MODELING AND CONTROL OF A SERVO HYDRAULIC MOTOR

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ABSTRACT

In this article, a mathematical model of a servo hydraulic motor is derived. In this model, factors such as fluid compressibility, leakage of hydraulic motor and friction have been considered. The experimental results have been used to determine the parameters of the model. The values of some parameters are defined in experimental results and the values of the other parameters are determined by comparing the results of model and experimental results. PID controller is used to control the motor speed and for the design of it, PID Control Block is used in Simulink. In this block, the design of PID controller is based on the step response of a linear system. Thus the linear model of the system is obtained then the PID controller are designed for it. Using of the designed controller for nonlinear model will cause overshoot in the response that this problem has been resolved with adjusting the coefficients of the PID controller.

KEYWORDS: Hydromotor, Servo Valve, Modeling, Pid Control

Hydraulic systems due to the ability to produce high forces and torques, have many applications in various industries. Hydraulic servo system is a hydraulic system that acts very accurate and generally fall into two groups – valve operated and pump operated servos (Ashby, 1989). Hydraulic servo systems are known to be nonlinear due to many factors such as leakage, friction, hysteresis, saturation, dead zone, etcetera. So modeling and control of this systems are important. In this paper, mathematical model of a valve operated servo hydromotor is derived. In modeling some factors such as leakage, friction, compressibility is considered. Because the number of system parameters is many and some of them are non-measurable, their values have been determined by using of experimental results.

Figure 1 represents an operated servo valve hydraulic motor. The inlet fluid pressure to servo valve is considered constant. When current passes through the coils of servo valve, the flapper is diverted to one of the two nozzles. This deviation reduces the flow through the nozzle so the pressure increases. The produced pressure difference on the both side of the spool valve changes its position so the flow through the valve will changes.

MATHEMATICAL MODEL AND SIMULATIOM IN SIMULINK SOFTWARE

By changing the position of the spool, the amount of fluid can pass through valve changes so the flow through the valve is controlled by this way. The dynamic of servo valve is much faster than the dynamic of the system so it can be considered as a first order transfer function like *Equation (1)* (Ashby, 1989). Both side of hydraulic motors is considered as two control volumes. Internal leakage in motor can be considered as a coefficient of the pressure difference on either side of the motor. *Equations (2) and (3)* is obtained by using of the flow continuity law (WU and LEE, 1996).



Figure 1. Construction of valve operated hydraulic motor (Watton, 2009) & (www.moog.com).

$$G_{v}(S) = \frac{X_{v}}{V} = \frac{K_{v}T}{TS+1}$$
$$Q_{lm} - \frac{V_{lm}}{\beta} \frac{dP_{lm}}{dt} - K_{ml}(P_{lm} - P_{rm}) = D_{m}\omega_{m}$$

(1)

(2)

$$Q_{rm} - \frac{V_{rm}}{\beta} \frac{dP_{rm}}{dt} + K_{ml} (P_{lm} - P_{rm}) = -D_m \omega_m$$

Sectional area of valve that the fluid passing through its, is considered to be a fixed width orifice and also flow is turbulent. According to the equation of flow through the

$$\begin{split} Q_{rm} &= -C_d W X_v \sqrt{\frac{2(P_{rm} - P_{Tank})}{\rho}} \qquad X_v > 0 \\ Q_{lm} &= C_d W X_v \sqrt{\frac{2(P_s - P_{lm})}{\rho}} \qquad X_v > 0 \\ Q_{rm} &= -C_d W X_v \sqrt{\frac{2(P_s - P_{rm})}{\rho}} \qquad X_v < 0 \\ Q_{lm} &= C_d W X_v \sqrt{\frac{2(P_{lm} - P_{Tank})}{\rho}} \qquad X_v < 0 \end{split}$$

When the flow is turbulent, discharge coefficient is almost constant and it is between 0.65 and 0.75. So the discharge coefficient is assumed to be equal to 0.7. Pressure of the oil reservoir is very low against of the system pressure so it is considered to be zero.

