PERFORMANCE EVALUATION OF MULTI-CARRIER DCSK SYSTEM OVER
MULTIPATH FADING

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ABSTRACT

In this Paper, we studied the Multi-carrier Differential Chaos Shift Keying (DCSK) modulation. DCSK system provides robustness, higher data rates and optimized the transmitted energy efficiency (EE) as compared to traditional multi-carrier spread spectrum modulations. In this scheme, each binary data is mapped to two chaotic sequences each of the transmitted bits consist of two parts, where one part is reference slot having M subcarrier is assigned to transmitting, while the other will carry the data slots. In this case, one chaotic reference is used to transmit M-1 bits, which is used to save the transmitted bit energy. DCSK transmitted structure is increases the spectral efficiency (SE) and uses less energy. The receiver circuit has easy to implement where no radio frequency (RF) delay circuit is needed to demodulate and receive original data. We computed and compare the bit error rate (BER) performance of MC-DCSK system under multipath Rayleigh fading and AWGN channels. Simulations results of MC-DCSK for M subcarriers where spreading factor values changed.

KEYWORDS: DCSK, EE, SE, RF, BER, CDMA, AWGN.

Various emerging challenges in computer and wireless communications computing devices become ubiquitous. In this perspective spectral efficiency (SE) and energy efficiency (EE), interference resistance, security and channel impairments has to be top requirements for wireless communications. Performance of mobile and wireless communications are characterized by transmitter, receiver hardware impairments and propagation environments [David et.al., 2010 & Mingo et.al., 2004]. Fading channel behaviors are typically changes in vehicle to vehicle communications and mobile communications [Karedal et.al., 2009].

Multi-carrier systems have been used to get optimal communication systems in varying channel, such as orthogonal frequency division multiplexing (OFDM), have resilience to multipath fading and improving SE and higher data rates. Non coherent system can outperform as compared to coherent in fast and time varying channels, because of the SE inefficiency inherent to the pilots insertion [Saux et.al., 2005]. Several Multicarrier techniques have been proposed in literature [Hanzo et.al., 2003 & Kondo and Milstein, 1996] such as multi-carrier code division multiple access (MC-CDMA), Multicarrier direct sequence CDMA (MC-DS-CDMA) and orthogonal frequency code division multiplexing (OFCDM). MC-CDMA defined as on bit chips are spread over M subcarriers in frequency domain which is used to increase the total processing gain, while MC-DS CDMA uses the time-frequency spreading domain, it is used to increase the processing gain in each subcarrier signals.

The chaotic signal allow the generation of theoretical infinite number of uncorrelated signals, these signal well suited for spread spectrum modulation because their inherent wideband characteristics [Lau and Tse, 2003 & Vali et.al., 2012]. DCSK system chaotic synchronization is not used on the receiver side to generate an exact replica of chaotic sequence but only requires frame or symbol rate sampling [Kaddoum et.al., 2012]. Hence DCSK delivers good performance in multipath channels [Kolumban et.al., 1998 & Xia et.al., 2004]. Further differential non-coherent systems are better than coherent for time and frequency selective channels.

In DCSK system each bit duration is divided into two equal slots, in the first slot reference chaotic signal is sent. When data bit transmitted 0, reference chaotic signal is transmitted within first half of the bit duration $\left[0, \frac{T}{2}\right]$. To represent this digital information (that is “0”), x(t) is again transmitted in the second half of the bit duration $\left[\frac{T}{2}, T\right]$. When the data to be transmitted is a “1”, x(t) is transmitted within the first half of the bit duration, but in the second half of the bit duration its additive inverse (negative waveform) is transmitted. This can be accounted as being energy inefficient and serious data rate reducer.

In this paper, we first studied the multicarrier DCSK system, on the transmitter end, one of the M subcarriers is assigned to transmitting the reference slot, while the other will carry the data slot. One chaotic reference sequence is used to transmit M-1 bits, which is used to save the transmitted bit energy and increases the data rates. After removal of subcarriers, a parallel demodulation process is involved to recover the transmitted bits. Therefore, a DCSK system provides
resistance to interference increases the data rates and optimizes the transmitted bit energy as compared to conventional multicarrier techniques. We analysis the BER performance of MC-DCSK under multipath Rayleigh fading and AWGN channels without neglecting the properties of chaotic sequences.

