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VISCOUS DISSIPATION AND CHEMICAL REACTION EFFECT ON NATURAL CONVECTION HEAT AND MASS TRANSFER FLOW ALONG AN ISOTHERMAL SPHERE WITH RADIATION AND HEAT LOSS

R. PANNEERSELVI[a1] AND P. DIVYA[b]

[ab]Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore, India

ABSTRACT

In this work, two dimensional free convection flow across a uniformly heated sphere immersed in a viscous incompressible fluid in the presence of species concentration with radiation and heat loss is investigated. The transformed non- linear equations are solved using Runge-Kutta method. The effects of few physical parameters like viscous dissipation parameter Vd, Radiation parameter Rd, Prandtl number Pr, heat generation parameter Q, Schmidt number Sc, local heat transfer and local mass transfer on velocity, temperature and concentration distribution are analyzed through graphs.

KEYWORDS: Natural Convection, Radiation Heat Loss, Skin Friction, Nusselt Number, Schmidt Number

Convection is the mode of energy transfer between a solid surface and in addition the adjacent liquid or gas that's in motion and it involves the combined effects of natural phenomenon and fluid motion. The faster the fluid motion, the larger the heat transfers as a result of convection. Free convection flow is usually encountered in cooling of nuclear reactors or at intervals the study of the structure of stars and planets. The study of temperature and heat transfer is of nice importance to the engineers as a result of its nearly universal prevalence in many branches of science and engineering. Applying of natural phenomenon techniques to mass transfer has been of substantial facilitating in developing the speculation of separation processes and chemical dynamics. Griffith (1964) analyzed velocity, temperature and Concentration distribution during fiber spinning.. Ericlcson, Fan and Fox (1966) analyzed moving continuous plate considering for the case where the transverse velocity component is nonzero at the surface of the plate. Gebhart and Pera (1971) studied the effects on velocity, heat and mass transfer and on laminar stability**.**

Hossain and Rees (1996) studied the combined heat and mass Transfer in natural convection. Effects of chemical reaction heat and mass Transfer laminar flow along a semi-infinite horizontal plate have been studied by Anjalidevi and Kandasamy (1999). Takhar, Chamkha and Nath (2000) analyzed the flow and mass transfer characteristics of a viscous electrically conducting fluid on a continuously stretching surface with non-zero velocity. Zakerullah (1972) analyzed viscous dissipation and pressure work affects in axisymmetric natural convection flows. Merkin (1977) studied mixed convection from a horizontal circular cylinder in a stream flowing vertically upwards in both the cases of heated and cooled cylinder. Free Convection boundary layers on an isothermal horizontal circular cylinder have been studied by Merkin (1978).

Ingham (1978) analyzed the free convection on a horizontal cylinder whose temperature is suddenly increased at large Grashof number. Nazar, Amin and Pop (2002) studied the steady convection boundary layer flow of a micro polar fluid about a sphere with constant surface temperature. Huang and Chen (1987) investigated influence of Prandtl number and surface mass transfer on a steady, laminar free convective flow over a sphere with non-uniform surface temperature. Gebhart and Mollendorf (1969) studied viscous dissipation in external natural convection flow.

Ibrahim (2013) investigated chemical reaction and radiation effects on free convection along a stretching surface with viscous dissipation and heat generation. Akther and Alim (2010) studied the effects of pressure work on natural convection flow around a sphere with radiation heat loss.

Alim, Rahman (2008) analyzed two dimensional laminar incompressible flow around a sphere in the presence of viscous dissipation. Hye, Molla and khan (2007) studied conjugate effect of heat and mass transfer on natural convection flow across an isothermal horizontal circular cylinder with chemical reaction.

Thermal Radiation heat transfer was studied by Siegel and Howell (1972). Cebeci and Bradshaw (1984) analyzed Physical and computational aspects of convective

[1]Corresponding Author

heat transfer. Radiation should be thought-about in calculative thermal effects in rocket nozzles, power plants, engines and hot temperature heat exchangers. Radiation will generally be necessary despite the fact that the temperature level is not elevated and alternative modes of warmth transfer are present. Radiation includes a nice impact within the energy equation that results in an extremely non-linear partial differential equation. Heat, mass and momentum transfer on an endlessly moving or a stretching sheet has many applications in chemistry and compound process.

