ONLINE PARAMETER ESTIMATION OF PERMANENT MAGNET SYNCHRONOUS MOTOR USING ORTHOGONAL PROJECTION ALGORITHM

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ABSTRACT

This paper presents parameter estimation of permanent magnet synchronous motor (PMSM) using orthogonal projection algorithm (OPA). Knowing accurate values of motor parameters is of great concern due to high efficiency of this kind of motors. It is shown in this paper that all the parameters of the motor can be estimated in a limited range. Besides, this method is compared to conventional recursive least square (RLS) method where it is deduced that our method has higher convergence speed in respect to RLS method. These two methods are implemented in a sample PMSM with high dynamics with and without noise. Simulation results show effectiveness of the proposed method.

KEYWORDS : Nonlinear System, Orthogonal Projection Algorithm (Opa), Permanent Magnet Synchronous Motor (Pmsm), Linear Regression Form, Recursive Least Square (Rls)

Permanent magnet synchronous motor (PMSM) is widely used in control systems specifically in servo systems due to high controllability, high torque, negligible torque ripple and other good characteristics (Liu Mingji, 2004). Therefore, knowing the parameters of the system is very effective in controller design (Raja Ramakrishnan, 2009), (KHov Makara, 2009). Besides, knowing the parameters of the system can be used in fault detection of the machine (Samuel J. Underwood, 2010). Electrical parameters are prone to be changed due to external factors such as high temperature, mechanical vibrations, loading condition, long duration of the motor service and environmental factors. Using incorrect parameters leads to an increase in output torque ripple (Samuel J. Underwood, 2010, Flah Aymen, 2013) and affects considerably on the static and dynamic performance of the system (Gao Junli, 2010). Therefore, online estimation of the electrical parameters of these systems is extremely necessary (Flah Aymen, 2013). Different methods are investigated about online parameter estimation of PMSM in books and journals such as extended Kalman filter (Thierry Boileau, 2008, S. Wang, 2009), neural networks (S. Wang, 2009), adaptive algorithms (Thierry Boileau, 2011), reference model based methods (Thierry Boileau, 2008, Thierry Boileau, 2011) and recursive least square method (Samuel J. Underwood, 2010).

All these methods have pros and cons. Extended Kalman filter is a method with complex calculation, neural network has a learning phase (at first off-line and then online) and the last method needs too many sampling for converging to the actual parameters. Convergence of these methods is proved only by experiment and there is no theory for proving the convergence of these estimators (Thierry Boileau, 2011). Theses methods also do not have the ability to estimate all the parameters. In this paper, we have presented two estimation methods i.e. orthogonal projection algorithm (OPA) and recursive least square (RLS). Both methods are converged to actual parameters but OPA is converged quicker. The rate of updating parameters is very effective in control of system.

The rest of paper is organized as follow:

Section 1 describes OPA. In the next section, selecting the PMSM model is presented. In section 3, the proposed algorithm is implemented for parameter estimation on the form of elected model. Simulation results are illustrated in section 5 and finally paper is concluded in section 5.

OPA is an online estimator with high convergence speed. This method tries to produce an algebraic sub-space where unknown parameters' vector can be projected on it. When projection of a vector on a co-dimensional sub-space is certain and unique, that vector can be defined as a unique vector (H. Agahi, 2006). The limitation of this method is that the model has to be rewritten in the linear regression form in order to apply the abovementioned estimator on it (Hamed Agahi, 2007). Structure of the non-linear model for implementing the estimation of this kind is written in the linear regression form:

\[
y[t] = \theta^T \Phi[t-1] + w[t]
\]

(1)

Here, \(y[t]\) is output at instant t, \(\Phi\) linear function of non-linear functions vector, \(\theta\) vector of unknown parameters and \(w[t]\) a bounded noise including disturbance, measurement error, modelling error, error of not modelled dynamics and rounding error.

OPA is started for equation (1) with initial estimation from \(\theta_0\), and for vector of parameters and unit square covariance matrix with \(p_0\). Then the estimation with \(\theta_t\) and \(p_t\) is started for \(t>1\) in a recursive form (Hamed Agahi, 2007).

\[
\begin{align*}
\Phi_{t+1} P_{t+1} \Phi_{t+1}^T \neq 0 \\
\theta_t &= \theta_{t-1} + \frac{P_{t-1} \Phi_{t-1}^T}{\Phi_{t-1} P_{t-1} \Phi_{t-1}^T} \left[y[t] - \theta_{t-1}^T \Phi_{t-1}\right] \\
P_t &= P_{t-1} + \frac{P_{t-1} \Phi_{t-1} \Phi_{t-1}^T P_{t-1}}{\Phi_{t-1}^T P_{t-1} \Phi_{t-1}} \\
\end{align*}
\]