DCSK SYSTEM

Figure 1 shows that DSK modulator, each bit each bit s_i = {-1,+1} is represented by two sets of chaotic signal samples, with first slot representing the reference and second carrying data. If +1 is transmitted the data bearing sequence is equal to the reference sequence and if -1 is transmitted, an inverted version of reference sequence is used as the data bearing sequence. In DCSK system 2β (where β is an integer) is representing as the spreading factor, which defined as the number of chaotic sample sent for each bit. The i_th bit duration, the output of the transmitter e_i,k is defined as

\[ e_{i,k} = \begin{cases} x_{i,k} & \text{for } 1 < k \leq \beta \\ s_{i,k} x_{i,k-\beta} & \text{for } \beta < k \leq 2\beta \end{cases} \]  

(1)

Where x_k is the chaotic sequence used as reference and x_k-β is the delayed version of the reference sequence. The received signal r_k is correlated to a delayed version of the received signal r_k+β and summed over a bit duration T_b (where T_b = βT_c and T_c is the chip time) to demodulate the transmitted bits is shown in Figure 1.

MULTI-CARRIER DCSK SYSTEM

Chaotic Generator

Second-order Chebyshev polynomial function (CPF) is used in this paper, it is defined as

\[ x_{k+1} = 1 - 2x_k^2 \]  

(2)

This map is chosen because it generates chaotic sequence ease and the performance is good [Kaddoum et al., 2009]. In addition, chaotic sequences are normalized and their mean values are zero and mean squared values are unity such that \( E(x_k) = 0 \) and \( E(x_k^2) = 1 \).

The Transmitter

In DCSK system, the input information sequence is converted into U parallel data sequences \( s_u \) for \( u = 1, 2, \ldots, U \).

\[ s_u = [s_{u,1}, \ldots, s_{u,i}, \ldots, s_{u,M-1}] \]  

(3)

Where \( s_{u,i} \) is the i_th bit of the u_th sequence data and M-1 is the number of data per u_th sequence.

\[ \beta \]

(a) Transmission of DCSK System

\[ \chi_{i,k} \quad \ldots \quad \chi_{i,k+\beta} \quad \sum_{k=1}^{\beta} r_k \quad \sum_{k=1}^{\beta} h(t-kT_c) \quad \text{Threshold} \]

(b) DCSK frame

\[ \beta \]

(c) Receiving of DCSK System

In Figure 2 reference chaotic code \( x_u(t) \) to be used as a reference signal and spreading code, after a serial to parallel conversion the M-1 bits of the u_th data sequence are spread due to multiplication in time with the same chaotic spreading code \( x_u(t) \).

\[ x_u(t) = \sum_{k=1}^{\beta} x_{u,k} h(t-kT_c) \]  

(4)

Where the spreading factor is \( \beta \), the square-root-raised cosine filter is \( h(t) \). This filter is band-limited and is normalized to have unit energy.
Let \( H(f) = F[h(t)] \), where \( F \) denotes a Fourier transform. It is assumed that \( H(f) \) is limited to \( \left[ \frac{B_c}{2}, \frac{B_c}{2} \right] \) which satisfies the Nyquist criterion with a roll off factor \( \alpha \) \((0 \leq \alpha \leq 1)\) where \( B_c = \frac{(1+\alpha)}{T_c} \).

The chaotic signal \( x_c(t) \) modulates the first subcarrier as reference, after the data signals spread by \( M-1 \) modulate the \( M-1 \) subcarriers. Therefore, the transmitted signal of the MC-DCSK is given by:

\[
e(t) = x_u(t) \cos \left( 2\pi f_1 t + \phi_1 \right) + \sum_{i=2}^{M} s_{u,i}(t) \cos \left( 2\pi f_i t + \phi_i \right)
\]  

(5)

Where \( \phi_i \) represents the phase angle introduced in the carrier modulation process. In this paper, we normalize the transmitted energy in every subcarrier.

