PAIR OF COPLANAR BARENBLATT CRACKS AT THE INTERFACE OF TWO BONDED DISSIMILAR MICROPOLAR ELASTIC HALF-PLANES

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ABSTRACT

The condition of finiteness of stresses at the end of a crack and smooth joining of the opposite sides of the crack were first proposed in hypothetical form by Khristianovich and proved on the basis of the principle of virtual displacement by Barenblatt.

KEYWORDS: Micropolar Poisson Ratio, Modulus of Rigidity, Classical Lame’s Constant

MATERIALS AND METHODS

Formulation of the Problem

In this chapter, we shall study the stress and displacement field in the vicinity of a pair of coplanar Barenblatt cracks located at the interface of two bonded dissimilar micropolar elastic half-planes. We consider a pair of coplanar Barenblatt cracks located at the interface of two bonded dissimilar micropolar elastic half-planes. We suppose that two half planes $y > 0$ and $y < 0$ be occupied by elastic constants $\mu_1$, $k_1$ and $\mu_2$, $k_2$ with $k_1 = 3 - 4\eta_i$ ($i = 1,2$) where $\eta_i$ denotes the Poison ratio of the two elastic materials and $\mu_i$ denotes the modulus of rigidity of two respective media.

Following Lowengrub and Sneddon (1969), we shall require that

$$u_y(a^+, 0^+) = u_y(a^+, 0^-) = 0$$
$$u_y(a^-, 0^+) = u_y(a^-, 0^-) = 0$$
$$u_y(b^+, 0^+) = u_y(b^+, 0^-) = 0$$
$$u_y(b^-, 0^+) = u_y(b^-, 0^-) = 0$$

where $y = (u_x, u_y, \phi)$. The component of stress, displacement and microrotation must satisfy the condition

$$\sigma_{yy}(x, 0^+) = 0(x^{-1})$$
$$\sigma_{xy}(x, 0^+) = 0(x^{-1}), \quad x \to \infty$$
$$m_{yy}(x, 0^+) = 0(x^{-1})$$

If we assume that the upper and lower surface of both cracks are subjected to prescribed pressures $p(x)$ and $q(x)$, then inside the crack following conditions are to be satisfied.

$$\sigma_{yy}(x, 0^+) = \sigma_{yy}(x, 0^-) = -p(x), -b \leq x \leq -a, a \leq x \leq b$$
$$\sigma_{xy}(x, 0^+) = \sigma_{xy}(x, 0^-) = -q(x), -b \leq x \leq -a, a \leq x \leq b$$
$$m_{yy}(x, 0^+) = m_{yy}(x, 0^-) = 0, -b \leq x \leq -a, a \leq x \leq b$$

where $p(x)$ and $q(x)$ are the internal pressure and shear applied to the faces of the crack. For the region of the interface not occupied by the crack, following continuity conditions must be satisfied.

$$u_x(x, 0^+) = u_x(x, 0^-), |x| < a, and |x| > b,$$
$$u_y(x, 0^+) = u_y(x, 0^-), |x| < a, and |x| > b,$$
$$\phi(x, 0^+) = \phi(x, 0^-), |x| < a, and |x| > b,$$
$$\sigma_{yy}(x, 0^+) = \sigma_{yy}(x, 0^-), |x| < a, and |x| > b,$$
$$\sigma_{xy}(x, 0^+) = \sigma_{xy}(x, 0^-), |x| < a, and |x| > b,$$
$$m_{yy}(x, 0^+) = m_{yy}(x, 0^-), |x| < a, and |x| > b.$$

Following Lowengrub and Sneddon (1969), we take the displacement field-

$$F_3 \left[ A_1 - p_1 \xi^{-1} B_1 + Q_1 y B_1 \right] e^{-\xi y} - L_1 \eta_1 c_1 e^{\eta_1 y}, y > 0$$
$$u_x(x, y) =$$
$$F_3 \left[ A_2 - p_2 \xi^{-1} B_2 + Q_2 y B_2 \right] e^{\xi y} - L_2 \eta_2 c_2 e^{\eta_2 y}, y < 0$$

