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PORTFOLIO OPTIMIZATION AND ASSET ALLOCATIONS USING DEA AND DEA/AHP

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ABSTRACT

The present study is an attempt toward evaluating the performance of portfolios and asset allocations usingData Envelopment Analysis (DEA) and DEA/AHP (Analytic Hierarchy Process) ranking model. Among many evaluation methods, DEA is one of the best methods for assessing the relative efficiency a group of homogenous Decision Making Units (DMUs) that use multiple inputs to produce multiple outputs.The problem consists of choosing an optimal set of assets in order to minimize the risk and maximize return. The purpose of this paper is to evaluate portfolio in two steps: asset selection using (DEA) and asset allocations using DEA/AHP and ranking mutual funds in practical example. Thus, DEA/AHP ranking model is compared with Anderson and Petersen (AP) ranking model. This method is illustrated by application in mutual funds and extremely weights are obtained via DEA/AHP for making efficient portfolio. The finding could be used for constructing efficient portfolio in stock companies, in various finance organization and public and private sector companies.

KEYWORDS : Portfolio, Data Envelopment Analysis (DEA), Analytic Hierarchy Process (AHP)

In financial literature, a portfolio is an appropriate mix investments held by an institution or private individuals. Evaluation of portfolio performance has created a large interest among employees also academic researchers because of huge amount of money are being invested in financial markets. The theory of mean variance, Markowitz (1952) is considered the basis of many current models and this theory is widely used to select portfolios. This model is due to the nature of the variance in quadratic form. Due to quadratic form, a study by Arditti (1975), Kane (1982) and Ho and Cheung (1991) shown that investors preferskewness which means that utility functions of investors are not quadratic. Other problem in Markowitz model is that increasing the number of assets will be developed the covariance matrix of asset returns and will be added to the content calculation. Due to these problems sharp one- factor model or market model is proposed by Sharp (1964). This method reduces the number of calculations required information for the decision. It can be seen that the beta coefficient in a security's market model measures the sensitivity of the security's returns to the market index's returns. Hence, assets with betas greater than one are more volatile than the market index and are known high-risk assets. In contrast, assets with betas less than one are less volatile than the market index and are known as low-risk assets. Data envelopment analysis (DEA) has proved the efficiency for assessing the relative efficiency of Decision Making Units (DMUs) that employing multiple inputs to produce multiple outputs (Charnes et al, 1978). One of the most important models for ranking extremeefficient units was proposed by Anderson and Petersen (AP) (1993). Morey and Morey (1999) proposed mean variance framework based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return is the output. Joro& Na (2005) introduced mean - variance skewness framework and skewness of returns are also considered as an output. The portfolio optimization problem is a well-known problem in financial real world. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. Since there are a large number of assets to invest in, this objective leads to two investment problems:

(1) Selecting assets to be included in the portfolio (asset selection)

(2) Determining the proportion or weights to be allocated to the selected assets (asset allocation).

AHP (Analytic Hierarchy Process) is a multi-objective decision analysis method first proposed by professor Saaty. It is designed for subjective evaluation of a set of alternatives based on multiple criteria, organized in a hierarchical structure. Sinuany-stem et al. introduced a method for ranking and allocation of weigh by using DEA/AHP .This paper later selection firstly constructs comparison matrix using DEA, then assets the alternatives using AHP.The rest of the paper is organized as follows. The next section represents DEA model, mean - variance model of Markowitz and DEA/AHP briefly. Third Section developsmethodology. Fourth Section is a real application in mutual funds. Finally conclusions are given.