![Figure 2: MC-DCSK system block diagram](image)

For the MC-DCSK, the modulated subcarriers are orthogonal over the chip duration. Hence, the baseband frequency corresponding to the \( i \)th subcarrier is \( f_i = f_p + \frac{i}{T_c} \) where \( f_p \) is the fundamental subcarrier frequency. The minimum spacing between two adjacent subcarriers equals \( \Delta = \frac{(1+\alpha)}{T_c} \) which is a widely used assumption [Kondo and Milstein, 1996].

The power spectral density (PSD) of MC-DCSK system is shown in figure 3. Let us \( B \) representing the total bandwidth of the system. When both bit duration \( T_b \) and \( B \) are set, chip duration \( T_c \), spreading factor \( \beta \) depend on the number of subcarrier \( M \), the bandwidth of each sub channel \( B_c \) or the subcarrier spacing \( \Delta \).

We divide the total band \( B \) into \( M \) equi-width frequency bands, as shown in Figure 3, where all bands are disjoint. The bandwidth of each subcarrier band \( B_c \) is given by:

\[
B_c = \frac{(1+\alpha)}{T_c}
\]

![Figure 3: The PSD of a band-limited MC-DCSK system](image)

Thus, the spreading factor function of the system parameters is given by:

\[
\beta = \frac{T_b}{T_c} = \frac{T_c B}{M (1+\alpha)}
\]  

(6)

Finally, the received signal is given by
\[ r(t) = \sum_{l=1}^{L} \lambda_l(t-r_l) e(t) + n(t) \]  

(7)

Where \( L \) is the number of path, \( \lambda_l(t) \) and \( r_l \) are the channel coefficient and the appropriate delay of the \( l \)th path respectively, * is the convolution operator, and \( n(t) \) is a wideband AWGN with zero mean and power spectral density of \( N_0 = 2 \). For our analysis, we choose a commonly used channel model in spread spectrum wireless communication systems [Xia et al., 2004, Proakis, 2001 & Rappaport, 1996].

We consider a slow fading multipath channel with \( L (L \geq 2) \) independent and Rayleigh distributed random variables as shown in figure 4. In this model, \( \lambda_l \) is the channel coefficient and \( r_l \) is time delay of the \( l \)th path (i.e., for \( L = 1 \) \( r_1 = 0 \) line-of-sight). The Rayleigh probability density function (PDF) of the channel coefficient is given by

\[ f_{\lambda}(z) = \frac{z^2}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}}, \quad Z > 0 \]  

(8)

Where \( \sigma > 0 \) is the scale parameter of the distribution.

In an AWGN case, the number of path is equal to one \( L = 1 \) with a unit channel coefficient \( \lambda(t) = 1 \)

![Figure 4: Multipath Rayleigh fading model](image)

C The Receiver

The MC-DCSK receiver is shown in Figure 2 this system provides a robust receiver and its gives better performance. We consider a set of matched filters, each demodulating the desired signal of the corresponding carrier frequency \( f_i \), and then the signals are sampled every \( kT_c \) time. The outputs discrete signals are stored in matrix memory. The matrix implementation of the receiver simplifies the parallel data recovery, where the decoding algorithm is described as follow under AWGN channel.

First, at the same time, the output of the first match is stored in matrix \( P \) and the M-1 data signals are stored in the second matrix \( S \), where

\[ P = \left( x_{u,1} + n_{u,1}, x_{u,2} + n_{u,2}, \ldots, x_{u,\beta} + n_{u,\beta} \right) \]

Where \( n_{uk} \) is the \( k \)th sample of additive Gaussian noise added to the reference signal, the matrix \( S \) is

\[
S = \begin{pmatrix}
s_{u,1}x_{u,1} + n_{u,1} & \ldots & s_{u,\beta}x_{u,\beta} + n_{u,\beta} \\
\vdots & \ddots & \vdots \\
s_{u,M-1}x_{u,1} + n_{u,1} & \ldots & s_{u,M-1}x_{u,\beta} + n_{u,\beta}
\end{pmatrix}
\]

Where \( n_{uk} \) is the \( k \)th sample of additive Gaussian noise added to the \( i \)th bit of \( u \)th data sequence.