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\[ F_c \left[ A_1 + Q_1 y B_1 \right] e^{-\gamma y} - L_1^2 \zeta c_1 e^{\eta_1 y}, \quad y > 0 \]
\[ u(x, y) = F_c \left[ A_2 + Q_2 B_2 \right] e^{\gamma y} + L_2^2 \zeta c_2 e^{\eta_2 y}, \quad y < 0 \] (2.11)

\[ \phi(x, y) = F_s \left[ B_1 e^{-\gamma y} + c_1 e^{-\eta_1 y} \right], \quad y > 0 \]
\[ F_s \left[ B_2 e^{\gamma y} + c_2 e^{-\eta_2 y} \right], \quad y < 0 \] (2.12)

where \( F_s \) and \( F_c \) are the Fourier sine and cosine transforms. We suppose that

\[ p_i = \frac{\lambda_i + 3 \mu_i}{\lambda_i + 2 \mu_i}, \quad q_i = \frac{\lambda_i + \mu_i}{\lambda_i + 2 \mu_i}, \quad L_i^2 = \frac{v_i}{2 \mu_i} \]

\[ \Gamma_1 = \frac{\mu_1}{2 v_2}, \quad \Gamma_2 = \frac{v_1}{v_2}, \quad \rho_i = 2 - Q_i \quad i = 1, 2 \]

Here \( \lambda_i \) and \( \mu_i \) are the classical Lame's constants and \( v_i \) is the micropolar moduli and \( Q_i \) is the micropolar poisson ratio. The micropolar moduli \( v_i \) and \( \mu_i \) have the dimensions of force and stress respectively. The internal characteristics length \( L_i \) of the medium given by

\[ L_i = \sqrt{\frac{v_i}{2 \mu_i}} \]

from equations (2.1) and (2.7) we see that \( \sigma_{yy}(x, 0 +) = \sigma_{yy}(x, 0 -) \) for all values of \( x \) and it is easily shown that this condition is equivalent to the single equation

\[ \xi A_2 + (1 - Q_2) B_2 + L_2^2 \zeta C_2 = \Gamma_1 \left\{ -\xi A_1 + (1 - Q_1) B_1 + L_1^2 \zeta \eta_1 C_1 \right\} \]

from equations (2.2) and (2.8) we see that \( \sigma_{xy}(x, 0 +) = \sigma_{xy}(x, 0 -) \) the boundary condition is equivalent to the single equation

\[ \xi A_2 + (1 - Q_2) B_2 + L_2^2 \zeta C_2 = \Gamma_1 \left\{ \xi A_1 - B_1 L_1^2 \zeta \eta_1 C_1 \right\} \]

Similarly from equation (2.3) and (2.9) we see that

\[ m_{\psi_2}(x, 0) = m_{\psi_2}(x, 0) \] for all values of \( x \) and it is easily shown that this condition is equivalent to the single equation

\[ \xi B_2 + \eta_2 C_2 = \Gamma_2 \left( \xi B_1 + B_1 \eta_1 C_1 \right) \]

Solving these equations for \( A_2, B_2, C_2 \) in terms of \( A_1, B_1, C_1 \) we find that

\[ A_2 = -\Gamma_1 \left[ 1 + 2 Q^{-1} \{ (1 - Q_2) \eta_2 + \eta_2 \xi^2 L_2^2 \} A_1 \right. \]
\[ + \xi^{-1} \left( \Gamma_1 (1 - Q_1) - Q^{-1} (1 - Q_2) \right) \left\{ \Gamma_1 L_2^2 \xi \eta_2 \eta_2 \right\} B_1 \]
\[ + \left. \left\{ \Gamma_1 \eta_2 L_2^2 - Q^{-1} (1 - Q_1) \right\} B_1 + \left\{ \Gamma_1 \eta_2 L_2^2 - Q^{-1} (1 - Q_1) \right\} C_1 \right\} \]

\[ B_2 = \xi^{-1} \left[ 2 \Gamma_1 \eta_2 \xi A_1 + \left\{ \Gamma_1 L_2^2 \xi \eta_2 \eta_2 \right\} B_1 + \left\{ \Gamma_1 L_2^2 \zeta \eta_2 \eta_2 \right\} C_1 \right\] \]

\[ C_2 = \xi^{-1} \left[ -L_2^2 \xi \eta_2 \eta_2 \right]\]

Now from equations (2.10 – 2.12) we see that the boundary conditions (2.4 – 2.6) are equivalent to the conditions

\[ F_s \left[ A_1 + A_2 \right] + \left\{ (2 - Q_2) B_2 - (2 - Q_1) B_1 \right\} + L_2^2 \xi \eta_2 C_2 \]
\[ - L_2^2 \zeta \eta_1 C_1 : x = 0 \] (2.13)

\[ F_s \left[ B_1 - B_2 + C_1 + C_2 : x \right] = 0 \]

Substituting the values of \( A_2, B_2, C_2 \) in the above equations (2.13) and applying the boundary conditions (2.1-2.3) we get.

