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PAIR OF COPLANAR BARENBLATT CRACKS AT THE INTERFACE OF TWO BONDED DISSIMILAR MICROPOLAR ELASTIC HALF-PLANES

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ABSTRACT

The condition of finiteness of stresses at the end of a crack and smooth joining of the opposite sides of the crack were first proposed in hypothetical form by Khirstianovich and proved on the basis of the principle of virtual displacement by Barenblatt.

KEYWORDS: Micropolar Poisson Ratio, Modulus of Rigidity, Classical Lame's Constant

Recent examples of such studies are proved by Sneddon (1969), Burnisten and Gurely (1973), Tresher and Smith (1973). In all these investigations however, the crack is embedded in a homogeneous medium.

MATERIALS AND METHODS

Formulation of the Problem

In this chapter, we shall study the stress and displacement field in the vicinity of a pair of coplanar Barenblatt cracks located at the interface of two bonded dissimilar micropolar elastic half planes. We consider a pair of coplanar Barenblatt cracks $a \le |x| \le b$, $y = 0$ located at the interface of two bonded dissimilar micropolar elastic half planes. We suppose that two half planes $y > 0$ and $y < 0$ be occupied by elastic constants μ_1, k_1 and μ_2, k_2 with $k_i = 3 - 4\eta_i$ $(i = 1,2)$ where η_i denotes the Poison ratio of the two elastic materials and μ_i denotes the modulus of rigidity of two respective media.

Following Lowengrub and Sneddon (1969), we shall require that

$$u_y(a^+, 0\ +) = u_y(a^+, 0-) = 0$$

$$u_y(a^-, 0\ +) = u_y(a^-, 0-) = 0$$

$$u_y(b^+, 0\ +) = u_y(b^+, 0-) = 0$$

$$u_y(b^-, 0\ +) = u_y(b^-, 0-) = 0$$

where $y = (u_x, u_y, \phi)$. The component of stress, displacement and microrotation must satisfy the condition

$$\sigma_{yy}(x, 0\ +) = 0(x^{-1})$$

$$\sigma_{xy}(x, 0\ +) = 0(x^{-1}),\ x \to \infty$$

$$m_{\phi y}(x, 0\ +) = 0(x^{-1})$$

If we assume that the upper and lower surface of both cracks are subjected to prescribed pressures p(x) and q(x), then inside the crack following conditions are to be satisfied.

$$\sigma_{yy}(x, 0\ +) = \sigma_{yy}(x, 0\ -) = -p(x), -b \le x \le -a, a \le x \le b \tag{2.1}$$

$$\sigma_{xy}(x, 0\ +) = \sigma_{xy}(x, 0\ -) = -q(x), -b \le x \le -a, a \le x \le b \tag{2.2}$$

$$m_{\phi y}(x, 0\ +) = m_{\phi y}(x, 0\ -) = 0, -b \le x \le -a, a \le x \le b \tag{2.3}$$

where p(x) and q(x) are the internal pressure and shear applied to the faces of the crack. For the region of the interface not occupied by the crack, following continuity conditions must be satisfied.

$$u_x(x, 0+) = u_x(x, 0-), |x| < a, and\ |x| > b, \tag{2.4}$$

$$u_y(x, 0+) = u_y(x, 0-), |x| < a, and\ |x| > b, \tag{2.5}$$

$$\phi(x, 0+) = \phi(x, 0-), |x| < a, and\ |x| > b, \tag{2.6}$$

$$\sigma_{yy}(x, 0+) = \sigma_{yy}(x, 0-), |x| < a, and\ |x| > b, \tag{2.7}$$

$$\sigma_{xy}(x, 0+) = \sigma_{xy}(x, 0-), |x| < a, and\ |x| > b, \tag{2.8}$$

$$m_{\phi y}(x, 0\ +) = m_{\phi y}(x, 0\ -), |x| < a, and\ |x| > b, \tag{2.9}$$

Following Lowengrub and Sneddon (1969), we take the displacement field-

$$u_x(x,y) = \begin{cases} F_s\left[A_1 - p_1\xi^{-1}B_1 + Q_1 y B_1\right]e^{-\xi y} - L_1^2\eta_1 c_1 e^{\eta_1 y}, & y > 0 \\ F_s\left[A_2 - p_2\xi^{-1}B_2 + Q_2 y B_2\right]e^{\xi y} - L_2^2\eta_2 c_2 e^{\eta_2 y}, & y < 0 \end{cases}$$
(2.10)

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$$u_y(x,y)=\begin{cases} F_c\left[[A_1+Q_1yB_1]e^{-\xi y}-L_1^2\xi c_1 e^{\eta_1 y}\right], & y>0 \\ F_c[A_2+Q_2B_2]e^{\xi y}+L_2^2\xi c_2 e^{\eta_2 y}, & y<0 \end{cases} \quad (2.11)$$

$$\phi(x,y)=\begin{cases} F_s\left[[B_1e^{-\xi y}+c_1e^{-\eta_1 y}]\right], & y>0 \\ F_s[B_2e^{\xi y}+c_2e^{-\eta_2 y}], & y<0 \end{cases} \quad (2.12)$$

where F_s and F_c are the Fourier sine and consine transforms. We suppose that

$P_i=\frac{\lambda_i+3\mu_i}{\lambda_i+2\mu_i},\ Q_i=\frac{\lambda_i+\mu_i}{\lambda_i+2\mu_i}, L_i^2=\frac{\nu_i}{2\mu_i}$