In this work, heat associate degreed mass transfer on free (or) natural convection flow across a sphere immersed in a viscous incompressible fluid within the presence of species concentration with radiation heat loss is studied.

FORMULATION OF THE PROBLEM

Consider a steady two-dimensional laminar free convective flow that flows across a uniformly heated sphere of radius a, which is immersed in a viscous and incompressible fluid. It is assumed that the surface temperature of the sphere isT_w, whereT_w>T_∞. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, r(x) is the radial distance from the symmetrical axis to the surface of the sphere and ($\hat{u}$, ,$\hat{v}$) are components along the($\hat{x}$,$\hat{y}$) axis. The basic governing equations are

Continuity Equation

$$\nabla \overline{\vee} = 0 \tag{2.1}$$

Momentum Equation

$$\rho\left[\frac{\partial\overline{\vee}}{\partial t} + (\overline{\vee}.\nabla)\,\overline{\vee}\right] = -\nabla\rho + \mu\nabla^2\overline{\vee} + \rho g\beta(T^1 - T^1\infty) + \overline{J}\times\overline{B} \tag{2.2}$$

Energy Equation

$$\rho c_p\left[\frac{\partial T}{\partial t} + (\overline{\vee}.\nabla)T\right] = -k\nabla^2 T - \rho(\nabla.\overline{\vee}) + S^*(T^1 - T^1_\infty) + \mu(\nabla.\overline{\vee})^2 \tag{2.3}$$

Equation of Concentration

$$\frac{\partial C^1}{\partial t^1} + V^1\frac{\partial C^1}{\partial y^1} = D\frac{\partial^2 C^1}{\partial {y^1}^2} \tag{2.4}$$

The coordinates system and the configuration are shown in Figure 1.

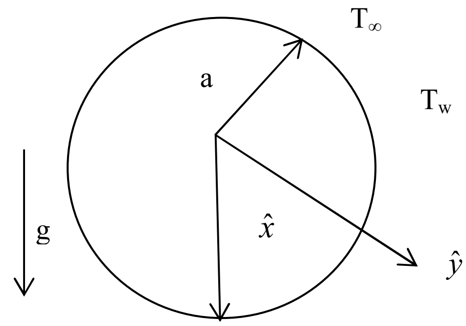

Figure 1: Physical model and coordinate system

Under the usual Boussinesq and boundary layer approximation, the governing equations of the flows are

$$\frac{\partial}{\partial\hat{x}}(r\hat{u}) + \frac{\partial}{\partial\hat{y}}(r\hat{v}) = 0 \tag{2.5}$$

$$\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{v}}{\partial\hat{y}}\right) = \mu\frac{\partial^2\hat{u}}{\partial\hat{y}^2} + \rho g\beta_T(T - T_\infty)Sin\left(\frac{\hat{x}}{a}\right) + \rho g\beta_c(C - C_\infty)\,Sin\left(\frac{\hat{x}}{a}\right) \tag{2.6}$$

$$\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{v}\frac{\partial T}{\partial\hat{y}} = \frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial\hat{y}^2} - \frac{1}{k}\frac{\partial q_r}{\partial\hat{y}}\right] + \frac{\nu}{\rho c_p}\left(\frac{\partial\hat{u}}{\partial\hat{v}}\right)^2 + \frac{Q_0}{\rho c_p}(T - T\infty) \tag{2.7}$$

$$\hat{u}\frac{\partial C}{\partial\hat{x}} + \hat{v}\frac{\partial C}{\partial\hat{y}} = D\frac{\partial^2 C}{\partial\hat{y}^2} - kr(C - C\infty) \tag{2.8}$$

The boundary conditions for the Equations (2.5)-(2.8) are

$$\hat{u} = \hat{v} = 0,\ T = T_W,\ C = C_W\ at\ \hat{y} = 0 \tag{2.9a}$$

$$\hat{u} \to 0,\ T \to T_W, C = C_W at\ \hat{y} \to \infty \tag{2.9b}$$

where r($\hat{x}$) = asin($\hat{x}$/a) is the radial distance from the center of the sphere, **g** is the acceleration due to gravity,β_T is the coefficient of thermal expansion, $\beta_C$is the coefficient of concentration expansion, μ is the viscosity of the fluid,C_p is the specific heat at constant pressure, ρ is the density, D is the molecular diffusivity of the species concentration and q_r is the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies that will employ a more detail representation for the radiative heat flux; here the optically dense radiation limit is taken into account.