\[ F_c \left[ -\xi A_1 + B_1 + L_1^2 \zeta C_1 : x \right] \]
\[ = \frac{p(x) - b \leq x \leq -a}{2 \mu_1} \]
\[ a \leq x \leq b \]

\[ F_s \left[ -\xi A_1 + (1 - Q_1) B_1 + L_1^2 \zeta C_1 : x \right] \]
\[ = \frac{p(x) - b \leq x \leq -a}{2 \mu_1} \]
\[ a \leq x \leq b \]
Putting the values of \( A \), \( B \), and \( C \) in terms of \( \Phi(\xi) \), \( \Psi(\xi) \), and \( X(\xi) \) through the equations:

\[
aA_1 = ((b_2c_3 - b_3c_2) c_2\phi(\xi) - (b_2c_3 - b_3c_2) c_1 + (b_1c_2 - b_2c_1) c_3) \Psi(\xi) + (b_1c_2 - b_2c_1) c_2 X(\xi) \]

\[
aB_1 = -(a_2c_3 - a_3c_2)c_2\phi(\xi) - (a_2c_3 - a_3c_2)c_1 + (a_1c_2 - a_3c_2)c_3) \Psi(\xi) + (a_1c_2 - a_3c_2)c_2 X(\xi) \]

\[
aC_1 = ac_i^{-1} \phi(\xi) - (a_1D_1 + b_1D_2) \]

Where:

\[
a_1A_1 + b_1B_1 + c_1C_1 = \phi(\xi) \]

\[
a_2A_1 + b_2B_1 + c_2C_1 = \Psi(\xi) \]

\[
a_3A_1 + b_3B_1 + c_3C_1 = X(\xi) \]

Putting the values of \( A_1 \), \( B_1 \), \( C_1 \) in the equations (2.14) which are further reduced to the following set of equations:

\[
a = (a_1c_2 - a_2c_1) (b_2c_3 - b_3c_2) - (a_2c_3 - a_3c_2) (b_1c_2 - b_2c_1) \]

\[
a_1 = 1 - \Gamma_1 + 2\Gamma_1Q^{-1}(\eta_2(2 - Q_2) - \xi^2L_2^2\eta_2 - \eta_2(1 - Q_2) + \xi^2L_2^2) \]

\[
b_1 = L_2^2\eta_2Q^{-1}\xi(\Gamma_2Q_2 + \Gamma_1(2 - Q_1) + \xi^{-1}\Gamma_1(1 - Q_1)) \]

\[
c_1 = \Gamma_1\eta_1L_2^2 - Q^{-1}\{1 - Q_2\}\{\eta_1\Gamma_1L_2^2(\eta_2 - \xi) - \Gamma_2\eta_2L_2^2(\eta_2 + \xi)\} + \eta_2L_2^2\{\Gamma_1\xi^2(\eta_1 + \xi)L_2^2 + \Gamma_2\eta_1Q_2\} + (2 - Q_2)Q^{-1}\{\eta_1\Gamma_2L_2^2(\eta_2 - \xi)\} \]

\[
d_1 = b_1c_3 - b_2c_2 \quad c_2\phi(\xi) - \{(b_2c_3 - b_3c_2)c_1 + (b_1c_2 - b_2c_1)c_3\} \psi(\xi) + (b_1c_2 - b_2c_1)c_2 X(\xi) \]

\[
b_2 = Q^{-1}\{1 - Q_2\}\{\Gamma_2\xi^2(\eta_2 - \xi) - \Gamma_1\xi^2\eta_2(2 - Q_1)\} + L_2^2\eta_2\{\Gamma_1\xi^2(\eta_1 + \xi)L_2^2 + \Gamma_2\eta_1Q_2\} + L_2^2\xi\{\Gamma_2Q_2\eta_1 + \Gamma_1\xi^2L_2^2(\eta_1 + \xi)\} + L_2^2\xi - \Gamma_1\eta_1L_2^2 \]

\[
a_2 = 1 + \Gamma_1 + 2Q^{-1}(\Gamma_1\eta_1(1 - Q_2) + \Gamma_1\eta_2\xi^2L_2^2 + \xi^3L_2^2) \]

\[
a_3 = -2\Gamma_1\xi Q^{-1}(\eta_2 + \xi) \]

\[
b_3 = 1 + Q^{-1}\{\Gamma_2Q_2\xi^2(2 - Q_1)\xi\Gamma_1 - \{\Gamma_2L_2^2\xi^2(\eta_2 - \xi) - \Gamma_1\eta_2(2 - Q_1)\}\} \]

\[
c_3 = 1 + Q^{-1}\{\Gamma_2Q_2\eta_1 + \Gamma_1\xi^2L_2^2(\eta_1 + \xi) - \xi\{\Gamma_2\eta_1L_2^2(\eta_2 - \xi) - \Gamma_1\eta_2(\eta_1 + \xi)\}\} \]

