta} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right) \\ \left(\frac{1}{2}-\rho-\sum_{i=1}^{r}k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right]$$

(3.9) $$\int_0^\infty \left(a+\frac{b}{x^2}\right)x^{\rho-1}L_n^{(\alpha)}\left[c_1x^{\delta_1}\right]\ \overline{H}_{P,Q}^{M,N}\left[zx^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]dx$$

$$=\sqrt{\pi}(4ab+c)^{\rho-\frac{1}{2}}\sum_{k_1=0}^{[n]}\frac{(-n)_{k_1}}{k_1!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}}c_1^{\ k_1}(4ab+c)^{k_1\delta_1}.$$

$$\overline{H}_{P+1,Q+1}^{M+1,N}\left[z(4ab+c)^\eta\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},\left(1-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right)\\\left(\frac{1}{2}-\rho-\sum_{i=1}^r k_i\delta_i,\eta\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

CONCLUSION

We have obtained the result namely Integral formulae (2.1), (2.2), (2.3) and equation of special cases satisfies all the condition which is mentioned the statement.

REFERENCES

Agrawa P. and Jain S., 2009. On unified finite integrals involving a multivariable polynomial and a generalized Mellin Barnes type contour integral having general argument. Nat. Acad. Letters, **32**: 8-9.

Agrawal P., 2011. On multiple integral relations involving eneralized Mellin-Barnes type of contour integral. Tamsui Oxford Journal of Information and Mathematical Sciences, Aletheia University, **27**(4): 449 – 462.

Buschman R.G. and Srivastava H.M., 1990. The H-function associated with a certain class of Feynman integrals. J. Phys. A. Math. Gen., **23**: 4707 – 4710.

Glöckle W.G. and Nonnenmacher T.F., 1993. Fox function representation of non-Debye relaxation processes. J Stat. Phys., **71**: 741-757.

Gupta K.C. and Soni R.C., 2006. On a basic integral formula involving the product of the H-function and Fox H-function. J. Raj. Acad. Sci., **4**(3): 157 – 164.

Gupta K.C., Jain R. and Agrawal R., 2007. On existence conditions for a generalizes Mellin-Barnes type integral. Acad Sci. Lett .Nat., **30**(5-6): 169 – 172.

Hussain Inaya A.A., 1987. New properties of hypergeometric series derivable from Feynman integrals, I. Transformation and reduction formulae. J. Phys. A. Math. Gen., **20**: 4109 – 4117.

Hussain Inayat A.A., 1987. New properties of hypergeometric series derivable from Feynman intergrals, II. A generalization of H - function. J. Phys. A. Math. Gen., **20**: 4119 – 4128.

Mainardi F.. Luchko Yu and Pagnini G., 2001. The fundamental solution of the space –time fractional diffusion equation. Frac. Calc Appl Anal., **4**: 153-192.

Meijer C.S., 1996. On the G- function. Proc. Nat Acad. Wetensch, **49**: 227.

Metzler R. and Klafter J., 2000. The random walk a guide to anomalous diffusion: A fractional dynamics approach. Phys Rep., **339**:333-347.

Qureshi M.I., Qureshi Kaleem A. and Pal R., 2011. Some definite integrals of Grudhsteyn-Ryzhil and other integrals. Global Journal of Science Frontier Research, **2**:75 – 8.

Sharm C.K. and Tiwari I.M., 1994. Certain Integrals and Series expansion involving the I-function. Acta Ciencica India, XXM (2):146-152.

Srivastava H.M., 1972. A contour integral involving Fox's H-function. India J. Math., **14**: 1 – 6.

Srivastava H.M., Gupta K.C. and Goyal S.P., 1982. The H - function of one and two variables with applications. South Asian Publishers, New Delhi, Madras.

Srivastava H.M. and Singh N.P., 1983. The integration of certain products of the multivariable H-function with a general class of polynomials. Rend. Circ. Math. Palermo, **2**(32):157 – 187.

Srivastava H.M. and Garg M., 1987. Some integrals involving a general class of polynomials and multivariable H-function. Rev Roumaine Phy., **32**:685 – 692.