Thus the Roseland diffusion approximation proposed by Siegel and Howell (1972) and is given by simplified radiation heat flux term as:

$$q_r = \frac{4\sigma}{3(a_r + \sigma_s)} \frac{\partial T^4}{\partial \hat{y}} \tag{2.10}$$

Consider the following non-dimensional variables:

$$x = \frac{\hat{x}}{a},\ q = Gr^{1/4}\frac{\hat{y}}{a},\ u = \frac{a}{v}Gr^{-1/2}\hat{u},\ v = \frac{a}{v}Gr^{-1/4}\hat{v}$$

$$\theta = \frac{T - T\infty}{T_N - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},\ Gr = \frac{g\beta_T (T_w - T_\infty)}{v^2}$$

$$Gm = \frac{g\beta_c (C_w - C_\infty)}{v^2},\ \theta_w = \frac{T_w}{T_\infty},$$

$$\Delta = \theta_w - 1 = \frac{T_w - T_\infty}{T_\infty} \tag{2.11}$$

Where $\upsilon = (\mu / \rho)$ is the reference kinematic viscosity,**Gr** is the Grashof number, **θ** is the non-dimensional temperature and ϕ is the non-dimensional species concentration.

Substituting the variables (2.11) into Equations (2.5)-(2.8) leads to the following non-dimensional equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.12}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + (\theta + N\phi)\sin x \tag{2.13}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial \eta} = \frac{1}{pr}\frac{\partial}{\partial \eta}\left[\left\{1 + \frac{4}{3}Rd(1+\Delta\theta)^3\right\}\frac{\partial \theta}{\partial \eta}\right] + Vd(\frac{\partial u}{\partial \eta}^2) + Q\theta \tag{2.14}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial \eta} = \frac{1}{SC}\frac{\partial^2 \phi}{\partial \eta^2} + Kr\phi \tag{2.15}$$

The boundary conditions (2.9) become

$u \to 0,\ \theta \to 0,\ \phi \to 0$ as $\eta \to \infty$ (2.16a)

$u=v= 0,\ \theta = 1,\ \phi =1$ at $\eta = 0$ (2.16b)

Where *Rd* is the radiation-conduction parameter, *Pr* is the Prandtl number and N is the ratio of the buoyancy forces due to the temperature and concentration are defined respectively as

$$Rd = \frac{4\sigma T_\infty^3}{k(a + \sigma_s)},\ pr = \frac{\mu c_p}{k},$$

$$Vd = \frac{v^2 G_r}{\rho a^2 C_p (T_w - T_\infty)}\ and\ N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)} \tag{2.17}$$

To solve Equations (2.12)-(2.15), subject to the boundary conditions (2.16), we assume the following variables

$$\psi = xr(x)\ f(x,\eta), \theta = \theta(x,\eta),$$
$$\phi = \phi(x,\eta), r(x) = \sin x \tag{2.18}$$

Where, φ is the non-dimensional stream function, which is related to the velocity component as:

$$u = \frac{1}{r}\frac{\partial \varphi}{\partial \eta}\ and\ v = \frac{-1}{r}\frac{\partial \varphi}{\partial x} \tag{2.19}$$

Substituting (2.18) into Equations (2.13)-(2.15) we get, after some algebra the following transformed equations

$$\frac{\partial^3 f}{\partial \eta^3} + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta}\right)^2 + (\theta + N\phi)\frac{\sin x}{x} = x\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right) \tag{2.20}$$

$$\frac{1}{pr}\left[\frac{\partial}{\partial \eta}\left\{1 + \frac{4}{3}Rd\,(1+\Delta\theta)^3\right\}\frac{\partial \theta}{\partial q}\right] + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial \theta}{\partial \eta} + Vd\left\{x\frac{\partial}{\partial \eta}\left(\frac{\partial f}{\partial \eta}\right)^2\right\} + Q\theta = x\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial x}\right) \tag{2.21}$$

$$\frac{1}{SC}\frac{\partial^2 \phi}{\partial \eta^2} + \left[1 + \frac{x}{\sin x}\cos x\right] f \frac{\partial \phi}{\partial \eta} - kr\left\{x\frac{\partial}{\partial \eta}\left(\frac{\partial f}{\partial \eta}\right)^2\right\} = x\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial \eta}\right) \tag{2.22}$$

