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IDENTITIES INVOLVING I-FUNCTION

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ABSTRACT

In this research work, we study and obtain new results on the following identities involving I-function of one variable and several special cases. Mathematics Subject classification -26A33,33C60, 44A15.

KEYWORDS : H-function , H-function of r-variables and I-function

1. The I- function of the one variable is defined by Saxena, 1982 and we will represent here in the following manner

$$I[z]=I^{m,n}_{p_i,q_i;r}[z]=I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}\ldots,\ldots\\ \ldots,\ldots\end{matrix}\right.\right]$$

$$=I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}(a_j,e_j)_{1,n};\quad (a_{ji},e_{ji})_{n+1,pi}\\ (b_j,f_j)_{1,m};\quad (b_{ji},f_{ji})_{m+1,qi}\end{matrix}\right.\right] \tag{1.1}$$

$$=\frac{1}{2\pi i}\int_L \theta(s)\, z^s ds \tag{1.2}$$

where $i=\sqrt{(-1)}, z(\neq 0)$ is a complex variable and (1.2) $z^s=\exp[s\{\log|z|+i\arg z\}]$. In which log |z| represent the natural logarithm of |z| and arg |z| is not necessarily the principle value. An empty product is interpreted as unity. Also,

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-f_js)\prod_{j=1}^{n}\Gamma(1-a_j+e_js)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+f_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-e_{ji}s)\right]} \tag{1.3}$$

m, n, pi and $q_i \forall i\in(1,\ldots r)$ are non-negative integers satisfying $0\le n\le p_i$, $0\le m\le q_i;\forall i\in\{1,\ldots,r\}$,

$e_{ji},(j=1,\ldots,p_i;i=1,\ldots r)$ and $f_{ji},(j=1,\ldots,q_i;i=1,\ldots r)$

are assumed to be positive quantities for standardization purpose. Also $a_{ji},(j=1,\ldots,p_i;\ i=1,\ldots r)$ and $b_{ji},(j=1,\ldots,q_i;\ i=1,\ldots,r)$ are complex numbers such that none of the points.

$$S=\{(b_n+v)|\ f_h\}h=1,\ldots,m;v=0,1,2,\ldots, \tag{1.4}$$

which are the poles of $\Gamma(b_h-f_hs), h=1,\ldots,m$

and the points

$$S=\{(a_l-n-1)|e_l\}=1,\ldots,n;\eta=0,1,2,\ldots, \tag{1.5}$$

Which poles are of $\Gamma(1-a_l+e_ls)$ coincide with one another, i.e. with

$$e_l(b_n+v)\neq b_n(a_l-\eta-1) \tag{1.6}$$

for

$$v,\eta=0,1,2,\ldots;\ h=1,\ldots,m;l=1,\ldots,n$$

Further, the contour L runs from $-i_\infty$ to $+i_\infty$. Such that the poles of $\Gamma(b_n-s), h=1,\ldots,m$; lie to the right of L and the poles $\Gamma(1-a_l+e_ls), l=1,\ldots,n$ lie to the left of L. The integral converges, if |argz|, $<\frac{1}{2}B\pi, B>0, A\le 0$ where

$$A=\sum_{j=1}^{p_i}e_{ji}-\sum_{j=1}^{q_i}f_{ji} \tag{1.7}$$

$$B=\sum_{j=1}^{n}e_j-\sum_{j=n+1}^{p_i}e_{ji}+\sum_{j=1}^{m}f_j-\sum_{j=m+1}^{q_i}f_{ji} \tag{1.8}$$

$$\forall i\in(1,\ldots,r)$$

2. Formula Used:

In the present investigation we required the following formula.

From (Rainville ,1960) .

$$\Gamma(z+1)=z\Gamma z \tag{2.1}$$

3. Results of Identities

$$I^{m+2,n}_{p_i+1,q_i+2;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots\ldots,(2-\mu,\upsilon)\\ (0,\upsilon),(3-\mu,\upsilon),\ldots\ldots\end{matrix}\right.\right]$$

$$=(2-\mu)I^{m+1,n}_{p_i,q_i+1;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots\ldots\\ (0,\upsilon),\ldots\ldots\end{matrix}\right.\right]-I^{m+1,n+1}_{p_i+1,q_i+1;r}\left[x\left|\begin{matrix}(1,\upsilon)\ldots\ldots\ldots\ldots\\ (0,\upsilon),\ldots\ldots(1,\upsilon)\end{matrix}\right.\right] \tag{3.1}$$

$$I^{m+1,n+1}_{p_i+1,q_i+2;r}\left[x\left|\begin{matrix}(\mu-1,\upsilon)\ldots\ldots\ldots\\ (1,\upsilon),\ldots\ldots,(\mu,\upsilon)\end{matrix}\right.\right]$$