$\Gamma_1=\frac{\mu_1}{2\mu_2},\ \Gamma_2=\frac{\nu_1}{\nu_2},\ p_i=2-Q_i\ \ i=1,2$

Here λ_i and μ_i are the classical Lames' constants and ν_i is the micropolar modulii and Q_i is the micropolar poisson ration. The micropolar modulii ν_i and μ_i have the dimensions of force and stress respectively. The internal characteristics length L_i of the medium given by

$$L_i=\sqrt{\frac{\nu_i}{2\mu_i}}$$

from equations (2.1) and (2.7) we see that $\sigma_{yy}(x,0+)=\sigma_{yy}(x,0-)$ for all values of x and it is easily shown that this condition is equivalent to the single equation

$$\xi A_2+(1-Q_2)B_2+L_2^2\xi C_2 = \Gamma_1\{-\xi A_1+(1-Q_1)B_1+L_1^2\xi\eta_1C_1\}$$

from equations (2.2) and (2.8) we see that $\sigma_{xy}(x,0+)=\sigma_{xy}(x,0-)$ the boundary condition is equivalent to the single equation

$$\xi A_2+(1-Q_2)B_2+L_2^2\xi^2C_2=\Gamma_1\{\xi A_1-B_1L_1^2\xi\eta_1C_1\}$$

Similarly from equation (2.3) and (2.9) we see that $m_{\phi y}(x,0+)=m_{\phi y}(x,0-)$ for all values of x and it is easily shown that this condition is equivalent to the single equation

$$\xi B_2+\eta_2C_2=\Gamma_2(\xi B_1+B_1\eta_1C_1)$$

Solving these equations for A_2, B_2, C_2 in terms of A_1, B_1, C_1 we find that

$A_2=-\Gamma_1[1+2Q^{-1}\{(1-Q_2)\eta_2+\eta_2\xi^2L_2^2\}]\,A_1$
$+\ \xi^{-1}(\Gamma_1(1-Q_1)-Q^{-1}(1-Q_2)\{\Gamma_2L_2^2\xi^2(\eta_2-\xi)-\Gamma_1\eta_2(2-Q_2)\}$ $B_2=Q^{-1}(2\Gamma_1\eta_2\xi A_1+\{\Gamma_2L_2^2\xi^2(\eta_2-\xi)-\Gamma_1\eta_2(2-Q_1)\}B_1+[\Gamma_1\eta_1L_1^2-Q^{-1}((1-Q_1)\{\Gamma_1\eta_1L_2^2(\eta_2-\xi)-\Gamma_2\eta_2L_1^2(\eta_1+\xi)+\eta_2L_2^2\{\Gamma_1\xi^2(\eta_1+\xi)L_1^2+\Gamma_2\eta_1Q_2\}\})]C_1$

$B_2=\ Q^{-1}[2\Gamma_1\eta_2\xi A_1+\{\Gamma_2L_2^2\xi^2(\eta_2-\xi)-\Gamma_1\eta_2(2-Q_1)\}B_1+\{\eta_1L_2^2\Gamma_2\xi(\eta_2-\xi)-\Gamma_1\eta_2L_1^2\xi(\eta_2+\xi)\}C_1]$

$C_2=\ Q^{-1}[-2\xi^2\Gamma_1A_1+\{\Gamma_2Q_2\xi+(2-Q_1)\xi\Gamma_1\}B_1+\{\Gamma_2Q_2\eta_1+\Gamma_1L_1^2\xi^2(\eta_1+\xi)\}C_1]$

$Q=L_2^2\xi^2(\eta_2-\xi)+Q_2\eta_2$

Now from equations (2.10 – 2.12) we see that the boundary conditions (2.4 – 2.6) are equivalent to the conditions

$$F_s[\xi(A_1+A_2)+\{(2-Q_2)B_2-(2-Q_1)B_1\}+L_2^2\xi\eta_2C_2 - L_1^2\xi\eta_1C_1 : x]=0$$

$$F_c[A_1-A_2-\xi(L_1^2C_1+L_2^2C_2):x]=0 \quad (2.13)$$

$$F_s[B_1-B_2+C_1+C_2:x]=0$$

Substituting the values of A_2, B_2, C_2 in the above equations (2.13) and applying the boundary conditions (2.1-2.3) we get.

$$F_c[-\xi A_1+B_1+L_1^2\xi^2C_1:x]=\frac{p(x)}{2\mu_1}, \quad \begin{matrix}-b\le x\le -a\\ a\le x\le b\end{matrix}$$

$$F_s[-\xi A_1+(1-Q_1)B_1+L_1^2\xi^2C_1:x]=\frac{p(x)}{2\mu_1}, \quad \begin{matrix}-b\le x\le -a\\ a\le x\le b\end{matrix}$$

(2.14)

$$F_c\left[B_1\xi + \eta_1 C_1 : x\right] = 0 \quad \begin{matrix} -b \le x \le -a \\ a \le x \le b \end{matrix}$$