Along with the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0 \tag{2.23a}$$

$$\frac{\partial f}{\partial y} \to 0, \theta \to 1, \phi \to 0 \text{ at } \eta = 0 \tag{2.23b}$$

It has been seen that the lower stagnation point of the sphere or $\varkappa \approx 0$, Equations (2.20)- (2.22) reduce to the following ordinary differential equations:

$$f''' + \left\{ 1 + \frac{x\left(1 - \frac{x^2}{2\,!} + \frac{x^4}{4\,!} ....\right)}{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} ....\right)} \right\} ff'' - f'^2 + (\theta + N\phi)\left(x \frac{-x^3}{3!} + \frac{x^5}{5!} ....\right)$$

$$= x\left(\frac{\partial f}{\partial \eta}\frac{\partial^2}{\partial x \partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right)$$

Since x=0 by stagnation point

$$f''' + (1+1)\, ff'' - f'^2 + (\theta + N\phi) = 0$$

$$f'''' + 2ff'' - f'^2 + (\theta + N\phi) = 0 \tag{2.24}$$

$$\frac{1}{\Pr}\left[\left\{1 + \frac{4}{3} Rd\ (1 + \Delta\theta)^3\right\}\theta'\right]' + 2f\theta' + Q\theta = 0 \tag{2.25}$$

$$\frac{1}{SC}\phi'' + 2f\phi' + Kr\phi = 0 \tag{2.26}$$

Subject to the boundary conditions at

$$f(0) = f'(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1 \text{ at } \eta = 0 \tag{2.27a}$$

$$f' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } \eta \to \infty \tag{2.27b}$$

In the above equations prime denote the differentiation with respect to η. In practical applications, the physical quantities of main interest are the shearing stress, the rate heat transfer and the rate of mass transfer in terms of the skin-friction coefficients $\mathbf{C_f}$, Nusselt number **Nu** and Sherwood number **Sh** respectively, which can be written as

$$C_f = \frac{Gr^{-3/4} a^2}{\mu\nu} \tau_w,\ Nu = \frac{aGr^{-1/4}}{k(T_w - T_\infty)}(q_c + q_r)$$

$$and\ Sh = \frac{aGr^{-1/4}}{k(C_w - C_\infty)} J_c \tag{2.28}$$

Where

$$\tau_w = \mu\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}\approx 0},\ q_c = -k\left(\frac{\partial T}{\partial \hat{y}}\right)_{\hat{y}=0}$$

$$J_c = -k\left(\frac{\partial C}{\partial \hat{y}}\right)_{\hat{y}=0} \tag{2.29}$$

Using the variables (2.11), (2.18) and the boundary condition (2.23a) into (2.28)-(2.29), we get

$$C_{fx} = xf''(x,0) \tag{2.30}$$

$$Nu_x = -\left(1 + \frac{4}{3} Rd\theta_w^3\right)\theta'(x,0) \tag{2.31}$$

$$Sh_x = -\phi'(x,0) \tag{2.32}$$

The values of the velocity, temperature and concentration distribution are calculated respectively from the following relations:

$$u = \frac{\partial f}{\partial \eta},\ \theta = \theta(x,y),\ \phi = \phi(x,y) \tag{2.33}$$

RESULTS AND DISCUSSION

The profile for velocity, temperature and species concentration are given in Fig 2 to Fig 9.The momentum, energy and concentration equations are characterized by the Prandtl number (Pr), Radiation parameter (Rd) and Schmidt number (Sc). We have calculated the coefficient of skin friction, rate of heat transfer and the rate of mass transfer by assigning specific values to the different values to the parameter involved in the Problem.

The velocity profile for various viscous dissipation parameter (Vd = 0.0, 1.0, 2.0, 3.0, 4.0) is shown in Figure 2. It is clear from the figure that the velocity profile decreases with the increase of viscous dissipation parameter (Vd) which indicated that Vd decreases the fluid motion slightly.