Finally, after \( \beta \) clock cycles, all the samples are stored, and the decoding step is activated. The transmitted M-1 bits are recovered in parallel by computing the sign of the resultant vector of the matrix product:

\[ \hat{s}_u = \text{sign}(P \times S^T) \]  

(9)

where \( \times \) is the matrix product and \( (\cdot)^T \) is the matrix transpose operator. In fact, this matrix product can be seen as a set of a parallel correlator where the reference signal multiplies each data slot, and the result is summed over the duration \( \beta T_c \).

In Figure 2 reference chaotic code \( x_u(t) \) to be used as a reference signal and spreading code, after a serial to parallel conversion the M-1 bits of the \( u \)th data sequence are spread due to multiplication in time with the same chaotic spreading code \( x_u(t) \).

\[ x_u(t) = \sum_{k=1}^{\beta} x_{u,k} h(t-kT_c) \]  

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Where the spreading factor is \( \beta \), the square-root-raised cosine filter is \( h(t) \). This filter is band-limited and is normalized to have unit energy.
Let \( H(f) = F\{h(t)\} \), where \( F \) denotes a Fourier transform. It is assumed that \( H(f) \) is limited to \( \mathbb{R} \).

**PERFORMANCE ANALYSIS OF MC-DCSK**

**BER Calculations**

To derive the analytical BER expression, the mean and the variance of the observation signal \( D_{u,i} \) must be evaluated. A chaotic generator is very sensitive to initial conditions, and we can deduce that the different chaotic sequences generated from different initial conditions are independent from each other. In addition, the independence between the chaotic sequence and the Gaussian noise is also true [Lau and Tse, 2003]. For the normalized chaotic map with zero mean, the variance \( \text{Var}(x) \) is equal to one, \( \text{Var}(x) = E(x^2) = 1 \).

In our analysis, we assume that the largest multipath time delay is shorter than the bit duration

\[
0 < \tau_L \beta \quad (10)
\]

In this case, the inter symbol interference (ISI) is negligible compared with the interference within each symbol due to multipath delay. However, when \( \tau_L \) increases, ISI increases and deteriorates the BER.

In most practical applications, the condition \( \tau_L \beta \) holds, making our assumption justifiable [Kolumban and Kis, 2000]. Not with standing, we approve in the next section that the negligible ISI hypothesis when \( \tau_L \beta \) is true, and we also show the limit of this negligible ISI assumption. Moreover a large spreading factor we have

\[
\sum_{k=1}^{\beta} \left( x_{u,k-\tau_l} x_{u,k-\tau_j} \right) = 0 \quad \text{for} \quad l \neq j \quad (11)
\]

Since the channel is assumed slow fading, the channel coefficients are assumed constant during the transmission time of a MC-DCSK frame, and change every data stream \( u \). The decision variable for the \( l \)th bit of the \( u \)th data stream at the output of the correlator is

\[
D_{u,i} \approx T_c \sum_{k=1}^{\beta} \left( \lambda_{u,l} x_{u,k-\tau_l} s_{u,i} + n_{u,k}^i \right)
\times \left( \lambda_{u,l} x_{u,k-\tau_l} + n_{u,k} \right) \quad (12)
\]

Where \( \lambda_{u,l} \) and \( \tau_l \) are the channel coefficient and the time delay of the \( l \)th path affecting the \( u \)th data stream respectively.

The components \( n_{u,k} \) and \( n_{u,k}^i \) are two independent zero Gaussian noises coming from the reference and the \( i \)th bit subcarrier. For mathematical simplification we set the time chip equal to one \((T_c = 1)\).

Finally, based on equation (11), the decision variable may be approximated as

\[
D_{u,i} \approx \sum_{k=1}^{\beta} \lambda_{u,l} x_{u,k-\tau_l} s_{u,i} + n_{u,k}^i + n_{u,k} \quad (13)
\]

The \( i \)th bit of the \( u \)th data stream is decoded by comparing the output \( D_{u,i} \) to a threshold of zero.