\[
D_1 = (b_2c_3 - b_3c_2) c_2\phi(\xi) - \{(b_2c_3 - b_3c_2)c_1 + (b_1c_2 - b_2c_1)c_3\} \psi(\xi) + (b_1c_2 - b_2c_1)c_2 X(\xi) \]
\[ D_2 = (a_1c_2 - a_2c_1) c_2 \phi(\xi) + \{(a_1c_2 - a_2c_1) c_3 + (a_2c_3 - a_3c_2) c_1\} \psi(\xi) \]

\[-(a_1c_2 - a_2c_1) c_2 X(\xi)\]

Putting the values of \( A_1, \ B_1, \ C_1 \) in the equations (2.14) which are further reduced to the following set of equations:

\[ F_c (a (\xi) \phi (\xi) + b(\xi) \Psi (\xi) + c(\xi) X(\xi) : x) = f_1 (x), \ a \leq x \leq b \]

\[ F_c (b(\xi) \phi (\xi) + c(\xi) \Psi (\xi) + a(\xi) X(\xi) : x) = f_2 (x), \ a \leq x \leq b \]

\[ F_c (c(\xi) \phi (\xi) + a(\xi) \Psi (\xi) + b(\xi) X(\xi) : x) = 0, \ a \leq x \leq b \]

(2.15)

Where

\[ a(\xi) = a^{-1} c_1 \{(a_2c_3 - a_3c_2) - \xi (b_2c_3 - b_3c_2)\} + L_1^2 \xi^2 c_1^{-1} \]

\[ b(\xi) = a^{-1} \{(a_2c_3 - a_3c_2) + \xi (b_2c_3 - b_3c_2)\} c_1 \]

\[ + a^{-1} \{(a_3c_1 - a_1c_3) + (b_3c_1 - b_1c_3)\} c_3 \]

\[ c(\xi) = a^{-1} \{(a_3c_1 - a_1c_3) - \xi (b_3c_1 - b_1c_3)\} c_2 \]

\[ f_1(x) = \frac{a \rho(x)}{2 \mu_1} \]

\[ f_2(x) = \frac{a \rho(x)}{2 \mu_1} \]

And

\[ F_c (\phi(\xi) : x) = 0, \ x > b \]

\[ F_c (\Psi(\xi) : x) = 0, \ x > b \] (2.16)

\[ F_c (X(\xi) : x) = 0, \ x > b \]

We proceed as in (5) and we define.

\[ F_c (\phi(\xi) : x) = \begin{cases} r_1(x), & a \leq x \leq b \\ 0, & 0 < x < a, \ x > b \end{cases} \]

\[ F_c (\Psi(\xi) : x) = \begin{cases} s_1(x), & a \leq x \leq b \\ 0, & 0 < x < a, \ x > b \end{cases} \]

(2.17)

\[ F_c (X(\xi) : x) = \begin{cases} w_1(x), & a \leq x \leq b \\ 0, & 0 < x < a, \ x > b \end{cases} \]

It is easily shown that if we make extensions \( r(u), \ s(u) \) and \( w(u) \) of \( r_1(u), s_1(u) \) and \( w_1(u) \) to \(-b \leq x \leq -a\) as follows:

\[ r(u) = \begin{cases} r_1(u), & a \leq u \leq b \\ r_1(-u), & -b \leq u \leq -a \end{cases} \]

\[ s(u) = \begin{cases} s_1(u), & a \leq u \leq b \\ s_1(-u), & -b \leq u \leq -a \end{cases} \]

\[ w(u) = \begin{cases} w_1(u), & a \leq u \leq b \\ w_1(-u), & -b \leq u \leq -a \end{cases} \]

then

\[ F_c (\phi(\xi) : x) = \frac{1}{\pi L} \int_{-u}^{u} \frac{r(u)}{x - u} \, du \]

\[ F_c (\Psi(\xi) : x) = \frac{1}{\pi L} \int_{-u}^{u} \frac{s(u)}{x - u} \, du \] (2.18)

\[ F_c (X(\xi) : x) = \frac{1}{\pi L} \int_{-u}^{u} \frac{w(u)}{x - u} \, du \]

where \( L = ((-b, \ -a) \ U \ (a, \ b)) \)

In like manner it is a simple matter to verify that

\[ s(x), \ a \leq x \leq b \]

\[ F_c (\phi(\xi) : x) = 0 \]
0, 0 < x < a, x > b

where \( s_i(x) = \int \frac{b}{x} s_i(u) \, du \),

\[
F_s(\Psi(\xi) : x) = 0
\]