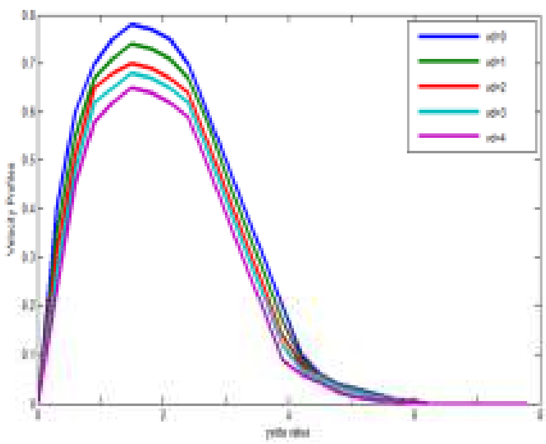

Figure 2: Velocity Profile for different viscous dissipation parameter (Vd[Rd = 1.0, Pr = 0.72, Q = 0.40]

The temperature profile for various viscous dissipation Parameter (Vd = 0.0, 1.0, 2.0, 3.0, 4.0) is shown in Figure 3. It is clear from the figure that the temperature profile decreases with the increase of viscous dissipation parameter Vd.

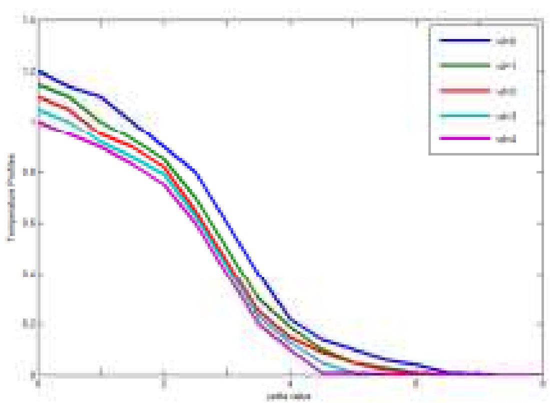

Figure 3: Temperature Profile for different viscous dissipation parameter (Vd)[Rd = 1.0, Pr = 0.72, Q = 0.40]

The velocity profile decreases with the increase of the heat generation parameter (Q = 0.20, 0.30, 0.40, 0.50, 0.60) is shown in Figure 4. It is clear from the figure that velocity decreases as well as the position moves upward the interface with increasing values of Q.

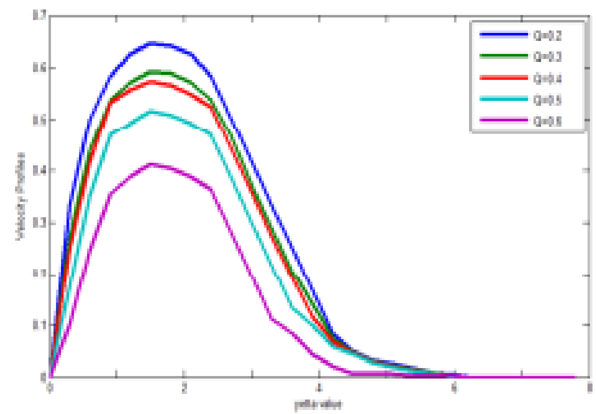

Figure 4: Velocity Profile for different heat generation parameter (Q) [Vd = 2.0, Pr = 0.72 and Rd = 1.0]

The temperature profile for various heat generation parameter (Q= 0.20, 0.30, 0.40, 0.50, 0.60) is shown in Figure 5. It is clear from the figure that temperature increases along with the increase of heat generation parameter.

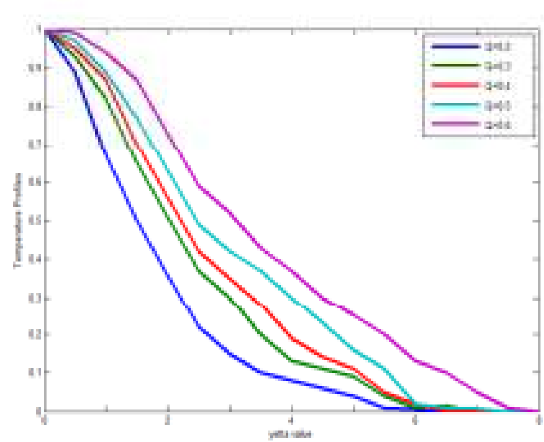

Figure 5: Temperature Profile for different heat generation parameter (Q)[Vd = 2.0, Pr = 0.72 and Rd = 1.0]

The concentration profile for various heat generation parameter (Q= 0.20, 0.30, 0.40, 0.50, 0.60) is shown in Figure 6. It is clear from the figure the species concentration increases with the increase of heat generation parameter.