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Background

Portfolio theory to investing is published by Markowitz, 1952. This approach starts by assuming that an investor has a given sum of money to invest at the present time. This money will be invested for a time as the investor's holding period. The end of the holding period, the investor will sell all of the assets that were bought at the beginning of the period and then either consume or reinvest. Since portfolio is a collection of assets, it is better that to select an optimal portfolio from a set of possible portfolios. Hence the investor should recognize the returns (and portfolio returns), expected (mean) return and standard deviation of return. This means that the investor wants to both maximize expected return and minimize uncertainty (risk). Rate of return (or simply the return) of the investor's wealth from the beginning to the end of the period is calculated as follows:

(end-of-period) - (begning-of-period)
Return=--
begning-of-period

Since Portfolio is a collection of assets, its return r_p can be calculated in a similar manner. Thus according to Markowitz, the investor should view the rate of return associated to any one of these portfolios as what is called in statistics a random variable. These variables can be described expected return (min or $\bar{r}_p$) and standard deviation of return. Expected return and deviation standard of return are calculated as follows:

Where:

$$\bar{r}_p = \sum_{i=1}^{n} \lambda_i \bar{r}_i \ , \ \sigma_p = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij} \right]^{1/2}$$

n=the number of assets in the portfolio
$\bar{r}_p$ =The expected return of the portfolio
$_i$ =The proportion of the portfolio's initial value invested in asset i
$\bar{r}_i$ =The expected return of asset i
$_p$ = The deviation standard of the portfolio
$_{ij}$ = The covariance of the returns between asset i and asset j

In the above, optimal portfolio from the set of portfolios will be chosen that maximum expected return for varying levels of risk and minimum risk for varying levels of expected return(Sharp 1985). Data Envelopment Analysis is a nonparametric method for evaluating the efficiency of systems with multiple inputs and multiple outputs. In this section we present some basic definitions, models and concepts that will be used in other sections in DEA. They will not be discussed in details. Consider DMU_j, (j=1,....,n) where each *DMU* consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $x_j=(x_{1j}, ..., x_{mj})$ and $y_j=(y_{1j}, ... y_{sj})$ respectively, and let $X_j \geq 0$ and $X_j \neq 0$, $Y_j \geq 0$ and $Y_j \neq 0$. A basic DEA formulation in input orientation is as follows:

$$\min \ \theta - \varepsilon \left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- \right)$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \qquad i = 1,..., m, \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \qquad r = 1,...,,s$$

$$\lambda \in \Lambda,$$

$$s^+, s^- \geq 0,$$

$$\varepsilon \geq 0$$

Where λ is a n-vector of λ variables, s^+ as-vector of output slacks, s^- an m-vector of input slacks and set Λ is defined as follows:

$$\Lambda = \begin{cases} \{\lambda \in R_+^n & \text{withconstantreturns to scale,} \\ \{\lambda \in R_+^n, 1\lambda \leq 1\} & \text{with non-increasing returns to scale,} \\ \{\lambda \in R_+^n, 1\lambda = 1\} & \text{with variable returns to scale} \end{cases}$$

Note that subscript 'o' refers to the unit under the evaluation. A DMU is efficient iff θ=1 and all slack variables s^-, s^+ equal zero; otherwise it is inefficient (Charnes et al., 1994). In the DEA formulation above, the left hand sides in the constraints define an efficient portfolio. is a multiplier defines the distance from the efficient frontier. The slack variables are used to ensure that the efficient point is fully efficient. This model is used for asset selection. To rank efficient DMU in BCC model, Anderson and Petersen (AP) model is solved for each DMU:

$$AP:\min\ \theta_o$$

$$S.t.\quad \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j x_{ij} \le \theta_o x_{io},\quad i=1,2,...,m$$

$$\sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j y_{rj} \le y_{ro},\qquad r=1,2,...,s \qquad (2)$$

$$\sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0,\quad j=1,...,n,\ j\neq 0$$