Theoretical torque that the hydraulic motor producing equals multiplication of the pressure difference on either side of the motor and the motor displacement. Friction in motor reduces the torque is produced. By using of the Euler law, the equation of load can be written as Equation (8).

$$D_m(P_{lm} - P_{rm}) = T_{friction} + T_{load} + J \frac{d\omega_m}{dt}$$
(8)

 (δ)

orifice, amounts of fluid passe through valve are calculated from Equations (4) to (7).

(4)
(5)
(6)
(7)

Friction in the system is a combination of coulomb and viscous frictions. In coulomb friction until the motor due to the presence of friction remains unmoved, friction torque equal to the torque applied to the system and when it is moving the friction torque is constant. In this model the friction is considered like Equation (9).

$$T_{friction} = \begin{cases} T & \omega_m = 0\\ B\omega_m + T_{cf} & \omega_m \neq 0 \end{cases}$$
(9)

Each of these equations is implemented in Simulink software. The linking together of these equations in Simulink environments, overall system model is obtained like Figure 2.



Figure 2. Model of valve operated hydraulic motor in Simulink

Model of system have a large number of parameters that some of them are non-measurable. All parameters should be known

in order to simulate the system. Experimental results have been used to obtain the values of them. A fixed displacement

pump is used to produce pressure and flow in the system in experiment. Maximum pressure of system is controlled by a relief valve. In experiments when the system is no load, for sinusoidal inputs with different frequencies as voltage of servo valve, the motor speed is measured. An example of the experimental results is given in *Figure 3* that the red curve is the input voltage to the servo valve and the blue curve is the speed of the motor.



Figure 3. Experimental data with 10.48(rad/s) frequency sinusoidal input

A number of system parameters are known on the experimental results that are listed in *Table 1*. Inlet pressure to the servo valve is considered to be constant and equal to 70 bar. The known parameters used in the modeling and other

parameters is obtained by comparing the results of simulation and experimental results. An example of the comparison between experimental and simulation results for the same input is given in *Figure 4*.

1 1	1
Parameter	value
D_m	4.5 millitre/rev
Flow and pressure of pump	0.115 <i>lit/s</i> at 70 <i>bar</i>
Operating pressure of relief valve	70 <i>bar</i>
J	$0.0034 \ kg.m^2$

Table 1. Specified parameters in the experimental results



Figure 4. Velocity comparation in 17.22 (rad/s) frequency input

Values of the unknown parameters that obtained by comparing simulation and experimental results are listed in Table 2.

Table 2.	The obtained	parameters fr	rom compai	ring simula	tion with e	xperiment

1	1 8
Parameter	Value

ρ	961.45 kg/m^3
В	N.m
	$\frac{0.002}{rad/s}$
F_{cf}	0.001 N.m
W	0.92 <i>mm</i>
$K_{_{ml}}$	$6.5 mm^3/Mpa$
eta	1.313 Gpa
V_{lm}	0.1 <i>lit</i>
V_{rm}	0.1 <i>lit</i>
Т	0.5
K_{v}	0.0004

PID CONTROLLER

PID control is one of the best and most widely used controller in industry. In Simulink software by using of PID Control Block can tune suitable controller for linear systems. Thus the equations of system linearized around the operating point by using Taylor series. Operating point conditions of the servo hydromotor is equal to Equation (10):

$$\begin{cases} X_v = 0\\ P_{lm} = P_{rm} = \frac{P_s}{2} \end{cases}$$
(10)

By taking laplace transformation from equations, a third order transfer function between the input voltage and the

output speed of the motor is obtained like Equation (11).

$$\frac{\omega_m}{V} = \frac{K_v T}{TS+1} \times \frac{2D_m \beta C_x}{JV_0 S^2 + (BV_0 + 2J\beta K_{ml})S + 2\beta (BK_{ml} + D^2)}$$
(11)
$$C_x = \frac{\partial Q_{lm}}{\partial X_v} \bigg|_{O.P_t} = \left[C_d W \sqrt{\frac{2(P_s - P_{lm})}{\rho}} \right]_{O.P_t}$$
(12)

By putting the values of the parameters in the transfer function, a linear model of the system is obtained between the voltage of servo valve and the motor speed. By using PID Control Block for this linear model in Simulink, two PID controllers with different response times are tuned for controlling the motor speed. Time integral of the squared errors of applying these controllers to the unit step input for the motor speed (radian per second) can be seen in Table 3.

