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SIXTH-ORDER SQUEEZING IN SUPERPOSITION OF TWO COHERENT STATES AT ARBITRARY LARGE INTENSITY OF RADIATION

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ABSTARCT

We study sixth-order squeezing of the Hermitian operator, $X_\theta \equiv X_1 \cos\theta + X_2 \sin\theta$ **in superposed coherent state,** $|\psi\rangle = K[\,|x\rangle + |xe^{i\gamma}\rangle\,]$ **of two coherent states** $|x\rangle$ **and** $|xe^{i\gamma}\rangle$**. Here operators $X_{1,2}$ are defined by $X_1 + i\,X_2 = a$, is annihilation operator. We find maximum sixth-order squeezing of X_θ in the superposed coherent state** $|\psi\rangle$ **with minimum value 0.044942 of** $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ **for an infinite combinations with** $2x\sin\frac{\gamma}{2} = 1.132$, $x^2 \sin\gamma = 2n\pi$ **and** $\theta = \frac{\gamma}{2}$ or $\frac{\gamma}{2} + \pi$ **. We conclude that sixth-order squeezing in superposed coherent state** $|\psi\rangle$ **can be obtained at arbitrary large intensity of radiation also but for its observations settings of the parameters become more demanding.**

KEYWORDS: - Non-classical Light, Squeezing, Higher-order Squeezing, Displacement Operator, Phase Shifting Operator

INTRODUCTION

States which cannot be explained on the basis of classical theory are called non-classical states (Walls, 1983; Dodonov, 2002). The non-classical nature of a quantum state can be manifested in different ways sub-Poissonian photon statistics, higher-order sub-Poissonian photon statistics and various kind of squeezing etc. In recent days applications of non-classical states in quantum information theory such as communication, quantum teleportation, dense coding and quantum cryptography have been well realized. It has been realized that non-classicality is the necessary input for entangled state.

Squeezing is a well-known non-classical effect, has been generalized to case of several variables (Hong and Mandel, 1985; Hillery, 1987). Hong and Mandel introduced the concept of higher-order squeezing by considering the n^{th} order (n is even integer) moments of the quadrature component and defined a state to be n^{th} order squeezed if the expectation value of the n^{th} power of the difference between a field quadrature and its average value is less than what it would be in a coherent state (Glauber, 1963). According to Hong and Mandel's definition, a state $|\psi\rangle$ is said to be n^{th}-order squeezed for the operator,

$$X_\theta = X_1 \cos\theta + X_2 \sin\theta, \quad (1)$$

if the n^{th}-order moment of X_θ,

$$\langle\psi|(\Delta X_\theta)^n|\psi\rangle < \frac{(n-1)!!}{2^n}, \quad (2)$$

where Hermitian operators $X_{1,2}$ are defined by $X_1 + iX_2 = a$, a is the annihilation operator, θ is an arbitrary angle and $\Delta X_\theta = X_\theta - \langle\psi|X_\theta|\psi\rangle$.

A coherent state does not exhibit non-classical effects but a superposition of coherent states can exhibit several non-classical effects such as squeezing, higher-order squeezing, sub-Poissonian statistics and higher-order sub-Poissonian statistics. Buzek *et al*. (Buzek, 1992) and Xia *et al.* (XIA, 1989) studied such effects in the superposition of two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ of identical amplitudes but opposite phases and reported that the even coherent states exhibit squeezing but not sub-Poissonian statistics while the odd coherent states exhibit sub-Poissonian statistics but not squeezing. Xia *et al.* studied such effects in even coherent states, odd coherent states and the states obtained by their displacement. The authors showed that the displaced even coherent states can exhibit both squeezing and sub-Poissonian statistics while the displaced odd coherent states can preserve sub-Poissonian statistics. Recently we studied(Prakash, 2011; Kumar, 2013) higher-order Hong-Mandel's squeezing in superposed coherent states and squeezing of both quadrature components in

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superposed coherent states. In practice, the superposition of coherent states can be generated in interaction of coherent state with nonlinear media and in quantum non-demolition techniques.