0, 0 < x < a, x > b \quad (2.19)

where \( r_i(x) \) is defined \( \frac{x}{a} \), \( a < x < b \)

\[
F_c(X(\xi) : x) = \int \frac{w(u)}{L}\frac{w(u)}{u-x} \, du
\]

If we substitute (2.17), (2.18) in the equation (2.15), we see that \( r, s, w \) must be solutions to the set of singular integral equations.

\[
a(\xi) r(x) + b(\xi) s(x) + c(\xi) w(x) = f_1(x), \quad a < |x| < b
\]

\[
a(\xi) s(x) + b(\xi) \frac{w(u)}{L}\frac{w(u)}{u-x} \, du + c(\xi) s(x) = f_2(x), \quad a < |x| < b
\]

(2.20)

\[
a(\xi) w(x) + b(\xi) r(u) \frac{r(u)}{u-x} \, du + c(\xi) s(x) = 0, \quad a < |x| < b
\]

where \( f_1(x) \) and \( f_2(x) \) are even functions defined on \( L \). The substitution

\[
\lambda(u) = s(u) - i r(u) + w(u)
\]

reduce the pair of equations (2.20) to the single integral equation

\[
a(\xi) \frac{\lambda(x)}{\pi i} \frac{\lambda(u)}{L, x - u} \, du = f(x), \quad \forall x \in L \quad (2.21)
\]

where \( L = (-b, -a) U (a, b) \),

\( F(x) = i f_1(x) + f_2(x) \)

If we now define

\[
\Lambda(z) = \frac{1}{2\pi i} \int \frac{\lambda(u)}{L(u-z)} \, du
\]

then using plemelj formulae.

\[
\Lambda^+(x) - \Lambda^-(x) = \lambda(x), \quad \Lambda^+(x) + \Lambda^-(x) = \frac{1}{\pi i} \int \frac{\lambda(u)}{L(u-x)} \, du,
\]

shows that (2.22) is equivalent to the condition

\[
\Lambda^+(x) = -K \Lambda^-(x) - \{c(\xi) - b(\xi) + a(\xi)\}^{-1} f(x), \quad x \in L
\]

(2.23)

Where

\[
K = \frac{c(\xi) - b(\xi) - a(\xi)}{c(\xi) - b(\xi) + a(\xi)} > 0
\]

Thus, we must find a sectionally holomorphic function \( \Lambda(z) \), vanishing at infinity and satisfying the condition (2.23). The solution to this problem is well known (cf p. 450 (6) and is given by

\[
\Lambda(z) = -K \Lambda^-(z) + X(z), \quad z \in L
\]

(2.25)

The homogeneous Riemann problem is known to have a solution (p.450) (3) given by

\[
x(z) = [(z-a)(z+b)]^{1/3}. [(z+a)(z-b)]^{-i\omega-1/2}
\]

(2.26)

where

\[
\omega = \frac{1}{2\pi} \log \left\{ \frac{c(\xi) - b(\xi) - a(\xi)}{c(\xi) - b(\xi) + a(\xi)} \right\}
\]

In the case in which \( f \) is a polynomial

\[
\int \frac{f(t)}{L^{2}(t-z)} = \frac{\pi i}{2} \left[ c(\xi) - b(\xi) + a(\xi) \right] \left\{ \frac{f(z)}{X(t)} - L(z) \right\}
\]

(2.27)

Where

\[
L(z) = \frac{1}{2\pi i} \lim_{\epsilon \to 0} \int_{c(\xi)}^{b(\xi)} \frac{f(Re^{i\theta})R e^{i\theta} d\theta}{X(Re^{i\theta}) (Re^{i\theta} - z)}
\]

(2.28)

Hence (2.214) yields

\[
\Lambda(z) = \frac{1}{2b(\xi)} [f(z) - X(z) L(z)] + P(z) X(z)
\]

(2.29)
for $0 < x < a$ and $x > b$

$$\sigma_{yy}(x, 0^+) = \sigma_{yy}(x, 0^-) = -\frac{2\mu_1 b(z)}{f_0} I_m \Lambda^+(x)$$

$$\sigma_{yy}(x, 0^+) = \sigma_{yy}(x, 0^-) = -\frac{2\mu_1 b(z)}{f_0} \text{Re} \Lambda^+(x)$$

$$m_{by}(x, 0^+) = m_{by}(x, 0^-) = \frac{2\mu_1 b(z)}{f_0} I_m \Lambda^+(x)$$

**The Case of Constant Internal Pressure**

We now consider the case in which the cracks are opened by constant normal and shearing pressure say $p(x) = q(x) = P_0$ so that,

$$f(x) = \frac{a P_0}{2\mu_1} = f_0$$

Thus, if $\beta = \frac{1}{2} - i\omega$, then

$$2\pi \int_{x}^{(\text{Re} + i\omega)} \left( \left( \frac{z^2 + 2bz\beta - 2a\beta z - bz + az - (\beta - 1) \frac{zh^2 + 4a^2}{2} + (4\beta^2 + 4\beta - 1) \right) C(R') \right)$$