In the decision variable given in equation (13), the first terms is the useful signal, whiles the second and third are zero-mean additive noise interferences. The output of the correlator for the MC-DCSK of equation (13) can be written in the form

\[
D_{u,i} = s_{u,i} E_b (u) \left( \frac{M-1}{M} \sum_{k=1}^{\beta} \lambda_{u,l} x_{u,k-\tau_l} \right) + W + Z \quad (14)
\]

Where \( E_b(u) \) is the transmitted bit energy for a given data sequence \( u \).

\[
W = \sum_{k=1}^{\beta} \lambda_{u,l} x_{u,k-\tau_l-1} \left( s_{u,i} n_{u,k} + n_{u,k}^i \right)
\]

\[
Z = \sum_{k=1}^{\beta} n_{u,k} n_{u,k}^i
\]

For a given \( i \)th bit of an \( u \)th data stream, the instantaneous mean and variance of the decision variable are derived as follows

\[
E\left(D_{u,i}\right) = s_{u,i} \frac{(M-1) \sum_{k=1}^{\beta} \lambda_{u,l}^2 E_b(u)}{M} \quad (15)
\]
Since the three terms of (14) are uncorrelated, the noise samples and channel coefficients are independent. The conditional variance of the decision variable for a given bit $i$ is

$$\text{Var}(D_{u,i}) = E \left( \frac{M-1}{M} \sum_{l=1}^{L} \lambda_{u,l}^2 E_b s_{u,i} \right)^2$$

$$+ E \left( \frac{\beta}{M} \sum_{k=1}^{L} \lambda_{u,k}^2 x_{u,k} n^i_{u,k} \right)^2$$

$$+ E \left( \frac{\beta}{M} \sum_{k=1}^{L} \lambda_{u,k}^2 x_{u,k} n_{u,k}^i \right)^2$$

$$- \left( \frac{M-1}{M} \sum_{l=1}^{L} \lambda_{u,l}^2 E_b s_{u,i} \right)^2$$

(16)

Finally after simplifications

$$\text{Var}(1) = \frac{(M-1)E_b^2}{M} \sum_{l=1}^{L} \lambda_{u,l}^2 N_0 \frac{2}{2} + \beta N_0^2$$

(17)

In order to compute the BER with our approach, the error probability must be evaluated first for a given received energy $E_b^2$ and channel coefficient $\lambda_{u,l}$.

Considering the bit energy (or chaotic chips) as a deterministic variable, the decision variable at the output of the correlator is necessarily a random Gaussian variable. Using equations (15) and (17), the bit error probability is

$$\text{BER} = \frac{1}{2} \text{Pr}(D_{u,i} < 0 | s_{u,i} = +1)$$

$$+ \frac{1}{2} \text{Pr}(D_{u,i} > 0 | s_{u,i} = -1)$$

$$= \frac{1}{2} \text{erfc} \left( \frac{E[D_{u,i}|s_{u,i} = +1]}{\sqrt{2 \text{Var}(D_{u,i})}} \right)$$

(18)

Where erfc(x) is the complementary error function defined by

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} \text{d}t$$

The BER expression for the MC-DCSK system is

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \frac{M N_0}{\beta E_b} + \frac{M^2 \beta N_0^2}{2(M-1)^2 \gamma_b^2} \right)^{1/2}$$

(19)

Many approaches have been considered for computing the BER of chaos-based communication systems, with the most widely used being the Gaussian approximation, which considers the transmitted bit energy $E_b^2$ as constant [Sushchik et al., 2000]. This assumption gives a good approximation of the performance for high spreading factors. Based on this fact, the overall BER expression of the MC-DCSK system can be simplified as

$$\text{BER} = \frac{1}{2} \text{erfc} \left( \frac{E_b}{(M-1)^2 \gamma_b} + \frac{M \beta}{2(M-1)^2 \gamma_b^2} \right)^{1/2}$$

(20)

Where $\gamma_b = \frac{E_b}{\sum_{l=1}^{L} \lambda_{u,l}^2 N_0}$

For high spreading factors the bit energy $E_b$ can be assumed to be constant [Kaddoum et al., 2009]. In this case, and for L independent and identically distributed (i.i.d) Rayleigh-fading channels, the PDF of the instantaneous $\gamma_b$ can be written as [Proakis, 2001]

$$f(\gamma_b) = \frac{(\gamma_b / \gamma_c)^{L-1}}{(L-1)! \gamma_c} \exp^{- \gamma_b / \gamma_c} = f(\gamma_b, \gamma_c, L)$$