In this paper we study sixth-order squeezing of the Hermitian operator, $X_\theta \equiv X_1 \cos\theta + X_2 \sin\theta$ in superposed coherent state, $|\psi\rangle = K\,[\,|x\rangle + |xe^{i\gamma}\rangle\,]$ of two coherent states $|x\rangle$ and $|xe^{i\gamma}\rangle$. Here operators $X_{1,2}$ are defined by $X_1 + i\,X_2 = a$, is annihilation operator. We find maximum sixth-order squeezing of X_θ in the superposed coherent state $|\psi\rangle$ with minimum value 0.044942 of $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ for an infinite combinations with $2x\sin\frac{\gamma}{2} = 1.132$, $x^2 \sin\gamma = 2n\pi$ and $\theta = \frac{\gamma}{2}$ or $\frac{\gamma}{2} + \pi$. We conclude that sixth-order squeezing can be occurred in superposed coherent state $|\psi\rangle$ at arbitrary large intensity of radiation but in this case parameters become more sensitive.

SIXTH-ORDER SQUEEZING IN SUPAERPOSED COHERENT STATE $|\psi\rangle$

According to Hong and Mandel's definition, a state $|\psi\rangle$ is said to be sixth-order squeezed for the operator, $X_\theta = X_1 \cos\theta + X_2 \sin\theta$, if

$$\langle\psi|(\Delta X_\theta)^6|\psi\rangle < \frac{15}{64}. \tag{3}$$

Here Hermitian operators $X_{1,\,2}$ are defined by $X_1 + iX_2 = a$, and $\Delta X_\theta = X_\theta - \langle\psi|X_\theta|\psi\rangle$.

Now we have

$$\langle\psi|(\Delta X_\theta)^6|\psi\rangle = \langle\psi|:(\Delta X_\theta)^6:|\psi\rangle + \frac{15}{4}\langle\psi|:(\Delta X_\theta)^4:|\psi\rangle + \frac{45}{16}\langle\psi|:(\Delta X_\theta)^2:|\psi_1\rangle + \frac{15}{64}. \tag{4}$$

Here

$$\begin{aligned}\langle\psi|:(\Delta X_\theta)^6:|\psi\rangle = [\,&\langle\psi|:X_\theta^6:|\psi\rangle - 6(\langle\psi|:X_\theta^5:|\psi\rangle)(\langle\psi|:X_\theta:|\psi\rangle) - 5(\langle\psi|:X_\theta:|\psi\rangle)^4 \\ &+ 15(\langle\psi|:X_\theta^4:|\psi\rangle)(\langle\psi|:X_\theta:|\psi\rangle)^2 + 15(\langle\psi|:X_\theta^2:|\psi\rangle)(\langle\psi|:X_\theta:|\psi\rangle)^4 \;, \\ &- 20\langle\psi|:X_\theta^3:|\psi\rangle\langle\psi|:X_\theta:|\psi\rangle^3\,]\end{aligned} \tag{5}$$

$$\begin{aligned}\langle\psi|:(\Delta X_\theta)^4:|\psi\rangle = [\,&\langle\psi|:X_\theta^4:|\psi\rangle + 6(\langle\psi|:X_\theta^2:|\psi\rangle)(\langle\psi|:X_\theta:|\psi\rangle)^2 - 3(\langle\psi|:X_\theta:|\psi\rangle)^4 \\ &- 4\langle\psi|:X_\theta^3:|\psi\rangle\langle\psi|:X_\theta:|\psi\rangle\,]\end{aligned} \;, \tag{6}$$

$$\langle\psi|:(\Delta X_\theta)^2:|\psi\rangle = \langle\psi|:X_\theta^2:|\psi\rangle - (\langle\psi|:X_\theta:|\psi\rangle)^2\,], \tag{7}$$

$$\begin{aligned}\langle\psi|:X_\theta^6:|\psi\rangle = \frac{1}{32}\{&\mathrm{Re}[\langle\psi|a^6|\psi\rangle e^{-6i\theta}] + 15\,\mathrm{Re}[\langle\psi|a^{+2}a^4|\psi\rangle e^{-2i\theta}] \\ &+ 6\,\mathrm{Re}[\langle\psi|a^{+}a^5|\psi\rangle e^{-4i\theta}] + 10(\langle\psi_2|a^{+3}a^3|\psi_2\rangle)\,\}\end{aligned} \;, \tag{8}$$

$$\begin{aligned}\langle\psi_1|:X_\theta^5:|\psi_1\rangle = \frac{1}{32}\{&\mathrm{Re}[\langle\psi|a^5|\psi\rangle e^{-5i\theta}] + 5\,\mathrm{Re}[\langle\psi|a^{+}a^4|\psi\rangle e^{-3i\theta}] \\ &+ 10(\langle\psi|a^{+2}a^3|\psi\rangle)\,e^{-i\theta}\},\end{aligned} \;, \tag{9}$$