If we now express A_1, B_1 and C_1 in terms of $\phi(\xi)$, $\Psi(\xi)$ and $X(\xi)$ through the equations

$$aA_1 = ((b_2c_3 - b_3c_2)\, c_2\phi(\xi) - \{(b_2c_3 - b_3c_2)\, c_1 + (b_1c_2 - b_2c_1)\, c_3\}\, \Psi(\xi) + (b_1c_2 - b_2c_1)\, c_2\, X(\xi))$$

$$aB_1 = -((a_2c_3 - a_3c_2)c_2\phi(\xi) - \{(a_1c_2 - a_2c_1)\, c_3 + (a_2c_3 - a_3c_2)\, c_1\}\, \Psi(\xi) + (a_1c_2 - a_2c_1)\, c_2\, X(\xi))$$

$$aC_1 = a c_1^{-1}\phi(\xi) - (a_1D_1 + b_1D_2)$$

Where

$$a_1A_1 + b_1B_1 + c_1C_1 = \phi(\xi)$$

$$a_2A_1 + b_2B_1 + c_2C_1 = \Psi(\xi)$$

$$a_3A_1 + b_3B_1 + c_3C_1 = X(\xi)$$

Putting the values of A_1, B_1, C_1in the equations (2.14) which are further reduced to the following set of equations:

$$a = (a_1c_2 - a_2c_1)(b_2c_3 - b_3c_2) - (a_2c_3 - a_3c_2)(b_1c_2 - b_2c_1)$$

$$a_1 = 1 - \Gamma_1 + 2\Gamma_1 Q^{-1}(\eta_2(2 - Q_2) - \xi^2 L_2^2 \eta_2 - \eta_2\{1 - Q_2\} + \xi^2 L_2^2)$$

$$b_1 = L_2^2\eta_2 Q^{-1}\xi\{\Gamma_2 Q_2 + \Gamma_1(2 - Q_1)\} + \xi^{-1}\Gamma_1(1 - Q_1)$$

$$\begin{aligned} -Q^{-1}\big[&(1 - Q_2)\{\Gamma_2 L_2^2 \xi^2(\eta_2 - \xi) - \Gamma_1\eta_2(2 - Q_1)\} \\ &+ \xi^2 L_2^2 \eta_2\{\Gamma_1(2 - Q_1) + \Gamma_2 Q_2\}\big] \\ &+ (2 - Q_2)\xi^{-1}Q^{-1}\{\Gamma_2 L_2^2\xi^2(\eta_2 - \xi) \\ &- \Gamma_1\eta_2(2 - Q_1)\} - \xi^{-1}(2 - Q_1) \end{aligned}$$

$$\begin{aligned} c_1 = \Gamma_1\eta_1 L_1^2 - Q^{-1}\big[&(1 - Q_2)\{\eta_1\Gamma_1 L_2^2(\eta_2 - \xi) \\ &- \Gamma_2\eta_2 L_1^2(\eta_2 + \xi)\} \\ &+ \eta_2 L_2^2\{\Gamma_1\xi^2(\eta_1 + \xi)L_1^2 + \Gamma_2\eta_1 Q_2\}\big] \\ &+ (2 - Q_2)Q^{-1}\{\eta_1\Gamma_2 L_2^2(\eta_2 - \xi)\} \end{aligned}$$

$$\begin{aligned} -\Gamma_1\eta_1 L_1^2\xi(\eta_1 + \xi)) \\ &+ L_2^2\eta_2 Q^{-1}\{\Gamma_2 Q_2\eta_1 + \Gamma_1\xi^2(\eta_1 + \xi) \\ &- L_1^2\eta_1\} \end{aligned}$$

$$a_2 = 1 + \Gamma_1 + 2Q^{-1}(\Gamma_1\eta_2(1 - Q_2) + \Gamma_1\eta_2\xi^2 L_2^2 + \xi^3 L_2^2$$

$$a_3 = -2\Gamma_1\xi Q^{-1}(\eta_2 + \xi)$$

$$\begin{aligned} b_2 = Q^{-1}\big[&(1 - Q_2)\{\Gamma_2\Gamma_2^2\xi(\eta_2 - \xi) - \Gamma_1\xi^{-1}\eta_2(2 - Q_1)\} \\ &+ L_2^2\eta_2\{\Gamma_1\xi(2 - Q_1) + \Gamma_2\xi Q_2\} \\ &+ L_2^2\xi^2\{\Gamma_2 Q_2 + (2 - Q_1)\Gamma_1\}\big] \\ &- \xi^{-1}\Gamma_1(1 - Q_1) \end{aligned}$$

$$\begin{aligned} c_2 = Q^{-1}\big[&(1 - Q_2)\{\Gamma_1\eta_1 L_2^2(\eta_2 - \xi) - \Gamma_2\eta_2 L_1^2(\eta_1 + \xi) \\ &+ \eta_2 L_2^2\}\{\Gamma_1\xi^2(\eta_1 + \xi)L_1^2 + \Gamma_2\eta_1 Q_2\} \\ &+ L_2^2\xi\{\Gamma_2 Q_2\eta_1 + \Gamma_1 L_1^2\xi^2(\eta_1 + \xi)\} \\ &+ L_1^2\xi - \Gamma_1\eta_1 L_1^2\big] \end{aligned}$$

$$a_3 = -2\Gamma_1\xi Q^{-1}(\eta_2 + \xi)$$

$$\begin{aligned} b_3 = 1 + Q^{-1}\big[&\Gamma_2 Q_2\xi(2 - Q_1)\xi\Gamma_1 \\ &- \{\Gamma_2 L_2^2\xi^2(\eta_2 - \xi) - \Gamma_1\eta_2(2 - Q_1)\}\big] \end{aligned}$$