So that

$$L(z) = f_0 \left[ z^2 + (2\beta - 1)(b - a)z - \beta(\beta - 1) \frac{4a^2 + 5b^2}{2} + \cdots \right]$$

(3.1)

If follows from (2.29) that

$$\Lambda(z) = \frac{f_0}{2a(z)} \left[ 1 - \{z^2 + h_1 z + h_2\} X(z) \right]$$

where $X(z)$ is already defined by the equation (2.26) and $h_1, h_2$ are arbitrary complex constants.

We obtain relation (2.26), the expressions for $X^+$ and $X^-$ as follows.

$$X^+(x) = -iK^{1/2}(x^2 - a^2)(b^2 - x^2)^{-1/2} \cos\omega \theta + i \sin\omega \theta$$

$$X^-(x) = iK^{1/2}(x^2 - a^2)(b^2 - x^2)^{-1/2} \cos\omega \theta + i \sin\omega \theta$$

(3.3)

Where

$$\theta = \log \left\{ \frac{(x-a)(x+b)}{(x+a)(b-x)} \right\}, \text{ while}$$

(i) for $b < x < -a,$

$$X^+(x) = iK^{1/2}(x^2 - a^2)(b^2 - x^2)^{-1/2} \cos\omega \theta + i \sin\omega \theta$$

$$X^-(x) = -iK^{1/2}(x^2 - a^2)(b^2 - x^2)^{-1/2} \cos\omega \theta + i \sin\omega \theta$$

(3.4)

(ii) for $0 < x < a$

$$X^+(x) = -((a^2 - x^2)(b^2 - x^2))^{-1/2} (\cos\omega \theta_1 + i \sin\omega \theta_1)$$

(iii) for $x > b$

$$X^+(x) = X^-(x) = ((x^2 - a^2)(x^2 - b^2))^{-1/2} \cos^2 \omega \theta_2 + i \sin\omega \theta_2$$

(3.6)

Where

$$\theta_1 = \log \left\{ \frac{(x-a)(x+b)}{(x+a)(b-x)} \right\}$$

$$\theta_2 = \log \left\{ \frac{(x-a)(x+b)}{(x+a)(x-b)} \right\}$$

Hence for $a < x < b$, we find that if we suppose

$$h_1 = h_1^1 + i h_2^1, h_2 = h_1^2 + i h_2^2$$

then,

$$\Lambda^+(x) - \Lambda^-(x) = \frac{F_0}{\sqrt{(b^2 - a^2)}} \left( (x^2 - a^2)(b^2 - x^2) \right)^{-1/2} \left( (x^2 + h_1^1 x + h_2^1) \cos\omega \theta \right)$$

$$+ \left( i (h_2^1 x + h_2^2) \sin\omega \theta \right)$$

$$\cos\omega \theta + (x^2 + h_1^1 x + h_2^1) \sin\omega \theta$$

(3.7)

While

$$\Lambda^+(x) + \Lambda^-(x) = \frac{F_0 b(z)}{a(z) \sqrt{a(z)^2 - b(z)^2}} \left( (x^2 - a^2)(b^2 - x^2) \right)^{-1/2}$$

$$\left\{ (x^2 + h_1^1 x + h_2^1) \cos\omega \theta - (h_2^1 x + h_2^2) \sin\omega \theta \right\}$$

$$\cos\omega \theta + (x^2 + h_1^1 x + h_2^1) \sin\omega \theta$$

(3.8)
On \( a < x < b \), the Plemelj relation yield,

\[
s_1(x) = \frac{-f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x + h_2^2) \cos \omega \theta + (h_1^2 x + h_2^2) \sin \omega \theta \}
\]

(3.9)

\[
r_1(x) = \frac{-f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(h_1^2 x + h_2^2) \cos \omega \theta - (x^2 + h_1^2 x + h_2^2) \sin \omega \theta \}
\]

(3.10)

\[
w_1(x) = \frac{f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x + h_2^2) \cos \omega \theta + (h_1^2 x + h_2^2) \sin \omega \theta \}
\]