Many researchers highlight the relationship between DEA and Multi-Criteria Decision Analysis (MCDA). Indeed DEA incorporates a process of assigning weights to criteria. Ranking is very common in MCDA literature, especially when we have a discrete list of elements or alternatives with single or multiple criteria which we wish to evaluate and compare or select. The evaluations are often carried out subjectively by the decision maker. Our current paper is another attempt to fully rank mutual funds in the DEA context, utilizing one of the more popular MCDM methods the AHP. The AHP is chosen for subjective evaluation of a set of alternatives (units) based on multiple criteria, organized in a hierarchical structure. At the top level, the criteria are evaluated and at the lower levels, the alternatives are evaluated by each criterion. AHP use a pairwise comparison matrix between criteria and between units, assessed subjectively by the decision maker, to rank the units. The DEA/AHP ranking model is utilized in this paper is two- stage for asset allocation. First, DEA is run for every pair of mutual funds, two mutual funds at a time, ignoring the others. From the results of the first stage, a pairwise comparison matrixis producedto which we apply a two level AHP, which provides a full ranking scale of all the mutual funds.

Methodology

Return of assets consist of money which we receive among period plus difference of buying and selling. Return is not definitely usually obvious. This uncertain in rate of expected return defined as deviation of return. Deviation of return is called risk. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. In this regard, mean-variance model Markowitz, expected return is treated as output and deviation of return as input. The methodology in this paper starts with asset selection then determines the proportion or weights to be allocated to the selected assets. The data used for both ofthe methodologiesare from 57 mutual funds (Joro and Na, 2005). In many cases similar to this example there are a lot of assets. It is better that starts with asset selection. The choice of the asset can be random or discrete. The random choice of assets is usually biased and do not promise an optimum portfolio; hence it is more rational to have an objective choice while selecting the assets to be included in the portfolio.Among many evaluation methods, Data Envelopment Analysis (DEA) is one of the best ways for assessing the relative efficiency a group of homogenous decision making units (DMUs) that use multiple inputs to produce multiple outputs, originated from the work by Charnes et al.,(1978). Selection of assets to be included in portfolio is followed by using DEA. Input oriented model with non- increasing return to scale is used for efficiency evaluation. With this approach, lending but no borrowing can be done by requiring the sum of s to be smaller than unity:

$$\sum \lambda_j \le 1$$

(Joro and Na, 2005). In the analysis, the variance of the assets is used as an input to the DEA and expected return is used as an output. For assets selection, input oriented model with non-increasing return to scale is used to analyze the given 57 mutual funds and four efficient mutual funds is obtained by this model. Then there are only four mutual funds that are potential candidates for portfolio formation. The later section determines the proportions or weights to the selected mutual funds obtained from DEA model for optimal portfolio. For the allocation of weights, DEA/AHP ranking model is used in this section. Thus, DEA/AHP ranking model is compared with AP ranking model.

DEA/AHP

This paper introduces a comprehensive evaluation method based on the DEA and AHP (Zhenhaa,2009) for the allocation of weights to assets, which is divided into two phases. The use of DEA methods on the effectiveness of every pair mutual funds According to the calculation results of the first phase, establish comparison matrix using DEA and then use a two level AHP approach.

A. Comparison Matrix

Suppose there are n mutual funds, each mutual fund have one output (mean or expected return) and one input (variance or risk). For example, two decision making units 1 and 2(without loss of generality) are selected and DEA method is used to calculate the efficiency of the two decision making units, h_{12} and h_{12} are respectively the optimal solution of the linear programing problem and LP_1 & LP_2.

LP_1 :

$$h_{12} = \max \sum_{r=1}^{s} u_r y_{r1}$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \quad j = 1, 2 \quad (3)$$

$$\sum_{i=1}^{m} v_i x_{i1} = 1$$

$$v_i \geq 0, u_r \geq 0, \; i = 1,\ldots,m, r = 1,\ldots,s$$

LP_2 :

$$h_{21} = \max \sum_{r=1}^{s} u_r y_{r2}$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \quad j = 1,2 \quad (4)$$

$$\sum_{i=1}^{m} v_i x_{i2} = 1$$

$$v_i \geq 0, u_r \geq 0, i = 1,\ldots, m, r = 1,\ldots, s$$

The efficiency ratio mutual funds selected 1,2 are the following:

$$a_{12} = \frac{h_{12}}{h_{21}} \quad (5)$$

In general, the efficiency ratio of every pair of mutual funds selected is the following:

$$a_{ij} = \frac{h_{ij}}{h_{ji}}, a_{ji} = \frac{1}{a_{ij}}, a_{ii} = 1$$

Structure comparison matrix is as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & 2a_n & \ldots & a_{nn} \end{bmatrix}$$