$$\begin{aligned} c_3 = 1 + Q^{-1}\big[&\Gamma_2 Q_2\eta_1 + \Gamma_1 L_1^2\xi^2(\eta_1 + \xi) \\ &- \xi\{\Gamma_2\eta_1 L_2^2(\eta_2 - \xi) - \Gamma_1\eta_2(\eta_1 + \xi)\}\big] \end{aligned}$$

$$D_1 = (b_2c_3 - b_3c_2)\, c_2\phi(\xi) - \{(b_2c_3 - b_3c_2)c_1 + (b_1c_2 - b_2c_1)\, c_3\}\psi(\xi) + (b_1c_2 - b_2c_1)\, c_2\, X(\xi)\}$$

$D_2 = (a_3c_2 - a_2c_3)\, c_2\phi(\xi) + \{(a_1c_2 - a_2c_1)\, c_3 + (a_2c_3 - a_3c_2)\, c_1\}\psi(\xi)$

$-(a_1c_2 - a_2c_1)\, c_2\, X(\xi)$

Putting the values of A_1, B_1, C_1 in the equations (2.14) which are further reduced to the following set of equations:

$$F_c\,(a(\xi)\,\phi(\xi) + b(\xi)\,\Psi(\xi) + c(\xi)\,X(\xi) : x) = f_1(x),\ \ a \le x \le b$$

$$F_s\,(b(\xi)\,\phi(\xi) + c(\xi)\,\Psi(\xi) + a(\xi)\,X(\xi) : x) = f_2(x),\ \ a \le x \le b$$

$$F_c\,(c(\xi)\,\phi(\xi) + a(\xi)\,\Psi(\xi) + b(\xi)\,X(\xi) : x) = 0,\ \ a \le x \le b \qquad (2.15)$$

Where

$$a(\xi) = a^{-1}\, c_2\{(a_2c_3 - a_3c_2) - \xi(b_2c_3 - b_3c_2)\} + L_1^2\xi^2 c_1^{-1}$$

$$b(\xi) = a^{-1}\,\{(a_2c_3 - a_3c_2) + \xi\,(b_2c_3 - b_3c_2)\}\, c_1 + a^{-1}\,\{(a_1c_2 - a_2c_1) + \xi\,(b_1c_2 - b_2c_1)\}\, c_3$$

$$c(\xi) = a^{-1}\,\{(a_2c_1 - a_1c_2) - \xi\{(b_1c_2 - b_2c_1)\}\, c_2$$

$$f_1(x) = \frac{aP(x)}{2\mu_1}$$

$$f_2(x) = \frac{a\,q(x)}{2\mu_1}$$

And

$$F_s\,(\phi(\xi) : x) = 0\ ,\ \ x > b$$

$$F_c\,(\Psi(\xi) : x) = 0\ ,\ \ x > b \qquad (2.16)$$

$$F_s\,(X(\xi) : x) = 0\ ,\ \ x > b$$

We proceed as in (5) and we define.

$$F_c\,(\phi(\xi) : x) = \begin{cases} r_1(x)\ , & a \le x \le b \\ 0, & 0 < x < a\ ,\ x > b \end{cases}$$

$$F_s\,(\Psi(\xi) : x) = \begin{cases} s_1(x)\ , & a \le x \le b \\ 0, & 0 < x < a\ ,\ x > b \end{cases} \qquad (2.17)$$

$$F_s\,(X(\xi) : x) = \begin{cases} w_1(x)\ , & a \le x \le b \\ 0, & 0 < x < a\ ,\ x > b \end{cases}$$

It is easily shown that if we make extensions r(u), s(u) and w(u) of $r_1(u)$, $s_1(u)$ and $w_1(u)$ to $-b \le x \le -a$ as follows:

$$r(u) = \begin{cases} r_1(u)\ , & a \le u \le b \\ r_1(-u), & -b \le u \le -a \end{cases}$$

$$s(u) = \begin{cases} s_1(u), & a \le u \le b \\ s_1(-u), & -b \le u \le -a \end{cases}$$

$$w(u) = \begin{cases} w_1(u), & a \le u \le b \\ w_1(-u), & -b \le u \le -a \end{cases}$$

then

$$F_s\,(\phi(\xi) : x) = \frac{1}{\pi}\int_L \frac{r(u)}{x-u}\,du$$

$$F_c\,(\Psi(\xi) : x) = \frac{1}{\pi}\int_L \frac{s(u)}{x-u}\,du \qquad (2.18)$$

$$F_s\,(X(\xi) : x) = \frac{1}{\pi}\int_L \frac{w(u)}{x-u}\,du$$

where L = ((-b, -a) U (a, b))

In like manner it is a simple matter to verify that

$$F_c\,(\phi(\xi) : x) = 0 \begin{cases} s_1(x)\ , & a \le x \le b \end{cases}$$

$0,\ 0 < x < a,\ x > b$

where $s_1(x) = \int_x^b s_1(u)\, du$,

$$F_s(\phi\Psi(\xi) : x) = \begin{cases} r_1(x), & a \le x \le b \\ 0, & 0 < x < a,\ x > b \end{cases} \qquad (2.19)$$

where $r_1(x) = \int_x^b r_1(u)\, du$

$$F_c(X(\xi) : x) = \frac{1}{\pi}\int_a^x dx \int_L \frac{w(u)}{u-x}\, du$$

If we substitute (2.17), (2.18) in the equation (2.15), we see that r, s, w must be solutions to the set of singular integral equations.