(3.11)

we may also note on \(-b < x < -a\)

\[
s(x) = \frac{f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x + h_2^2) \cos \omega \theta \}
\]

(3.12)

\[
r(x) = \frac{-f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x + h_2^2) \sin \omega \theta - (h_1^2 x + h_2^2) \cos \omega \theta \}
\]

(3.13)

\[
w(x) = \frac{-f_0}{\sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x + h_2^2) \cos \omega \theta + (h_1^2 x + h_2^2) \sin \omega \theta \}
\]

Hence, the relation \( s(x) = -s_1(-x) \), \( r(x) = r_1(-x) \)

and \( w(x) = w_1(-x) \) on \(-b < x < -a\) be satisfied, we must choose \( h_1^2 \) and \( h_2^2 \) so that \( h_1^2 = h_2^2 = 0 \).

Another use of the Plemelj formulae yield

\[
\frac{1}{L} \int_{x}^{L} \frac{r(u)}{a(u-x)} \, du = \frac{f_0 b(\xi)}{a(\xi) \sqrt{a(\xi^2-b(\xi)^2)}} ((x^2 - a^2)(b^2 - x^2))^{-\frac{1}{2}} \{(x^2 + h_1^2 x) \cos \omega \theta - h_1^2 x \sin \omega \theta \}
\]

(3.14)

We can deduce from the equation (3.12) and (3.9) that

\[
\frac{1}{L} \int_{x}^{L} \frac{r(u)}{a(u-x)} \, du = -\frac{b(\xi)}{a(\xi)} s_1(x), \quad a < x < b
\]

(3.15)

It is simple matter to show that for \( x > b \),

\[
\Lambda^+(x) = \frac{f_0}{2 a(\xi)} - \frac{f_0}{2 a(\xi)} ((x^2 - a^2)(x^2 - b^2))^{-\frac{1}{2}} \{(x^2 + h_1^2) \cos \omega \theta_2 \sin \omega \theta_2 - (x^2 + h_2^2) \sin \omega \theta_2 + h_1^2 x \cos \omega \theta_2 \}
\]

(3.16)

and hence, for \( x > b \)

\[
\sigma_{yy}(x, 0 +) = -P_0 (1 + i) (1 - \{(x^2 - a^2)(x^2 - b^2)\})^{-\frac{1}{2}} \{(x^2 + h_1^2) \cos \omega \theta_2 - h_1^2 x \sin \omega \theta_2 \}
\]

(3.17)

\[
\sigma_{xy}(x, 0 +) = -P_0 (1 + i) ((x^2 - a^2)(x^2 - b^2))^{-\frac{1}{2}} \{(x^2 + h_1^2) \sin \omega \theta_2 + h_1^2 x \cos \omega \theta_2 \}
\]

(3.18)

\[
\mu_{xy}(x, 0 +) = -P_0 (1 + i) (1 - \{(x^2 - a^2)(x^2 - b^2)\})^{-\frac{1}{2}} \{(x^2 + h_1^2) \sin \omega \theta_2 + h_1^2 x \cos \omega \theta_2 \}
\]

(3.19)

we see from (3.17), (3.18) and (3.19) that as \( x \to \infty \)

\[
\sigma_{yy}(x, 0 +) = -P_0 (1 + i) 0 (x^{-1})
\]

(3.20)

Hence, it follows that the condition \( \sigma_{yy}(x, 0 +) = 0 (x^{-1}) \) as \( x \to \infty \) is automatically satisfied while that \( \sigma_{xy}(x, 0 +) = 0 (x^{-1}) \) as \( x \to \infty \) will only be satisfied if we choose \( h_1^2 = -2 \omega (b - a) \). Thus it only remains to determine the constant \( h_2^2 \). This
is determined from the condition that at the end point \( x = a \),
\[ u_x (0, 0+) = 0 = u_y (0, 0-). \]
From (2.10), (2.19), (3.9) and (3.15) we see that
\[ u_x (x, 0+) = s_{1(x)} \frac{c}{a(x)} a^2, \quad a < x < b \]
\[ u_y (x, 0-) = s_{1(x)D} \frac{1}{a(x)} a^2, \quad a < x < b \]
where \( C \) and \( D \) can be calculated.