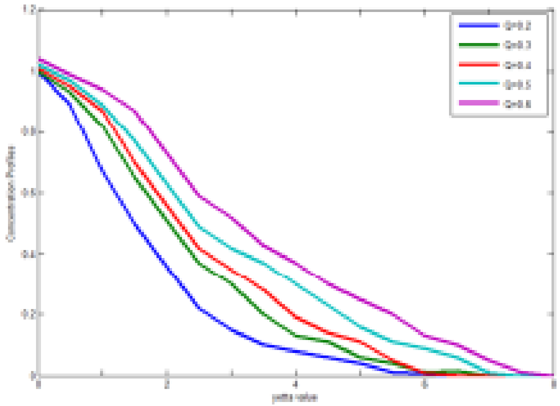

Figure 6: Concentration Profile for different heat generation parameter (Q) [Vd = 2.0, Pr = 0.72 and Rd = 1.0]

The velocity profile for various chemical reaction parameter (Kr = 0.0, 0.20, 0.40, 0.60, 0.80) is shown in Figure 7. It is clear from figure that velocity decreases sharply with increase of chemical reaction Kr.

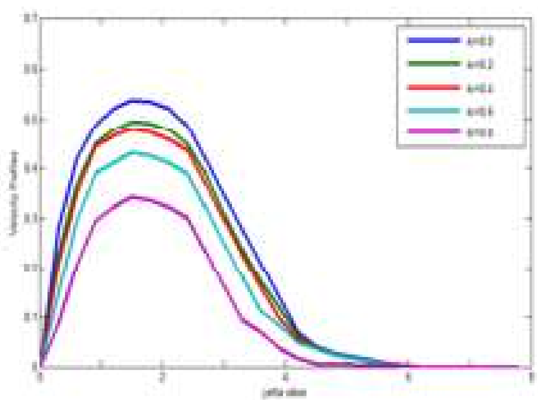

Figure 7: Velocity Profile for different chemical reaction parameter (Kr)[Pr = 0.72 and Sc = 0.22]

The temperature profile for various chemical reaction parameter (Kr = 0.0, 0.20, 0.40, 0.60, 0.80) is shown in Figure 8. It is clear from figure that temperature increases significantly with the increase of chemical reaction Kr.

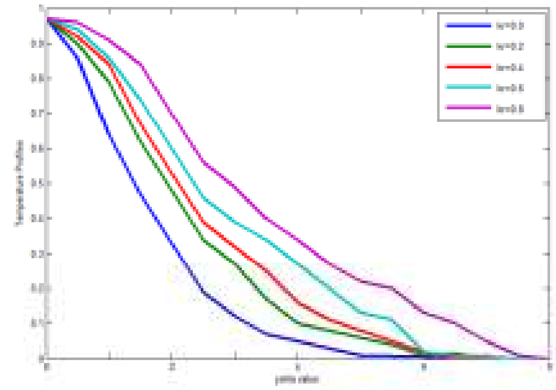

Figure 8: Temperature Profile for different chemical reaction parameter (Kr)[Pr = 0.72 and Sc = 0.22]

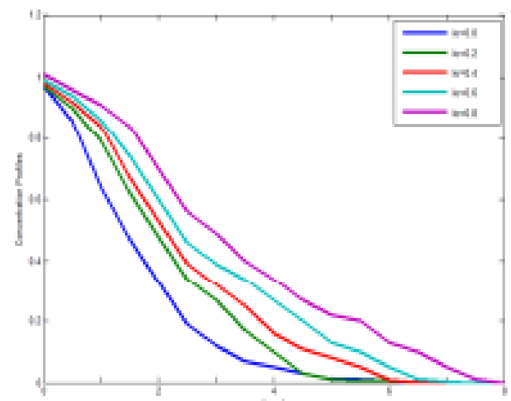

Figure 9: Concentration Profile for different chemical reaction parameter (Kr)[Pr = 0.72 and Sc = 0.22]

The concentration profile for various chemical reaction parameter (Kr = 0.0, 0.20, 0.40, 0.60, 0.80) is shown in Figure 9. It is clear from figure that species concentration increases with the increase of chemical reaction.

The local skin friction coefficient with variation in Schmidt number (Sc = 0.22, 0.80, 1.00, 1.10, 1.30) is shown in Figure 10. It is clear from the figure that the local skin friction coefficient cf_x decreases uniformly with increasing values of Sc.

