

## PHENOMENOLOGICAL DESCRIPTION OF SPIN COULOMB DRAG CONSIDERING SPIN-FLIP POSSIBILITY

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### ABSTRACT

The phenomenological equations for spin coulomb drag using the spin-drag co-efficient  $\gamma$  is discussed in this paper. Expression for transresistivity  $\rho$  is also given and it is shown that  $\gamma$  is directly proportional to  $\rho$ . In brief electron-electron collision and electron-impurity collision with spin-flip process are discussed. Finally, the collision-integral equation is given which interprets spin coulomb drag as damping mechanism for spin-current and a source for power loss in spintronic devices or circuits.

**KEYWORDS:** Spin coulomb drag, spin-current, transresistivity, spin-drag co-efficient, drift velocity, spin-flip, and collision -integral equation.

Ordinary Coulomb Drag is caused by momentum exchange between electrons residing in two separate 2-dimensional layers and interacting via the Coulomb interaction (Rojo; 1999). The Spin Coulomb (SCD) is the single layer analogue of the ordinary Coulomb Drag. In this case spin-up and spin-down electrons play the role of electrons in different layers and the friction arises due to Coulomb interaction when spin-up and spin-down electrons move within one single layer with different drift velocities (D' Amico and Vignale; 2000). The simplest description of the SCD is given in terms of a phenomenological friction co-efficient  $\gamma$ .

### THEORY

Let us start with the equation of motion for the drift velocity of spin  $\sigma$  electron.

$$m^*N_\sigma \dot{\vec{v}}_\sigma = -eN_\sigma \vec{E}_\sigma + \vec{F}_{\sigma\bar{\sigma}} - \frac{m^*}{\tau_\sigma} N_\sigma \vec{v}_\sigma + \frac{m^*}{\tau_\sigma} N_{\bar{\sigma}} \vec{v}_{\bar{\sigma}} \quad (1)$$

Where  $\vec{v}_\sigma$  =velocity of electron with spin  $\sigma$ ;

$\sigma$  = up spin                       $\bar{\sigma}$  = down spin

$\gamma$  = spin-drag co-efficient

$m^*$  = effective mass of the carrier

$N_\sigma$  = number of electrons with spin,

$\vec{F}_{\sigma\bar{\sigma}}$  = the net force exerted by spins on  $\bar{\sigma}$  spins

$\frac{1}{\tau_\sigma}$  = the rate of change of momentum of electrons with spin  $\sigma$  due to electron-impurity scattering, and is basically the Drude scattering rate.

$\frac{1}{\tau_\sigma}$  = the rate of change of momentum due to electron-impurity scattering in which an electron flips its spin from  $\bar{\sigma}$  to  $\sigma$ .

It is to be noticed that the net force exerted by spins of the same orientation vanishes by virtue of Newton's third law. For exactly the same reason we must have,

$$\vec{F}_{\sigma\bar{\sigma}} = -\vec{F}_{\bar{\sigma}\sigma} \quad (2)$$

By Galilean invariance this force can only depend on the relative velocity of the two components. Hence in the linear approximation, we can write

$$\vec{F}_{\sigma\bar{\sigma}} = -\gamma m^* N_\sigma \frac{n_{\bar{\sigma}}}{n} (\vec{v}_\sigma - \vec{v}_{\bar{\sigma}}) \quad (3)$$

Here  $n_\sigma$  is the density of electrons with spin  $\sigma$  and  $n = n_\sigma + n_{\bar{\sigma}}$  is the total density,  $\gamma$  is the spin-drag co-efficient. Putting the value of  $\vec{F}_{\sigma\bar{\sigma}}$  in equation (1) the equation of motion takes the form

$$m^*N_\sigma \dot{\vec{v}}_\sigma = -eN_\sigma \vec{E}_\sigma - \gamma m^* N_\sigma \frac{n_{\bar{\sigma}}}{n} (\vec{v}_\sigma - \vec{v}_{\bar{\sigma}}) - \frac{m^*}{\tau_\sigma} N_\sigma \vec{v}_\sigma + \frac{m^*}{\tau_\sigma} N_{\bar{\sigma}} \vec{v}_{\bar{\sigma}} \quad (4)$$

Using the current density or the spin component of the current

$$\vec{J}_\sigma = -en_\sigma \vec{v}_\sigma \quad (5)$$

and applying Fourier transformation to equation (4) we have

$$i\omega \vec{J}_\sigma(\omega) = \frac{-n_\sigma e^2 \vec{E}_\sigma(\omega)}{m^*} + \left( \gamma \frac{n_{\bar{\sigma}}}{n} + \frac{1}{\tau_\sigma} \right) \vec{J}_\sigma(\omega) - \left( \gamma \frac{n_\sigma}{n} + \frac{1}{\tau_\sigma} \right) \vec{J}_{\bar{\sigma}}(\omega) \quad (6)$$

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The resistivity matrix  $\rho_{\sigma\bar{\sigma}}$  is defined as the co-efficient of proportionality between the electric field and the current,

$$\vec{E}_{\sigma} = \sum_{\bar{\sigma}} \rho_{\sigma\bar{\sigma}} \vec{J}_{\bar{\sigma}} \quad (7)$$

Inverting equation (5) gives us the electric field

$$\vec{E}_{\sigma}(\omega) = \left( \frac{-i\omega m^*}{n_{\sigma} e^2} + \frac{m^*}{n_{\sigma} e^2 \tau_{\sigma}} + \gamma \frac{m^* n_{\bar{\sigma}}}{n n_{\sigma} e^2} \right) \vec{J}_{\sigma}(\omega) - \left( \frac{m^*}{n_{\bar{\sigma}} e^2 \tau_{\bar{\sigma}}} + \gamma \frac{m^*}{n e^2} \right) \vec{J}_{\bar{\sigma}}(\omega) \quad (8)$$