B. The use of a two-level AHP method to find weights (Asset Allocation)

After the comparison matrix is composed,using eigenvector the two- level AHP method is used to account the weights of mutual funds selected. These means that, proportion or weights of mutual funds in portfolio is obtained by using eigenvalue

λ_{max} and eigenvector $\vec{w}$ in $A\vec{w} = \lambda\vec{w}$

Application in mutual funds

We illustrate our approach using a data set 57 mutual funds . A list of funds used is provided in Table 1. In this report, there is expected return and variance of mutual funds which expected return is considered as output and variance is as input.

Firstly, asset selection is started. Selection of mutual funds to be included in portfolio is followed by using DEA. Input oriented model with non-increasing returns to scale is used for efficiency evaluation. The detailed results are presented in Table 2. Four efficient mutual funds are obtained by this model.

Efficient mutual funds data is in Table 3. Then the comparison matrix can be composed from the formula (3), (4), (5):

$$\begin{bmatrix} 1 & 0.68 & 0.29 & 0.57 \\ 1.47 & 1 & 0.43 & 0.84 \\ 3.44 & 2.32 & 1 & 1.96 \\ 1.75 & 1.19 & 0.51 & 1 \end{bmatrix}$$

Then proportion or weights of mutual funds in portfolio is obtained by using eigenvalue λ_{max} and eigenvector $\vec{w}$ in $A\vec{w}=\vec{w}$. Then, we have:

$$w_{ACEGX} = 0.132 \quad w_{RYOCX} = 0.18 \quad w_{UMBIX} = 0.45 \quad w_{JANSX} = 0.23$$

We summarize in Table, 4 eigenvector and the final ranking. Results or weights are obtained for four efficient mutual funds via DEA/AHP. Also mutual funds are sorted using DEA/AHP and AP ranking model.

Table 1 : Descriptive Statistics of The Mutual Funds

Mutual fund	Expected return	Variance	Mutual fund	Expected return	Variance	Mutual fund	Expected return	Variance
ACEFX	2.671	64.173	IDEUX	1.874	24.913	RYOSX	2.690	42.951
ACEGX	2.734	64.254	IGLBX	1.858	25.178	SCGEX	1.656	18.530
ACEMX	2.668	64.098	IGLCX	2.488	39.982	SRGFX	2.224	40.412
AELAX	1.241	22.976	JAMRX	1.991	35.740	SSGWX	2.044	26.771
AELGX	1.287	22.970	JAOSX	1.870	23.264	TALGX	1.368	30.664
AGTHX	1.902	23.445	JAVLX	2.237	30.771	TRGEX	1.786	25.739
ARCGX	2.017	30.697	JAWWX	1.946	20.705	TWIEX	1.773	23.208
AVLFX	1.615	19.817	LMVTX	1.735	19.463	UMBIX	1.823	12.379
BJBIX	1.506	33.203	MAFGX	1.849	20.490	USBOX	2.093	24.542
CUCAX	1.990	24.858	MBFGX	1.923	20.626	VGHCX	1.334	26.919
FAGIX	0.533	5.471	MCFGX	2.463	60.247	VPMCX	1.513	20.243
FAIAX	1.404	39.099	MCOBX	1.959	19.729	WAGEX	1.625	17.619
FAICX	1.420	39.095	MCOFX	1.875	18.821	WBIGX	0.979	25.015
FDEGX	2.445	53.059	MDFGX	1.942	18.833	FMAGX	1.681	14.225
FEURX	1.696	30.655	MGCAX	1.789	31.720	JANSX	2.643	35.453
FIUIX	1.398	13.813	MSEQX	1.842	31.348	VFINX	1.690	27.731
FSUTX	1.697	18.829	NAWCX	1.318	24.631	VWNDX	1.889	21.968
GEGTX	1.727	19.483	NAWGX	1.384	24.676			
HRCPX	2.376	34.856	POVBX	2.579	149.234			
IDETX	2.356	35.331	POVSX	1.619	21.647			