$$a(\xi)\, r(x) - \frac{b(\xi)}{\pi}\int_L \frac{s(u)}{u-x}\, du + c(\xi) w(x) = f_1(x),\ a \le |x| \le b$$

$$a(\xi)\, s(x) + \frac{b(\xi)}{\pi}\int_L \frac{w(u)}{u-x}\, du + c(\xi) s(x) = f_2(x),\ a \le |x| \le b$$

(2.20)

$$a(\xi)\, w(x) - \frac{b(\xi)}{\pi}\int_L \frac{r(u)}{u-x}\, du + c(\xi) s(x) = 0,\ a \le |x| \le b$$

where $f_1(x)$ and $f_2(x)$ are even functions defined on L. The substitution

$$\lambda(u) = s(u) - i\, r(u) + w(u) \qquad (2.21)$$

reduce the pair of equations (2.20) to the single integral equation

$$a(\xi)\, \lambda(x) + \frac{c(\xi) - b(\xi)}{\pi i}\int_L \frac{\lambda(u)}{x-u}\, du = f(x),\ x \,\varepsilon\, L \qquad (2.22)$$

where

$L = ((-b, -a) \cup (a, b))$,

$F(x) = i\, f_1(x) + f_2(x)$

If we now define

$$\Lambda(z) = \frac{1}{2\pi i}\int_L \frac{\lambda(u)}{(u-z)}\, du$$

then using plemelj formulae.

$$\Lambda^+(x) - \Lambda^-(x) = \lambda(x),\ \Lambda^+(x) + \Lambda^-(x) = \frac{1}{xi} - \int_L \frac{\lambda(u)}{(u-x)}\, du,$$

shows that (2.22) is equivalent to the condition

$$\Lambda^+(x) = -K\,\Lambda^-(x) - \{c(\varepsilon) - b(\varepsilon) + a(\varepsilon)\}^{-1} f(x),\ x \,\varepsilon\, L$$

(2.23)

Where

$$K = \left(\frac{c(\xi) - b(\xi) - a(\xi)}{c(\xi) - b(\xi) + a(\xi)}\right) > 0$$

Thus, we must find a sectionally holomorphic function $\Lambda(z)$, vanishing at infinity and satisfying the condition (2.23). The solution to this problem is well known (cf p. 450 (6) and is given by

$$\Lambda(z) = \frac{X(z)}{2\pi i\,\{c(\varepsilon) - b(\varepsilon) - a(\varepsilon)\}}\left[\int_L \frac{f(t)}{K^+(t)(t-z)}\, dt + P(z)\, X(z)\right] \qquad (2.24)$$

where $P(z) = h_1 z + h_2$, h_1 and h_2 are arbitrary complex constants and X(z) is the solution to the homogeneous Riemann boudary value problem.

$$X^+(t) = -K\, X^-(t),\ t \,\varepsilon\, L \qquad (2.25)$$

The homogeneous Riemann problem is known to have a solution (p.450) (3) given by

$$x(z) = [(z-a)(z+b)]^{i\omega}\,.\ [(z+a)(z-b)]^{-i\omega-½} \qquad (2.26)$$

where

$$\omega = \frac{1}{2\pi}\log\left\{\frac{c(\xi) - b(\xi) - a(\xi)}{c(\xi) - b(\xi) + a(\xi)}\right\}$$

In the case in which f is a polynomial

$$\int_L \frac{f(t)}{X^+(t)(t-z)} = \frac{\pi i\,\{c(\xi) - b(\xi) + a(\xi)\}}{c(\xi) - b(\xi)}\left[\frac{f(z)}{X(t)} - L(z)\right]$$

(2.27)

Where

$$L(z) = \frac{1}{2\pi}\lim_{R\to\infty}\int_0^{2\pi} \frac{f(Re^{i\theta})R\, e^{i\theta}\, d\theta}{X(Re^{i\theta})(Re^{i\theta} - z)} \qquad (2.28)$$

Hence (2.214) yields

$$\Lambda(z) = \frac{1}{2b(\xi)}[f(z) - X(z)\, L(z)] + P(z)\, X(z) \qquad (2.29)$$

for $0 < x < a$ and $x > b$

$$\sigma_{yy}(x, 0+) = \sigma_{yy}(x, 0-) = -\frac{2\mu_1\, b(\xi)}{f_0} I_m \Lambda^+(x)$$

$$\sigma_{xy}(x, 0+) = \sigma_{xy}(x, 0-) = -\frac{2\mu_1\, b(\xi)}{f_0} Re\Lambda^+(x)$$

$$m_{\phi y}(x, 0+) = m_{\phi y}(x, 0-) = \frac{2\mu_1\, b(\xi)}{f_0} I_m \Lambda^+(x)$$