Therefore, we have condition
\[ s_1(a) = \int_a^b s_1(u) \, du = 0 \quad (3.22) \]
This gives
\[ h_1 = -\frac{I_2 + 2\omega(b - a) I_1}{I_0} \quad (3.23) \]
Where
\[ I_0 = \int_a^b \frac{\cos \omega\theta \, du}{\sqrt{(u^2 - a^2)(b^2 - u^2)}} = \frac{\pi a}{(a + b) \cosh \pi \omega} \]

\[ I_1 = \int_a^b \frac{u \sin \omega\theta \, du}{\sqrt{(u^2 - a^2)(b^2 - u^2)}} = \frac{\pi a^2}{(a + b) \cosh \pi \omega} \]

\[ I_2 = \int_a^b \frac{u^2 \cos \omega\theta \, du}{\sqrt{(u^2 - a^2)(b^2 - u^2)}} = \frac{n a^2}{(a + b) \cosh \pi \omega} \]

\[ + \frac{\pi (b - a)^2}{2 \cosh \pi \omega} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \frac{(5/2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1, 2, z - z)}{} \]

For \( x > b \) stresses are given by
\[ \sigma_{xy} (x, 0+) = \sigma_{yy} (x, 0-) = -P_0 (1 + i) \left( 1 - \{(x^2 - a^2)(x^2 - b^2)^{1/2} \right) \{(x^2 + h_2^2) \cos \omega \theta - 2\omega(b - a) x \sin \omega \theta \} \quad (3.24) \]

\[ m_{xy} (x, 0+) = m_{yy} (x, 0-) = P_0 (1 + i) \left( -1 + \{(x^2 - a^2)(x^2 - b^2)^{1/2} \right) \{(x^2 + h_2^2) \cos \omega \theta - 2\omega(b - a) x \sin \omega \theta \} \quad (3.25) \]

The stresses for \( 0 < x < a \) are given by
\[ \sigma_{yy} (x, 0+) = \sigma_{yy} (x, 0-) = P_0 (1 + i) \left( 1 + \{(a^2 - x^2)(b^2 - x^2)^{1/2} \right) \{(x^2 + h_2^2) \cos \omega \theta - 2\omega(b - a) x \sin \omega \theta \} \quad (3.26) \]

\[ \sigma_{xy} (x, 0+) = \sigma_{xy} (x, 0-) = P_0 (1 + i) \left( 1 + \{(a^2 - x^2)(b^2 - x^2)^{1/2} \right) \{(x^2 + h_2^2) \sin \omega \theta - 2\omega(b - a) x \cos \omega \theta \} \quad (3.27) \]

\[ m_{xy} (x, 0+) = m_{yy} (x, 0-) = P_0 (1 + i) \left( -1 + \{(a^2 - x^2)(b^2 - x^2)^{1/2} \right) \{(x^2 + h_2^2) \cos \omega \theta - 2\omega(b - a) x \sin \omega \theta \} \quad (3.28) \]

where
\[ \theta_1 = \log \left( \frac{(a - x)(b - x)}{(a + x)(b + x)} \right) \]

\[ h_2 \] can be calculated from (3.22)

**The Stress Intensity Factors**

If \( N_{1b}, N_{2b} \) and \( N_{3b} \) are the normal and shear stress intensity factors at the crack tip \( x = b \) then
\[ N_{1b} = \lim_{\omega \to 0} [(x - b)^{1/2} \sigma_{yy} (x, 0+) \] (4.1)
\[ N_{2b} = \lim_{\omega \to 0} [(x - b)^{1/2} \sigma_{xy} (x, 0+) \] (4.2)

and \( F_3 \) is hypergeometric function of two variables defined in (112, p 274).
\[ N_{3b} = \lim_{x \to -} [(x - b)^{1/2}m_0(x, 0 +)] \quad (4.3) \]

then from the equations (3.23), (3.24) and (3.25), we get

\[ N_{1b} + N_{2b} + N_{3b} = \frac{p_0(1 + i)}{2b(b^2 - a^2)} \left\{ 4ob^2(b-a)^2 + (b^2 + h_1^2) \right\} \quad (4.4) \]

Similarly, the normal and shear stress intensity factor at the crack tip \( x = a \) are given by

\[ N_{1a} + N_{2a} + N_{3a} = \frac{p_0(1 + i)}{2b(a^2 - b^2)} \left\{ 4oa^2(b-a)^2 + (a^2 + h_1^2) \right\} \quad (4.5) \]

**RESULTS AND DISCUSSION**

In this paper, we consider a pair of coplanar Barenblatt cracks at the interface of the two bonded dissimilar micropolar elastic half planes. The components of stress and displacement have been calculated. The problem is reduced to the system of simultaneous dual integral equations which are further transformed to a Riemann boundary value problem. Calculations for evaluating the stress intensity factors at the crack tip are derived.

**ACKNOWLEDGEMENT**

I would like to acknowledge the help received from Dr. P.K. Tripathi, the department of Mathematics, CSJMU Kanpur.

**REFERENCES**