$$\langle\psi_1|:X_\theta^4:|\psi_1\rangle = \frac{1}{8}\{\mathrm{Re}[\langle\psi|a^4|\psi\rangle e^{-4i\theta}] + 4\,\mathrm{Re}[\langle\psi|a^{+}a^3|\psi\rangle e^{-2i\theta}] + 3(\langle\psi|a^{+2}a^2|\psi\rangle)\,\}, \tag{10}$$

$$\langle\psi_1|:X_\theta^3:|\psi_1\rangle = \frac{1}{4}\{\mathrm{Re}[\langle\psi|a^3|\psi\rangle e^{-3i\theta}] + 3\,\mathrm{Re}[\langle\psi_2|a^{+}a^2|\psi_2\rangle e^{-i\theta}]\}\;, \tag{11}$$

$$\langle\psi_1|:X_\theta^2:|\psi_1\rangle = \frac{1}{2}\{\mathrm{Re}[\langle\psi|a^2|\psi\rangle e^{-2i\theta}] + \langle\psi_2|a^{+}a|\psi_2\rangle\}, \tag{12}$$

and : : denotes the normal form of the operator within colons. Tedious but straight forward calculation leads to

$$\mathrm{Re}[\langle\psi|a^n|\psi\rangle e^{-ni\theta}] = K^2x^n[\cos n\theta + \cos n(\gamma-\theta) + C_1\cos(n\theta + C_2) + C_1\cos(n\gamma - n\theta + C_2)], \quad (13)$$

$$\mathrm{Re}[\langle\psi|a^{+m}a^n|\psi\rangle e^{i(m-n)\theta}] = K^2x^{m+n}[\cos(m-n)\theta + \cos(m-n)(\gamma-\theta) + C_1\cos(n\gamma + (m-n)\theta + C_2) + C_1\cos(m\gamma - (m-n)\theta + C_2)], \quad (14)$$

$$\text{and } \langle\psi|a^{+n}a^n|\psi\rangle = 2K^2x^{2n}[1 + C_1\cos(2n\gamma + C_2)]. \quad (15)$$

Here n and m are integers $C_1=e^{-x2(1-\cos\gamma)}$ and $C_2=x^2\sin\gamma$. If we minimize $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ with respect to x, γ and θ, we get minimum value 0.044942 of $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ using C++ Programming for a number of combination of x, γ and $\theta= \gamma/2$. For example we get approximately the same minimum value of $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ using C++ Programming at x = 566, γ=0.002 and $\theta= \gamma/2$ but sensitive variations of $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$ with parameters x, γ and θ.

CONCLUSION

The results obtained in the previous section can be explained on the basis of our recent results (Kumar, 2010) for maximum sixth-order squeezing of X_θ in the most general superposition state, $|\psi_1\rangle = Z_1|\alpha\rangle + Z_2|\beta\rangle$ of two coherent states $|\alpha\rangle$ and $|\beta\rangle$. We reported maximum sixth-order squeezing of X_θ with the absolute minimum value 0.044942 of $\langle\psi_1|(\Delta X_\theta)^6|\psi_1\rangle$ for infinite numbers of combinations with $\alpha - \beta$ =1.132 exp[$\pm$ i(π/2) + iθ], $\frac{Z_2}{Z_1} = \exp[\frac{1}{2}(\beta\alpha^* - \beta^*\alpha)]$ and with arbitrary values of ($\alpha + \beta$) and θ. This result has been generalized to case of higher-orders in our research work (Prakash, 2011). According to the results obtained previously, we find that the state $|\psi\rangle$ may have maximum sixth-order squeezing with the absolute minimum value 0.044942 of $\langle\psi|(\Delta X_\theta)^6|\psi\rangle$, if

$$x(1-e^{i\gamma}) = 1.132\exp[i(\pm\tfrac{\pi}{2}+\theta)], \text{i.e., } 2x\sin\frac{\gamma}{2} = 1.132; \;\; \theta = \frac{\gamma}{2} \text{ or } \frac{\gamma}{2}+\pi, \quad (18)$$

and

$$e^{ix^2\sin\gamma} = 1 \text{ i.e., } x^2\sin\gamma = 2n\pi;\ n = 0, 1, 2, \ldots.. \quad (19)$$

Hence we conclude from Eqs. (18) and (19) that sixth-order squeezing in superposed coherent state $|\psi\rangle$ can be obtained at high-intensity also but for its observations settings of the parameters are more demanding.

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