(2)

\[
\begin{align*}
\Phi_{t+1} P_{t+1} \Phi_{t+1}^T = 0 \\
\theta_t &= \theta_{t-1} \\
P_t &= P_{t-1}
\end{align*}
\]

(3)

This algorithm in noise-free condition has the ability to start from any initial estimation which needs \(N\) data (here \(N\) is number of unknown parameters) for determining the accurate value of the parameters. This is happened only if data could produce an independent vector in the estimation space that covers the estimation space with those unknown parameters (Hamed Agahi, 2007).
MODEL OF PMSM

The selected model for PMSM is a fourth rank model (A K. Parvathy, 2008). The reason for using this model is considering its high dynamics for accurate estimation of parameters.

\[
\dot{x} = Ax + Bu + f(x)
\]

\[
x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\delta \ \omega \ i_q \ i_d]^T
\]

\[
u = [v_1 \ v_2]^T = [v_q \ v_d]^T
\]

\[
A = 
\begin{bmatrix}
0 & 1 & 1.5p \lambda & 0 \\
0 & 0 & -\lambda & 0 \\
0 & 0 & 0 & -R/L_q \\
1.5p(L_d - L_q) & \lambda & -R/L_q & 0
\end{bmatrix}
\]

\[B = 
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 1/L_d \\
1/L_d
\end{bmatrix}
\]

\[
f = 
\begin{bmatrix}
0 \\
-\lambda p \omega i_d/L_q \\
L_p p \omega i_d/L_d \\
J
\end{bmatrix}
\]

From (R. Krishnan, 2010) we have:

\[i_q = i_m \sin(\delta) \quad (5)\]

\[i_d = i_m \cos(\delta) \quad (6)\]

Parameters of the model are \( R, L_q, L_d, \lambda, p, j \):

\(
\delta \quad \text{rotor position angle} \\
\omega \quad \text{angular velocity} \\
i_q \quad \text{direct-axis current in dq-framework} \\
i_d \quad \text{quadrature-axis current in dq-framework} \\
p \quad \text{nominal current} \\
p \quad \text{number of pole-pairs} \\
\lambda \quad \text{magnetic flux} \\
J \quad \text{inertia coefficient} \\
R \quad \text{stator resistance} \\
L_q \quad \text{q-axis inductance} \\
L_d \quad \text{d-axis inductance}
\)

The aim is to rewrite this model as linear regression form in order to implement our estimator on it. Model is changed into a \( T_s \) time sampling in discrete time form:

\[
\dot{x} \approx \frac{1}{T_s} [x(t-T_s) - x(t)]
\]

\[
x_{i}[k+1] = x_{i}[k] - T_s x_{i}[k] + T_s A_i x_{i}[k] + T_s A_2 \sin(\alpha) x_{i}[k] + T_s A_3 \cos(\alpha) x_{i}[k] + i_m \cos(\alpha)
\]

Here, \( A_1, \ldots, A_7 \) are factors of constants in (4) where are embedded in the parameters. In (8) model is rewritten in a form of \( x_i[k] \) (which is power angle) by substituting the time of equations and changing the variables. By considering \( T_s = 1 \)

Equation (8) is written in two forms because we do not face to non-linear equation in changing the factors to parameters.

Finally, complete content and organizational editing before formatting. Please take note of the following items when proofreading spelling and grammar:

A. first form

The second equation in (8) is written as follow:

\[
x_2 = [k+1] = A_1 x_1[k] + A_4 \sin(2\alpha)
\]

Now by substituting in equation (9), the third equation in (8) is transformed and by continuing this in the first equation of equation (8), we will reach to a kind of equations which is written as \( x_i[k] \) that is the power angle of PMSM.

\[
\psi[k + 3] - \psi[k + 2] = A_2 A_4 \sin(\alpha) + A_1 A_4 \sin(\alpha) x_1[k]
\]

\[
A_2 A_4 \sin(\alpha) + A_1 A_4 \sin(\alpha) x_1[k]
\]

Here we have \( \psi[k] = x_1[k] \), \( \alpha = x_1[k] \).

In the next section, equation (10) is changed into a complete linear regression.

\[
B. \text{second form}
\]

The second equation of (8) is written as follow:

\[
x_2[k+1] = A_1 \sin(\alpha) + A_2 \sin(\alpha) x_1[k]
\]

Now by substituting in equation (11), the fourth equation in (8) is transformed and by continuing this in the first equation of equation (8), we will reach to a kind of equations which is written as \( x_i[k] \) that is the power angle of PMSM.

\[
\gamma[k+3] - \gamma[k+2] = A_2 \sin(\alpha) x_1[k] + A_4 \sin(\alpha) \cos(\alpha)
\]

\[
A_2 \sin(\alpha) x_1[k] + A_4 \sin(\alpha) \cos(\alpha)
\]

Here we have \( \gamma[k] = x_1[k] \), \( \alpha = x_1[k] \).