(21)

Where $\gamma_c$ is the average SNR per channel defined as

$$\gamma_c = \frac{E_b}{N_0}$$

(22)

For dissimilar channels, the PDF of $\gamma_b$ can be written as [40]
\[ f(\gamma_b) = \frac{L}{l} \frac{\rho_l}{\gamma_l} \exp\left(\frac{-\gamma_b}{\gamma_l}\right) = \frac{L}{l} \frac{\rho_l}{\gamma_l} f(\gamma_b, \gamma_l) \] (23)

Where \( \rho_l = \prod_{j=1, j \neq l}^{L} \frac{\gamma_l}{\gamma_j} \) (24)

in which \( \gamma_l \) is the average value of \( \gamma_b = \frac{\lambda^2 E_b}{N_0} \), which is the instantaneous SNR on the lth channel.

Finally, the BER expression of the MC-DCSK system under multipath Rayleigh fading channel is

\[ BER = \frac{1}{2} \text{erfc} \left( \frac{M}{(M-1) \gamma_b} + \frac{M^2 \beta}{2(M-1) \gamma^2_b} \right)^{1/2} f(\gamma_b, \gamma_l) \] (25)

BER Calculations under AWGN Channel

In this section, the performance of the MC-DCSK under an AWGN channel will be evaluated for low and high spreading factors. The aim of this analysis is to highlight the non constant bit energy problem when the spreading factor is very low. In this case, one path is considered \( L = 1 \) within a channel coefficient equal to one \( \lambda = 1 \) and \( \gamma_b = \frac{E_b}{N_0} \).

For high spreading factors, the transmitted bit energy \( E_b \) can be considered constant. The BER expression of the MCDCSK system may then be approximated by

\[ BER = \frac{1}{2} \text{erfc} \left( \frac{MN_0}{(M-1)E_b} + \frac{M^2 N_0^2 \beta}{2(M-1)^2 E_b^2} \right)^{1/2} \] (26)

SIMULATIONS RESULTS

To evaluate the BER performance of MC-DCSK system over AWGN and multipath Rayleigh fading channels. The results obtained are for different numbers of subcarriers \( M \) and spreading factors \( \beta \). MC-DCSK system uses the square root raised cosine wave form with a roll-off factor \( \alpha \) equal to 0.25 is assumed.

Figure 5 shows that the effect of the number of subcarriers on the system performance under AWGN channel, we assumed the spreading factor is \( \beta = 5 \) and the bit duration \( T_b \), and then bandwidth \( B \) is wide enough to support any number of subcarriers \( M \). In fact, for a given spreading factor, when the number of subcarriers \( M \) increases, less reference energy is used to transmit one bit. In other words, the reference energy is shared among \( M-1 \) bits. This performance improvement, proven in the BER expression, means that for a high number of subcarriers \( M \), we need less energy to reach a given BER. We also show the performance improvement by simulation for \( M = 2 \) and \( M = 64 \), with a fixed spreading factor equal to \( \beta = 5 \). In the case of \( M = 2 \), the MC-DCSK system is equivalent to a DCSK system. We observe a degradation in performance between the MC-DCSK system for \( M = 64 \) and the coherent BPSK one. This degradation comes from the two noise sources added to the reference and data carrier signals.

The Figure 6 evaluates the effect of the value of the spreading factor on the performance of the MC-DCSK under an AWGN channel. The simulated bit error rate is plotted for different values of the spreading factor \( \beta \) with a fixed \( E_b/N_0 \) and a number of subcarriers \( M = 2 \).
CONCLUSION

The performance of DCSK the bit error rate expressions are derived for an AWGN and multipath Rayleigh fading channels. Simulation results match the theoretical BER expressions, justifying our approximations and demonstrating the accuracy of our approach. To compare the performance of the proposed system with that of the DCSK, the simulated BERs are plotted with the same spreading factor, where results show an increase in performance as compared to the conventional DCSK.

REFERENCES