Table 2 : Efficiency Measure of 57 Mutual Funds

Mutual funds	MV efficiency measure	Mutual funds	MV efficiency measure	Mutual funds	MV efficiency measure
ACEFX	0.62	IDEUX	0.55	RYOSX	1.000
ACEGX	1.000	IGLBX	0.53	SCGEX	0.61
ACEMX	0.62	IGLCX	0.78	SRGFX	0.59
AELAX	0.37	JAMRX	0.48	SSGWX	0.69
AELGX	0.38	JAOSX	0.59	TALGX	0.30
AGTHX	0.62	JAVLX	0.78	TRGEX	0.47
ARCGX	0.58	JAWWX	0.77	TWIEX	0.52
AVLFX	0.55	LMVTX	0.61	UMBIX	1.00
BJBIX	0.31	MAFGX	0.64	USBOX	0.81
CUCAX	0.69	MBFGX	0.74	VGHCX	0.34
FAGIX	0.66	MCFGX	0.50	VPMCX	0.51
FAIAX	0.24	MDFGX	0.82	WAGEX	0.63
FAICX	0.25	MGDAX	0.74	WBIGX	0.27
FDEGX	0.56	MSEQX	0.84	FMAGX	0.80
FEURX	0.38	MCOBX	0.38	JANSX	1.000
FIUIX	0.69	MCOFX	0.41	VFINX	0.41
FSUTX	0.61	NAWCX	0.36	VWNDX	0.65
GEGTX	0.60	NAWGX	0.38		
HRCPX	0.86	POVBX	0.23		
IDETX	0.77	POVSX	0.51		

Table 3 : Mutual funds selections, DEA model is used to analyze given 57 mutual funds and four efficient mutual funds is obtained

Mutual funds	Expected return(Output)	Variance(Input)
ACEGX	2.739	64.254
RYOSX	2.690	42.951
UMBLX	1.823	12.379
JANSX	2.643	35.453

CONCLUSION

In this paper, a new method is suggested for asset selection and asset allocations for selecting efficient portfolio with one input (variance) and one output (expected return) in the DEA context, integrated with AHP. In this case, the pairwise comparison matrix is perfectly consistent. Since there are a large number of assets to invest in, this objective leads to two investment problems. First, the assets are selected for making portfolio and second, the proportion or weights are determined to be allocated to the selected assets.Selection of assets to be included in portfolio is followed by using DEA. Input oriented modelwith non-increasing returns to scale is used for this purpose. In this regard, this model is used to analyze the given 57 mutual funds and four efficient mutual funds are obtained by this model.The DEA/AHP ranking model is used for four efficient mutual funds in this paper is two- stage for asset allocation. First, DEA is run for every pair of mutual funds, ignoring the others. From the results of the first stage, a pairwise comparison matrix is created to which we apply a two level AHP, which provides a full ranking scale of all the mutual funds. Thus, Anderson and Petersen (AP) ranking model is solved for this purpose In Table 4, the weights and rankings have been designated for four efficient mutual funds via DEA/AHP and AP ranking models. It can be seenthat resultsare same of two models for ranking.

Table 4 : Weights And Rankings

Mutual funds	Weights (eigenvector)	DEA/AHP ranks	AP ranks
ACEGX	0.132	4	infeasible
RYOSX	0.18	3	3
UMBLX	0.45	1	1
JANSX	0.23	2	2

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