Equation (12) like equation (10) is changed into a complete linear regression which is desirable. In the next section, a complete process for this will be discussed.

II. PARAMETER ESTIMATION OF PMSM

A. Parameter estimation of the first form

Equation (10) is written as:

\[
\psi[k+3] = \psi[k+2] + \Theta \Lambda[k]
\]

\[
\alpha = x_1[k]
\]
Here

\[
\Theta = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \end{bmatrix}^T
\]

\[
\Lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 \end{bmatrix}^T
\]

\[
\theta = [A_1 A_2 A_3 A_4 A_5]^T
\]

\[
\theta = [A_1 A_2 A_3 A_4 A_5]^T
\]

\[
\theta = [A_1 A_2 A_3 A_4 A_5]^T
\]

\[
\lambda = [\psi[k] \psi[k+1] \psi[k+2] \psi[k+3] \psi[k+4]]^T
\]

By selecting the initial covariance matrix \( P_{0[5*5]} \) and an initial estimation of parameters \( \Pi_{0[5*1]} \), estimation is started in \( k>1 \).

For parameter estimation of first and second forms, different algorithms can be applied. In this paper, OPA and conventional RLS are considered for comparison. In the next section, it is shown that OPA in noise-free situation has higher convergence comparing to its counterpart i.e. RLS method.

### III. SIMULATION

Simulation is performed in MATLAB software. Accidental signal (PRBS) is applied to the input of system and it is considered that power angle is known. Parameter estimation is done with and without the presence of noise in the system. After estimation of the factors, parameters are simply extracted from the factors by mathematical relations.

Characteristics of the under study system is as follow:

<table>
<thead>
<tr>
<th>Table 1: parameters for system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>2.875</td>
</tr>
</tbody>
</table>

A. Noise-free system \((W[t]=0)\)

Parameters of the system are estimated by two estimator i.e. OPA and RLS methods. As shown in the figures for physical parameter estimation of the considered system, we need less than 10 samples for OPA (Figure 1) but RLS have not converged with more than 200 samples until it was converged with 800 samples.

![Figure 1: Norm of parameter vector Error for OPA, \( W[t]=0 \)]
Figure 2: Norm of parameter vector Error for RLS-1, $W[t]=0$

![Figure 2](image2)

Figure 3: Norm of parameter vector Error for RLS-1, $W[t]=0$

![Figure 3](image3)

Table 2: compare parameters to estimations

<table>
<thead>
<tr>
<th>$R$</th>
<th>$l_q$</th>
<th>$l_d$</th>
<th>$\lambda_f$</th>
<th>$j$</th>
<th>$P$</th>
<th>S.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>2.875</td>
<td>9</td>
<td>0.175</td>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>OPA</td>
<td>2.875</td>
<td>8.9962</td>
<td>7.001</td>
<td>2</td>
<td>0.174</td>
<td>7.999</td>
</tr>
<tr>
<td>RLS</td>
<td>8.463</td>
<td>9</td>
<td>9.208</td>
<td>5</td>
<td>0.216</td>
<td>5.130</td>
</tr>
<tr>
<td>-1</td>
<td>2.876</td>
<td>8.9961</td>
<td>0.174</td>
<td>4</td>
<td>7.999</td>
<td>3.994</td>
</tr>
<tr>
<td>RLS</td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 compares the accuracy of the parameter estimation in the two methods. S.T in the eighth column is the number of sampling. In OPA, convergence to the actual parameters is obtained with 10 sampling but RLS have not converged to the actual parameters with more than 200 samples until it was converged with 800 samples. Simulation results showed that OPA is a high speed estimator in noise-free situation.

V. CONCLUSION

In this paper a new method for parameter estimation of PMSM is presented. OPA method can be applied for systems that are rewritten with linear and non-linear regression form. Space state is a fourth rank system. Then it is rewritten into a linear regression form based on power angle and it was considered that if power angle is accessible, then all the parameters of the PMSM can be specified. OPA is converged to actual parameters with limited samples. This method is proposed for systems with online specification and systems with high convergence speed. It was shown in the simulation that OPA has higher convergence speed in noise-free system. In a limited noise situation, it has agreeable results. However, by increasing noise in the system, estimation would be pulsating which is not desirable. It is first converged to actual parameters that we will employ it in the future. Therefore, the proposed estimator is proper for noise-free systems and systems with limited noise.

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