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$$= (1-\mu) I^{m+1,n}_{p_i,q_i+1;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots\ldots \\ (1,\upsilon),\ldots\ldots\end{matrix}\right.\right] + I^{m+1,n+1}_{p_i+1,q_i+2;r}\left[x\left|\begin{matrix}(0,\upsilon)\ldots\ldots\ldots \\ (1,\upsilon),\ldots,(1,\upsilon)\end{matrix}\right.\right], \tag{3.2}$$

$$I^{m+1,n}_{p_i+1,q_i;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots,(\mu,\upsilon) \\ (\mu+1,\upsilon),\ldots\ldots\end{matrix}\right.\right] - I^{m+1,n}_{p_i+1,q_i+1;r}\left[x\left|\begin{matrix}\ldots\ldots,(\mu-\upsilon-1,\upsilon) \\ (\mu-\upsilon,\upsilon),\ldots\ldots\ldots\end{matrix}\right.\right]$$

$$= (1+\upsilon) I^{m,n}_{p_i,q_i;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots \\ \ldots\ldots\ldots\end{matrix}\right.\right]. \tag{3.3}$$

$$\mu\ I^{m+1,n+1}_{p_i+1,q_i+2}\left[x\left|\begin{matrix}(1-\mu,\upsilon)\ldots\ldots\ldots\ldots \\ (1,\upsilon),\ldots\ldots,(-\mu,\upsilon)\end{matrix}\right.\right]$$

$$= I^{m+1,n}_{p_i,q_i+1;r}\left[x\left|\begin{matrix}\ldots\ldots\ldots\ldots \\ (1,\upsilon),\ldots\ldots\end{matrix}\right.\right] - I^{m+1,n+2}_{p_i+2,q_i+2;r}\left[x\left|\begin{matrix}(0,\upsilon),(1-\mu,\upsilon)\ldots\ldots \\ (1,\upsilon),\ldots\ldots\ldots,(-\mu,\upsilon)\end{matrix}\right.\right], \tag{3.4}$$

$$|\arg z| = \tfrac{\pi}{2} B$$

Where B given in equation (1.8).

4. Proof of Identities

Proof of equations (3.1),(3.2),(3.3) and (3.4) can be easily prove using equations (1.1),(1.2) and (2.1).

5. Particular Cases

(Saxena and Singh ,1994), (Rathi et al., 1960), (Agarwal, 1997), (Goyal and Agarwal 1980), (Srivastava 1992), (Srivastava, 1994) and Several other authors have studied identities involving I-function. On choosing in main integrals, we get following integrals in terms of H- function of one variable:

$$H^{m+2,n}_{p+1,q+2}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p},(2-\mu,\upsilon) \\ (0,\upsilon),(3-\mu,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right]$$

$$= (2-\mu) H^{m+1,n}_{p,q+1}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p} \\ (0,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right] - H^{m+1,n+1}_{p+1,q+1}\left[x\left|\begin{matrix}(1,\upsilon),(a_j,e_j)_{1,p} \\ (0,\upsilon),(b_j,f_j)_{1,q},(1,\upsilon)\end{matrix}\right.\right], \tag{5.1}$$

$$H^{m+1,n+1}_{p+1,q+2}\left[x\left|\begin{matrix}(\mu-1,\upsilon),(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q},(\mu,\upsilon)\end{matrix}\right.\right]$$

$$= (1-\mu) H^{m+1,n}_{p,q+1}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right] + H^{m+1,n+1}_{p+1,q+2}\left[x\left|\begin{matrix}(0,\upsilon),(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q},(1,\upsilon)\end{matrix}\right.\right], \tag{5.2}$$

$$H^{m+1,n}_{p+1,q}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p},(\mu,\upsilon) \\ (\mu+1,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right] - H^{m+1,n}_{p+1,q+1}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p},(\mu-\upsilon-1,\upsilon) \\ (\mu-\upsilon,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right]$$

$$= (1+\upsilon) H^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p} \\ (b_j,f_j)_{1,q}\end{matrix}\right.\right], \tag{5.3}$$

$$\mu\, H^{m+1,n+1}_{p+1,q+1}\left[x\left|\begin{matrix}(1-\mu,\upsilon),(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q},(-\mu,\upsilon)\end{matrix}\right.\right]$$

$$= H^{m+1,n}_{p,q+1}\left[x\left|\begin{matrix}(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q}\end{matrix}\right.\right] - H^{m+1,n+2}_{p+2,q+2}\left[x\left|\begin{matrix}(0,\upsilon),(1-\mu,\upsilon),(a_j,e_j)_{1,p} \\ (1,\upsilon),(b_j,f_j)_{1,q},(-\mu,\upsilon)\end{matrix}\right.\right], \tag{5.4}$$

DISCUSSION

We have obtained the result namely (3.1),(3.2), (3.3),(3.4),(5.1), (5.2), (5.3), and (5.4) which satisfied all the conditions of I- function and H-function.

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