The Case of Constant Internal Pressure

We now consider the case in which the cracks are opened by constant normal and shearing pressure say $p(x) = q(x) = P_0$ so that,

$$f(x) = \frac{a\, P_0}{2\mu_1} = f_0$$

Thus, if $\beta = \frac{1}{2} - i\omega$, then

$$\int_0^{2\pi} \frac{f(Re^{i\theta})R\, e^{i\theta}\, d\theta}{X(Re^{i\theta})(Re^{i\theta}-z)} = 2\pi f_0 \left\{z^2 + 2bz\beta - 2a\beta z - bz + az - (\beta - 1)\frac{5b^2 + 4a^2}{2} + \left(4\beta^2 + 4\beta - 1\right)\right\} C(R^{-1})$$

So that

$$L(z) = f_0\left[z^2 + (2\beta - 1)(b-a)z - \beta(\beta-1)\frac{4a^2+5b^2}{2} + \ldots\right] \tag{3.1}$$

If follows from (2.29) that

$$\Lambda(z) = \frac{if_0}{2a(\xi)z}\left[1 - \{z^2 + h_1 z + h_2\}\, X(z)\right] \tag{3.2}$$

where $X(z)$ is already defined by the equation (2.26) and $h_1,\ h_2$ are arbitrary complex constants.

We obtain relation (2.26), the expressions for X^+ and X^- as follows.

$$X^+(x) = -iK^{1/2}((x^2-a^2)(b^2-x^2))^{-1/2}(\cos\omega\theta + i\sin\omega\theta)$$

$$X^-(x) = iK^{-1/2}((x^2-a^2)(b^2-x^2))^{-1/2}(\cos\omega\theta + i\sin\omega\theta)$$

(3.3)

Where

$$\theta = \log\left\{\frac{(x-a)\,(x+b)}{(x+a)\,(b-x)}\right\},\ \text{while}$$

(i) for $-b < x < -a$,

$$X^+(x) = iK^{1/2}((x^2-a^2)(b^2-x^2))^{-1/2}(\cos\omega\theta + i\sin\omega\theta)$$

$$X^-(x) = -iK^{1/2}((x^2-a^2)(b^2-x^2))^{-1/2}(\cos\omega\theta + i\sin\omega\theta)$$

(3.4)

(ii) for $0 < x < a$

$$X^+(x) = X^-(x) = -((a^2-x^2)(b^2-x^2))^{-1/2}(\cos\omega\,\theta_1 + i\sin\omega\theta_1) \tag{3.5}$$

Where

$$\theta_1 = \log\left\{\frac{(a-x)\,(b+x)}{(a+x)\,(b-x)}\right\}$$

(iii) for $x > b$

$$X^+(x) = X^-(x) = ((x^2-a^2)(x^2-b^2))^{-1/2}(\cos\omega\theta_2 + i\sin\omega\theta_2) \tag{3.6}$$

Where

$$\theta_2 = \log\left\{\frac{(x-a)\,(x+b)}{(x+a)\,(x-b)}\right\}$$

Hence for $a < x < b$, we find that if we suppose

$$h_1 = h_1^1 + ih_1^2,\ h_2 = h_2^1 + ih_2^2$$

then,

$$\Lambda^+(x) - \Lambda^-(x) = \frac{F_0}{\sqrt{\{b(\xi)^2 - a(\xi)^2\}}}((x^2-a^2)(b^2-x^2))^{-1/2}\{(x^2 + h_1^1 x + h_2^1)$$

$$.\cos\omega\theta'\,(h_1^2 x + h_2^2)\sin\omega\theta\} + i\,((h_1^2 x + h_2^2)$$

$$.\cos\omega\theta + (x^2 + h_1^1 x + h_2^1)\sin\omega\theta) \tag{3.7}$$

While

$$\Lambda^+(x) + \Lambda^-(x) = \frac{F_0\, b(\xi)}{a(\xi)\sqrt{a(\xi)^2 - b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}$$

$$\{(x^2 + h_1^1 x + h_2^1)\cos\omega\theta - (h_1^2 x + h_2^2)\sin\omega\theta\}$$

$$+ i\left(\frac{f_0}{a(\xi)} + \frac{f_0\, b(\xi)}{a(\xi)\sqrt{a(\xi)^2 - b(\xi)^2}}[(x^2-a^2)(b^2-x^2)]^{-1/2}\right. \tag{3.8}$$

$$\{(x^2 + h_1^1 x + h_2^2)\sin\omega\theta + (h_1^2 x + h_2^2)\cos\omega\theta\})$$

On a $< x <$ b, the Plemelj relation yield,

$$s_1(x) = \frac{-f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_1^2x+h_2^1)\cos\omega\theta+(h_1^2x+h_2^2).\sin\omega\theta\}) \quad (3.9)$$

$$r_1(x) = \frac{-f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(h_1^1x+h_2^2)\cos\omega\theta-(x^2+h_1^1x+h_2^1).\sin\omega\theta\}) \quad (3.10)$$

$$w_1(x) = \frac{f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_1^1x+h_2^1)\cos\omega\theta+(h_1^2x+h_2^2).\sin\omega\theta\}) \quad (3.11)$$

we may also note on –b $< x <$ -a

$$s(x) = \frac{f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_1^1x+h_2^1)\cos\omega\theta-(h_1^2x+h_2^1).\sin\omega\theta\}$$

$$r(x) = \frac{-f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_1^1x+h_2^1)\sin\omega\theta-(h_1^2x+h_2^2).\cos\omega\theta\}$$

$$w(x) = \frac{-f_0}{\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_1^1x+h_2^1)\cos\omega\theta+(h_1^2x+h_2^2).\sin\omega\theta\}$$

Hence, the relation s(x) = $-s_1$ (-x), r(x) = r_1(-x) and w(x) = w_1(-x) on – b $< x <$ -a be satisfied, we must choose h_1^1 and h_2^2 so that $h_1^1 = h_2^2 = 0$.

