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NON - LINEAR OPTIMALITY MODELS WITH CONSTRAINTS

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ABSTRACT

The present work aims to decribe as simple as possible to evaluate several of the more effective methods of non linear and non smooth programming. If the function becomes non smooth with non linearty the solution techniques becomes more complicated. There is a rigorous development of first and second order optimality conditions. This work deals with the necessary and sufficient conditions for non linear and non smooth models (Ferris and Pang, 1995). Apart form the development of the theoritical foundations of non linear and non smooth programming. The emphasis will be careful and detailed presentation of a number of solution algorithms, that have shown themselves to be of continuing importance and practical utility. We also derive necessary and sufficient conditions which are applicable to some special type of non linear and non smooth problems.

KEY WORDS : Non linear optimal conditions, interpolation etc.

In this work we shall discuss necessary and sufficient optimality conditions. Optimality conditions are of two types :necessary conditions, which must hold at any minimizer for a model and sufficiency conditions, which is statisfied at a point, guarantee that the point is a minimizer (Baer, 1992 and Pazman, 2013). For the constrained case additional quantities are required. Conn, (1973) discussed about constrained optimization. We first develop the necessary conditions that require no constraint qualifications (Bertsekas 1982 and Cohen, 1972) suggested restriction which would be able to modify the development of such problems in which unconstrained local optima and their associated Lagrange multiplier values. The very Important Lagrange multipliers are generalised Lagrange function in the statement of the first order conditions (Evertt, 1963).

METHODOLOGY

Algebraic Derivation

Considerthe mathematical programming model including one or more equality constraints. We can reduce this model to one of the unconstrained optimization model by using the constraints to eliminate variables (Flether, 1987).Todevelop basic understanding of the method of Lagrange multiplier let us consider indetail the following model.

Max $f(x) = f(x_1, x_2, x_3)$

Subject to $g(x_1, x_2, x_3) = 0$ ……………. (1)

and $h(x_1, x_2, x_3) = 0$

The two constraints $g(x) = 0$, and $h(x) = 0$ Describe two surface in three dimensional Space.

Now consider the following mathematical model

Max $f(x)$ ………………… (2)

Subject to $g_i(x) = b_i$ $i = 1, 2, \ldots\ldots m$,

Where x is an n – component vactor and $m < n$.

Such that the jacobian matrix of first partial derivatives.

$$J = \begin{pmatrix} \frac{\delta g_1}{\delta x_1} & \cdots\cdots & \frac{\delta g_1}{\delta x_m} \\ \frac{\delta g_m}{\delta x_1} & \cdots\cdots & \frac{\delta g_m}{\delta x_m} \end{pmatrix}$$

Also since gradient vector ∇f is normal to the surface $f = k_{max}$ then ∇f must also be normal. The two gradient ∇g and ∇h are normal to the surface $g = 0$, and $h=0$ respectively Thus there must exist three real numbers α_1, α_2, $\alpha_3$not all zero (Mc Gree, 2008). Such that

$\alpha_1 \nabla f + \alpha_2 \nabla g + \alpha_3 \nabla h = 0$ at p

∇g and∇h must also intersect being normal

$\alpha_2 = \alpha_3 = 0$

$\alpha_2 \nabla g + \alpha_3 \nabla h = 0$

Hence α_1 can not be equal to zero and we may write

$\nabla f - \lambda_1 \nabla g - \lambda_2 \nabla h = h$ at p

Where $\lambda_1 = \frac{\alpha_2}{\alpha_1}$, and$\lambda_2 = \frac{\alpha_3}{\alpha_1}$

the $\lambda i$are said to be the Lagrange multipliers.

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INTER POLATION OF LAGRANGE MULTIPLIERS

Suppose that the global optimum of the general equality constrained mathematical problem

max f(x) : x = (x_1………x_n)

Subject to gi (x) = bi , i = 1, ………. m

Occurs at x* = (x_1* ……. x_n*) with associated Lagrange multipliers value

λ * = (λ_1* ………..λ_m*)

The partial derivatives of the global optimum Z* =f(x*) mustbe

$$\frac{\delta z^*}{\delta b_i} = \sum_{j=1}^{n} \frac{\delta \mathrm{f}}{\delta x_j^*} \cdot \frac{\delta x_j^*}{\delta b_i}$$

While the partial derivatives of the constraints $g_k(x^*) = b_k$ becomes

$$\frac{\delta g_k}{\delta b_i} = \sum_{j=1}^{n} \frac{\delta g_k}{\delta x_j^*} \cdot \frac{\delta g_j^*}{\delta b_i} = \begin{cases} 1 & \text{if } k = i \\ 0 & \text{if not} \end{cases}$$

For any specific i, this represents mequations multiplying each by the respective λ_k* , k = 1 …….. m and summing over k produces.

$$\sum_{k=1}^{m} \lambda_k * \sum_{j=1}^{n} \frac{\delta g_k}{\delta x_j^*} \cdot \frac{\delta x_j^*}{\delta b_i}$$

$$= \sum_{j=1}^{n} \frac{\delta x_j^*}{\delta b_i} \sum_{k=1}^{m} \lambda_k * \frac{\delta g_k}{\delta x_j^*} = \lambda_i *$$

The necersary condition which is known to hold at x*, can now be putting in above equations.

$$\sum_{j=1}^{n} \frac{\delta x_j^*}{\delta b_i} \cdot \frac{\delta \mathrm{f}}{\delta x_j^*} = \lambda_i * \quad \text{……………(3)}$$

ALGORITHM

The general inequality constrained mathematical program is.

Max f(x), x = (x_1 ……….x_n)

Subject to gi(x) ≥bi, i = 1, ……..m ………………(4)

The constraints may be converted to equations by the addition of non negative slack variables si, i = 1, …………. M Producing the following model

Max f(x)

Subject to g_i(x) + si = bi, i = 1, ……….m

And si ≥ 0 i = 1………m ………………… (5)

This suggests restriction that we would be able to modify the development of such problems in which unconstrained local optima and their associated Lagrange multipliers values were found to satisfy certain relationships with these expectation then we write the Lagrangian function.

$$f(x_i \mathrm{s}, \lambda) = f(x) - \sum_{i=1}^{\mathrm{m}} \lambda_i \; (\mathrm{g}_i(\mathrm{x}) + \mathrm{si} - \mathrm{bi})$$

At any constrained local maximum the partial derivatives of the x_i and λ_i should equal zero while for the slack variables, equations.

$$\frac{\delta f(x)}{\delta x_2} \leq 0 \text{ and } x_2 \frac{\delta f(x)}{\delta x_2} = 0 \text{ should hold}$$

Since one of the constrained local maxima is global, the following relationship must all be satisfied by the global maximum (x*, s*, λ*)

CONCLUSION

In this work first of all we introduced the basic concepts of Lagrange Multipliers andtheir algebraic derivation in detail. After discussion first order necessary conditions, we deal with another important necessary condition derivable when the functions are twice continuously differentiable (Iasc, 1997). Second derivative information is required to take into account the curvature of the problem functions. If the second derivative is strictly positive at a point. More over the gap between the sufficient conditions and their necessary counter parts is minimal (Pang, 1994). The optimality criteria for the special cases of non linear non smooth problems are easy corollaries of our general theorems.

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