Coefficients of the PID controller with a response time of 0.244 seconds:

$$\begin{cases} K_p = 0.33 \\ K_i = 0.75 \\ K_d = -0.00035 \end{cases}$$

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Coefficients of the PID controller with a response time of 0.353 seconds:

$$\begin{cases} K_p = 0.26 \\ K_i = 0.48 \\ K_d = -0.022 \end{cases}$$

Table 3. Time integral of the squared errors of controllers to step input in linear model

to step input in incur mouel		
Controller	$\int e^2(t)dt$	
The first PID controller	0.087	
The second PID controller	0.129	

The first PID controller has smaller time response and integral of the squared error so has better performance. This adjusted controller is used to control the motor speed in the nonlinear model. It can be seen when the designed controller for the linear model is applied to the nonlinear model, it does not work well and causes overshoot in response. So in order to

reduce this problem, PID controller is tuned for the nonlinear model. Time integral of the squared errors of these controllers to unit step input can be seen in *Table 4*.

PID controller coefficients tuned for non-linear models:

 $\begin{cases} K_p = 0.61 \\ K_i = 0.78 \\ K_d = 0.012 \end{cases}$

Table 4. Time integral of the squared errors of controllers to step input in nonlinear model

Controller	$\int e^2(t)dt$
PID tuned for linear model	0.063
PID tuned for nonlinear model	0.048

PID controller tuned for nonlinear models has less error and also significantly reduces overshoot. The overshoot of the system to the step input equals to 3 percent when the controller is tuned to linear model is applying to nonlinear models but it is 0.27 percent with the controller tuned for nonlinear model.



Figure 5. Comparison of tuned controllers for linear and nonlinear model when used for nonlinear model

RESULT

In this paper, a mathematical model of a servo hydraulic motor was derived and some factors such as fluid compressibility, leakage of hydraulic motors and friction had been considered. Because the model of the system had a large number of parameters and some of them were nonmeasurable, experimental results had been used to obtain the values of them. The PID controller was used to control the hydromotor speed. PID Control Block in Simulink were used to design PID controller. Simulink PID controller design based on linear models, so the first linear model was obtained then PID controller was applied for it. The controller was designed for linear model. In this case, there was overshoot in response that this problem resolved with adjusting the PID controller coefficients.

REFERENCES

Ashby J. G.; 1989. *Power Hydraulics*", First ed., Cambridge, Prentice Hall.

Watton J.; 2009. *Fundamentals of Fluid Power Control*", Cambridge University Press.

"a technical look of electrohydraulic valves", www.moog.com

H. W. WU, C. B. LEE.; 1996. Influence of a relief valve on the performance of a pump/inverter controlled hydraulic motor system", Elsevier, Mechatronics, **6**: 1-19.

Calarasu D., E. Serban, D. Scurtu.; 2004. *Dynamic model of the rotative hydraulic motor under constant pressure"*, Hydraulic Machinery and Hydrodynamics Conference, Timisoara, Romania.

Nomenclature

- B: Coefficient of viscose friction
- C_d : Discharge coefficient
- D_m : Hydraulics motor displacement
- T_{cf} : Coulomb friction
- J: Hydraulics motor and load inertia
- K_{ml} : Leakage coefficient
- K_{v} : Servo valve constant gain
- P_s : Supply pressure
- P_{Tank} : Tank pressure
- P_{lm} : Pressure in left side of motor
- P_{rm} : Pressure in right side of motor
- Q_{lm} : Flow rate in/out of left side of motor
- Q_{rm} : Flow rate in/out of right side of motor
- T: Time constant of servo valve
- V: Servo valve voltage input
- V_{lm} : Oil volume in left side of motor
- V_{rm} Oil volume in right side of motor
- W: Width of servo valve orifice opening area
- X_{v} : Servo valve spool position
- ρ : Fluid mass density
- β : Fluid bulk modulus
- ω_m : Hydraulics motor angular speed