Another use of the Plemelj formulae yield

$$\frac{1}{\pi}\int_L \frac{r(u)}{(u-x)}\,du = \frac{f_0\, b(\xi)}{a(\xi)\sqrt{a(\xi)^2-b(\xi)^2}}((x^2-a^2)(b^2-x^2))^{-1/2}\ \{(x^2+h_2^1)\cos\omega\theta - h_1^2x\sin\omega\theta\} \quad (3.12)$$

$$\frac{1}{\pi}\int_L \frac{s(u)}{(u-x)}\,du = \frac{f_0}{a(\xi)}\left[1+\frac{b(\xi)}{\sqrt{a(\xi)^2-b(\xi)^2}}\{(x^2-a^2)(b^2-x^2)\}\right]^{-1/2} \{(x^2+h_2^1)\sin\omega\theta - h_1^2x\cos\omega\theta\} \quad (3.13)$$

$$\frac{1}{\pi}\int_L \frac{w(u)}{(u-x)}\,du = \frac{-f_0}{a(\xi)}\left[1+\frac{b(\xi)}{\sqrt{a(\xi)^2-b(\xi)^2}}\{(x^2-a^2)(b^2-x^2)\}\right]^{-1/2} \{(x^2+h_2^1)\sin\omega\theta - h_1^2x\cos\omega\theta\} \quad (3.14)$$

We can deduce from the equation (3.12) and (3.9) that

$$\frac{1}{\pi}\int_L \frac{r(u)}{(u-x)}\,du = -\frac{b(\xi)}{a(\xi)}s_1(x),\quad a < x < b \quad (3.15)$$

It is simple matter to show that for $x > b$,

$$\Lambda^+(x) = \frac{i\,f_0}{2\,a(\xi)} - \frac{f_0}{2\,a(\xi)}((x^2-a^2)(x^2-b^2))^{-1/2}\{(x^2+h_2^1)\cos\omega\theta_2 h_2^1x\sin\omega\theta_2) - ((x^2+h_2^1)\sin\omega\theta_2 + h_2^1x\cos\omega\theta_2)\} \quad (3.16)$$

and hence, for $x > b$

$$\sigma_{yy}(x, 0+) = -P_0(1+i)(1-\{(x^2-a^2)(x^2-b^2)\}^{-1/2}\{(x^2+h_2^1)\cos\omega\theta_2 - h_2^1x\sin\omega\theta_2\}) \quad (3.17)$$

$$\sigma_{xy}(x, 0+) = -P_0(1+i)(\{(x^2-a^2)(x^2-b^2)\}^{-1/2}\{(x^2+h_2^1)\sin\omega\theta_2 - h_2^1x\cos\omega\theta_2\}) \quad (3.18)$$

$$m_{\phi y}(x, 0+) = -P_0(1+i)(1-\{(x^2-a^2)(x^2-b^2)\}^{-1/2}\{(x^2+h_2^1)\sin\omega\theta_2 + h_1^2x\cos\omega\theta_2\}) \quad (3.19)$$

where

$$\theta_2 = \log\left\{\frac{(x-a)\,(x+b)}{(x+a)\,(x-b)}\right\}$$

We see from (3.17), (3.18) and (3.19) that as $x \to \infty$

$$\sigma_{yy}(x, 0+) = -P_0(1+i)\,0\,(x^{-1})$$

$$\sigma_{xy}(x, 0+) = P_0(1+i)\{2\omega(b-1) + h_1^2\}(x^{-1}) + 0(x^{-1})$$

$$m_{\phi y}(x, 0+) = P_0(1+i)\,0\,(x^{-1}) \quad (3.20)$$

Hence, it follows that the condition $\sigma_{yy}(x, 0+) = 0\,(x^{-1})$ and $m_{\phi y}(x, 0+) = 0\,(x^{-1})$ as $x \to \infty$ is automatically satisfied while that $\sigma_{yy}(x, 0+) = 0\,(x^{-1})$ as $x \to \infty$ will only be satisfied if we choose $h_2^1 = -2\omega$ (b – a). Thus it only remains to determine the constant h_2^1. This is determined from the condition that at the end point x = a, u_y (a, 0 +) = 0 = u_y (a,0-). From (2.10), (2.19) (3.9) and (3.15) we see that

$$u_y(x, 0+) = \frac{s_1(x).c}{a(\xi).a^2}, \quad a < x < b$$

$$u_y(x, 0-) = \frac{s_1(x).D}{a(\xi).a^2}, \quad a < x < b \qquad (3.21)$$

where C and D can be calculated.