Comparing equation (7) and (8) the complete form of the resistivity matrix  $\rho_{\sigma\bar{\sigma}}$  is written as:

$$\begin{bmatrix} \frac{-i\omega m^*}{n_{\sigma} e^2} + \frac{m^*}{n_{\sigma} e^2 \tau_{\sigma}} + \gamma \frac{m^* n_{\bar{\sigma}}}{n n_{\sigma} e^2} & -\frac{m^*}{n_{\sigma} e^2 \tau_{\sigma}} - \gamma \frac{m^*}{n e^2} \\ \frac{m^*}{n_{\bar{\sigma}} e^2 \tau_{\bar{\sigma}}} - \gamma \frac{m^*}{n e^2} & \frac{-i\omega m^*}{n_{\bar{\sigma}} e^2} + \frac{m^*}{n_{\bar{\sigma}} e^2 \tau_{\bar{\sigma}}} + \gamma \frac{m^* n_{\sigma}}{n n_{\bar{\sigma}} e^2} \end{bmatrix} \quad (9)$$

## DISCUSSION

Several feature of this matrix are noteworthy. First, the matrix is symmetric due to the relation

$$\frac{1}{n_{\bar{\sigma}} \tau_{\bar{\sigma}}} = \frac{1}{n_{\sigma} \tau_{\sigma}} \quad (10)$$

Second, the off diagonal terms are negative, which can be explained as the appearance of a finite transresistivity. The transresistivity is induced in the up-spin channel by a current flowing in the down-spin channel when the up-spin current is zero. This is completely analogous to the transresistivity measured in the Coulomb drag experiments with electrons in two separate layers. Since a down-spin current in the positive direction tends to drag along the up-spins, a negative electric field is needed to maintain the zero value of the up-spin current.

Finally, the SCD appears in both diagonal and off-diagonal terms so the total contribution cancels out, if the drift velocities of up and down spins are equal. This is in agreement with the Newton's law enunciated in Equation (2). Due to the extreme smallness of the spin-flip rate  $\frac{1}{\tau_{\sigma}}$  the off-diagonal resistivity is controlled almost entirely by the term of Coulomb interaction. i.e. we can safely assume

$$\rho_{\sigma\bar{\sigma}} = -\gamma \frac{m^*}{n e^2} \quad (11)$$

and  $\gamma$  is directly proportional to the spin-transresistivity (D' Amico and Vignale, 2002).

At very low temperature spin-flip process

dominates because in this limit the Coulomb scattering is suppressed by phase space restrictions (Pauli's exclusion Principle) and  $\gamma$  tends to zero as  $T^2$  in 3D and  $T^2 \log T$  in 2D (D' Amico and Vignale, 2003). However, the spin-flip process from electron-impurity collisions does not effectively contribute to momentum transfer between the two spin channels. An up-spin electron that collides with an impurity and flips its spin-orientation from up to down is almost equally likely to emerge in any direction, so the momentum transfer from the up-to the down-spin orientation is minimal and independent of the down-spin. But, the situation looks quite different for electron-electron collisions. The collision of an up-spin electron with a down-spin electron leads to a momentum transfer that is preferently oriented against the relative velocity of the two electrons and is proportional to the latter (D' Amico and Vignale, 2002).

In further studies (Badalyan, et al., 2008; Hankiewicz and Vignale, 2006) using Boltzmann's approach in absence of spin-orbit interactions, the electron-electron contribution to the collision derivative takes the form

$$\begin{aligned} \dot{f}_{\sigma}(\vec{k})_{c,e-e} \simeq & -\frac{1}{k_B T} \sum_{k' p p'} W_c(k_{\sigma}, p - \sigma; k'_{\sigma}, p' - \sigma) \\ & \times [\hbar v_{\sigma} - \hbar v_{-\sigma}](k - k') f_{0\sigma}(\epsilon_k) f_{0-\sigma}(\epsilon_p) f_{0\sigma}(-\epsilon_{k'}) \\ & \times f_{0-\sigma}(-\epsilon_{p'}) \delta_{k+p, k'+p'} \delta(\epsilon_{k\sigma} + \epsilon_{p-\sigma} - \epsilon_{k'\sigma} - \epsilon_{p'-\sigma}) \end{aligned} \quad (12)$$

Where  $T$ = temperature,  $k_B$  = Boltzmann's constant. This equation is known as collision-integral equation which is proportional to the difference of velocities for spin-up and spin-down electrons. Therefore, if a finite spin current is set up through the application of an external field then the coulomb interaction will tend to equalize the net momenta of the two spin components causing  $(V_{\sigma} - V_{\bar{\sigma}})$  to decay. Thus, it can be interpreted as a damping mechanism for spin-current. As SCD provides an intrinsic decay mechanism for spin-polarized current it is therefore a source for power loss in a spintronic circuit or device (D' Amico and Ullrich, 2006). We find that if the external driving force couple in a different way to the two spins components such that the average spin-velocities are different, the SCD term

contributes to the charge channel too. In this particular case the two spin populations may be considered distinguishable, characterized by a spin dependent frequency both in the charge and spin channel. Hence the Coulomb drag force exerted by one population onto the other can be regarded as an external force.

## CONCLUSION

The spintronic devices are considered as future devices having extremely high speed and vast memory or storage capacity. Hence it is necessary to explain and examine its every aspect particularly at the experimental level. From theoretical point of view, the most urgent open challenge is the calculation of the influence of the interplay between spin-orbit coupling and spin coulomb drag in spintronic devices.

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