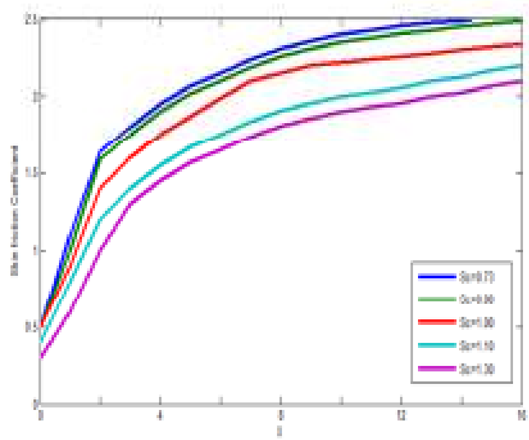

Figure 10: Local skin friction coefficient for different Schmidt number (Sc) [Pr = 0.72 and Rd = 0.30]

The Nusselt number distribution for various Radiation parameter (Rd = 0.0, 0.30, 0.50, 0.80, 1.00) is shown in Figure 11. It is clear from figure that an increase in Radiation parameter decreases the local Nusselt number $\mathrm{N}u_x$.

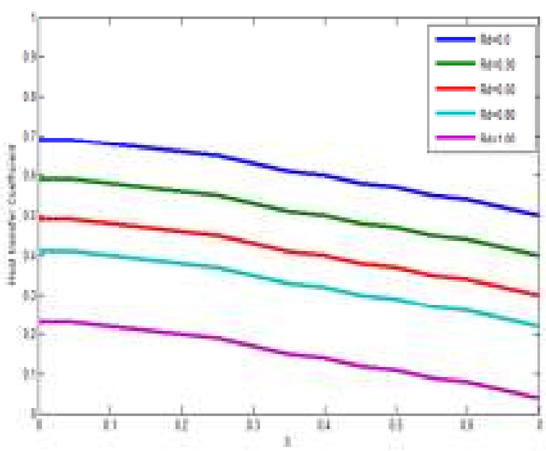

Figure 11: local Nusselt number for different Radiation Parameter (Rd)[Pr = 0.72 and Sc = 0.22]

The Sherwood number distribution for various Prandtl number (Pr = 0.72, 1.0, 3.0, 5.0, 7.0) is shown in Figure 12.It is clear from the figure that an increase in Prandtl number, leads to a decrease in the local Sherwood number $\mathrm{S}h_x$.

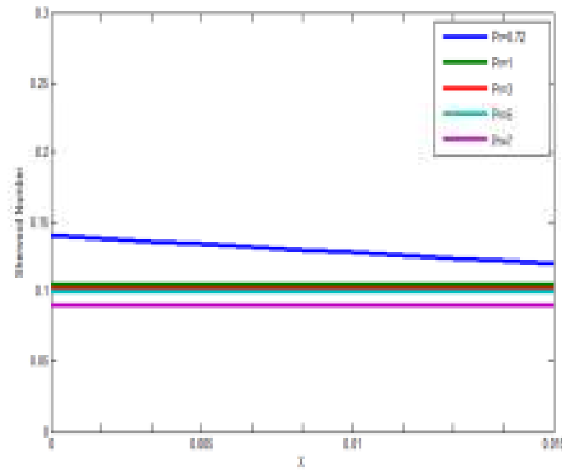

Figure 12: Sherwood number for different Prandtl number (Pr)[Rd = 0.30 and Sc = 0.22]

CONCLUSION

In this study, exact solution for the velocity, temperature and concentration field in the presence of viscous dissipation, heat generation parameter, Prandtl number, Schmidt number and Radiation parameter are constructed. Runge-Kutta method is employed to solve the equations governing the flow. The solution so obtained depending on the initial and boundary conditions are presented as sum of the non-dimensional parameters which occurs in problem under the study.

The following conclusions are made:

i) For the increase value of viscous dissipation parameter (Vd), the velocity and temperature profile decreases monotonically.
ii) Velocity decreases with increase in heat generation parameter andchemical reaction parameter.
iii) Temperature increases with the increase in heat generation parameterand chemical reaction parameter.
iv) Concentration increases with the increase of chemical reaction parameter.
v) The skin friction coefficient decreases with increase in Schmidt number.
vi) The Sherwood number decrease with increases in Prandtl number.

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