Therefore, we have condition

$$s_1(a) = \int_a^b s_1(u)\, du = 0 \qquad (3.22)$$

This gives

$$h_2^1 = \frac{-I_2 + 2\omega(b-a)\, I_1}{I_0}$$

Where

$$I_0 = \int_a^b \frac{\cos\omega\theta\ du}{\sqrt{(u^2-a^2)\,(b^2-u^2)}} = \frac{\pi}{(a+b)\cosh\pi\omega}\ 2F_1(½ + i\omega, ½, 1,\ z^2)$$

$$I_1 = \int_a^b \frac{u\sin\omega\theta\ du}{\sqrt{(u^2-a^2)\,(b^2-u^2)}} = \frac{\pi\omega z}{\cosh\pi\omega}\ F_3(½ + i\omega, ½ - i\omega, ½ - i\omega, ½ + i\omega,\ 2, z-z)$$

$$I_2 = \int_a^b \frac{u^2\cos\omega\theta\ du}{\sqrt{(u^2-a^2)\,(b^2-u^2)}} = \frac{\pi a^2}{(a+b)\cosh\pi\omega} F_3(½ - i\omega, ½ + i\omega, ½ + i\omega, ½ - i\omega,\ 1, z-z)$$

$$+ \frac{\pi(b-a)}{2\cosh\pi\omega}\ F_3(½ - i\omega, ½ + i\omega, ½ + i\omega, ½ + i\omega,\ 2, z-z)$$

$$-\frac{\pi(b-a)^2}{4(b+a)\cosh\pi\omega}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(½ - i\omega)_p(½ + i\omega)_q(½ + i\omega)_p(½ - i\omega)_q}{3!\, q!\, (3)_{p+q}}$$

$$.\ (2q + 1)\ (q - p)\ z^p(-z)^p$$

and F_3 is hypergeometric function of two variables defined in (112, p 274).

For x > b stresses are given by

$$\sigma_{yy}(x, 0+) = \sigma_{yy}(x, 0-) = -P_0(1+i)(1 - \{(x^2 - a^2)(x^2 - b^2)\}^{-½}\{(x^2 + h_2^1)\cos\omega\theta_2 + 2\omega(b-a)x\sin\omega\theta_2\}) \quad (3.23)$$

$$\sigma_{xy}(x, 0+) = \sigma_{xy}(x, 0-) = P_0(1+i)(\{(x^2 - a^2)(x^2 - b^2)\}^{-½}\{(x^2 + h_2^1)\sin\omega\theta_2 - 2\omega(b-a)x\cos\omega\theta_2\}) \quad (3.24)$$

$$m_{\phi y}(x, 0+) = m_{\phi y}(x, 0-) = P_0(1+i)(-1 + \{(x^2 - a^2)(x^2 - b^2)\}^{-½}\{(x^2 + h_2^1)\cos\omega\theta_2 - 2\omega(b-a)\, x\sin\omega\theta_2\}) \quad (3.25)$$

The stresses for 0 < x < a are given by

$$\sigma_{yy}(x, 0+) = \sigma_{yy}(x, 0-) = P_0(1+i)\ (1 + \{(a^2 - x^2)(b^2 - x^2)\}^{-½}$$

$$\{(x^2 + h_2^1)\cos\omega\theta_2 + 2\omega(b-a)\, x\sin\omega\theta_1\} \quad (3.26)$$

$$\sigma_{xy}(x, 0+) = \sigma_{xy}(x, 0-) = P_0(1+i)\{(a^2 - x^2)(b^2 - x^2)\}^{-½}$$

$$\{(x^2 + h_2^1)\sin\omega\theta_1 - 2\omega(b-a)\, x\cos\omega\theta_1\} \quad (3.27)$$

$$m_{\phi y}(x, 0+) = m_{\phi y}(x, 0-) = P_0(1+i)\ (-1 + \{(a^2 - x^2)(b^2 - x^2)\}^{-½}\{(x^2 + h_2^1)\cos\omega\theta_1 - 2\omega(b-a)\, x\sin\omega\theta_1\} \quad (3.28)$$

where

$$\theta_1 = \log\left\{\frac{(a-x)\,(b-x)}{(a+x)\,(b+x)}\right\}$$

h_2^1 can be calculated from (3.22)

The Stress Intensity Factors

If N_{1b}, N_{2b} and N_{3b} are the normal and shear stress intensity factors at the crack tip x = b then

$$N_{1b} = \lim_{x\to b}\left[(x-b)^{½}\sigma_{yy}(x, 0+)\right] \qquad (4.1)$$

$$N_{2b} = \lim_{\square\to\square}\left[(x-b)^{½}\sigma_{xy}(x, 0+)\right] \qquad (4.2)$$

$$N_{3b} = \lim_{\square \to \square}\left[(x-b)^{1/2} m_{\phi y}(x, 0+)\right] \tag{4.3}$$

then from the equations (3.23), (3.24) and (3.25), we get

$$N_{1b} + N_{2b} + N_{3b} = \frac{p_0(1+i)}{2b\left(b^2 - a^2\right)}\left\{4\omega b^2(b-a)^2 + (b^2 + h_2^1)\right\} \tag{4.4}$$

Similarly, the normal and shear stress intensity factor at the crack tip x = a are given by

$$N_{1a} + N_{2a} + N_{3a} = \frac{p_0(1+i)}{2b\left(b^2 - a^2\right)}\left\{4\omega a^2(b-a)^2 + (a^2 + h_2^1)\right\} \tag{4.5}$$

RESULTS AND DISCUSSION

In this paper, we consider a pair of coplanar Barenblatt cracks at the interface of the two bonded dissimilar micropolar elastic half planes. The components of stress and displacement have been calculated. The problem is reduced to the system of simultaneous dual integral equations which are further transformed to a Riemann boundary value problem. Calculations for evaluating the stress intensity factors at the crack tip are derived.

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