# A NOVEL APPROACH TO QUALITY ENHANCEMENT OF GRAYSCALE IMAGE USING PARTICLE SWARM OPTIMIZATION

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## ABSTRACT

Evolutionary Approach found countless applications in varieties fields due to their simplicity in understanding, flexibility in implementation and ease of adapting to the problem domain. In this paper we propose a novel approach to enhance the quality of grayscale image using particle swarm optimization (PSO) algorithm. Here, we treat image enhancement as a non-linear optimization problem and adopt PSO to solve it. The prime objective of this work is to devise an algorithm to improve the visual appearance quality of an image captured by the camera in a microcontroller based embedded systems, e.g., image captured by drone, hand-held medical imaging devices, robotic image capture application, etc. Results of simulation based experimental study indicate that the proposed algorithm yield acceptable result in terms of speed and image quality with respect to its implementation in a restrictive microcontroller based environment.

KEYWORDS: Image Enhancement; Particle Swarm Optimization (PSO) Algorithm; Evolutionary Algorithm

Image Processing involves converting an image into digital form and performing some operations on it. Image Enhancement is one of the most important and difficult techniques in image research. The aim of image enhancement is to improve the visual appearance of an image, or to provide a "better transform representation" for future automated image processing [Gorai and Ghosh, 2009]. The truthfulness of the results of image processing depends upon how flawlessly the incoming image is presented to the system. Real-life images are contaminated with varieties of errors, distortion and noise and also suffer from poor contrast. Therefore, image enhancement plays a significant role in extraction of useful information from it [Kaur and Kaur, 2015], enhance the contrast and remove the noise to increase image quality. The fundamental reason of image enhancement is to bring out detail that is hidden in an image that is covered up in a picture or to expand contrast in a low contrast picture. Image enhancement involves transformation techniques to devise new image enhancement method.

One of the most important stages in digital images processing application is the image enhancement technique which improves the quality (clarity) of images for human viewing, removing blurring and noise, increasing contrast, and revealing details are examples of enhancement operations. The enhancement technique differs from one field to another according to its objective. The existing techniques of image enhancement can be classified into two categories: Spatial Domain and Frequency domain enhancement [Umamageswari et. al., 2014]. Histogram transformation is considered as one of the fundamental processes for contrast enhancement of gray level images in spatial domain [Gorai and Ghosh, 2009 & Behera et. al., 2015] which facilitates subsequent higher level operations such as detection and identification. Linear contrast stretching employs a linear transformation that maps the gray-levels in a given image to fill the full range of values.

Evolutionary algorithms have been previously used to perform image enhancement [Kaur and Kaur, 2015] and in this work, we have applied a global graylevels image contrast enhancement technique using particle swarm optimization (PSO) so as to best fit the demands of the human interpreter [Singh et. al., 2017]. The key motivation behind selecting PSO as the tool to perform image enhancement is due to the fact that PSO requires less number of operation stapes as compared to GA (require selection, crossover and mutation operations) and also PSO takes less time to converge to better optima [Clerc and Kennedy, 2002]. The resulted gray-level enhanced images by PSO are found to be better compared with other automatic image contrast enhancement techniques. Both objective and subjective evaluations are performed on the resulted image which says about the goodness of PSO.

The paper is organized as follows. In section II, we have presented a review of literatures in the image enhancement problem domain. In Section III, we discuss necessary background concepts in detail and present basics of the PSO algorithm in Section IV. In Section V, we present the proposed PSO based methodology for simulation study. Experimental results are discussed in Section VI and Section VII presents concluding remarks on the work presented through this paper.

#### SURVEY OF LITERATURE

There are many approaches to manipulate the images to enhance its quality or serve as means of superior input in terms of image quality for other automated image processing techniques.

Maini and Aggarwal [2010] in their paper presented a comprehensive review of various digital image enhancement techniques adopted to provide a multitude of choices for improving the visual quality of images. The paper provides an overview of factors suitable to make appropriate choice of image enhancement techniques influenced by the imaging modality, task at hand and viewing conditions. Paper by Gogna and Tayalm [2012] presents a comprehensive review of the study of suitability of various evolutionary algorithms based metaheuristic techniques such as Genetic Algorithm (GA) and Differential Evolution Algorithm (DEA) for image enhancement applications. The results of image enhancement obtained using different evolutionary algorithms are compared amongst themselves and also with the output of histogram equalization method. Kaur and Kaur [2015] in their paper have presented comprehensive review of a couple of techniques such as Histogram Equalization, Adaptive Histogram Equalization, etc. that seek to improve the visual appearance of an image or uplift the image to a level that is better fitted to further analysis by an individual or machine. Authors Bedi and Khandelwal [2013] have presented a critical review of various image enhancement techniques in spatial domain and classify them with an attempt to evaluate shortcomings and fulfil the general needs in the field of active image processing research. Umamageswari et. al., [2014] have presented application of various transformation techniques to devise new image enhancement method to enhance the image as well as to preserve the naturalness of an image from the sensations of color perspective.

In the paper by Lee and Cho [2012] have proposes application of DEA based automatic image enhancement technique to achieve improvement in the image captured by the camera sensor in mobile devices. Subjective test shows the usefulness of the DEA based approach proposed.

Khunger et al. [2016] in their paper presented an embedded system application where image

enhancement techniques are adopted for bringing online corrections to the images captured by autonomous underwater vehicles that get blurred due to shadows Proposed approach adopts generated. contrast equalization by contrast limited adaptive histogram equalization followed by Homomorphic filtering, wavelet de-noising, bilateral filter and then applying contrast and intensity stretching for normalizing the color values. Performance of the proposed approach was assessed using Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), and Signal to Noise Ratio (SNR) of the images and established the validity of the proposed approach. Author Nichal et. al., [2013] demonstrated a novel approach of enhancing contrast quality of gray scale image in medical imaging, remote sensing, and electron microscopy and in many real life applications that suffer from poor picture contrast. The purposed approach attempts different image enhancement methods to employs such as Histogram Equalization (HE) and Discrete Wavelet Transform (DWT) to increase image visibility and details and their performance was comparatively studied considering common factors such as enhancement efficiency, computational requirements, noise amplification, user intervention and application suitability. Sahani et. al., [2015] in their paper presented a study referring to image processing application in designing an embedded system with simple interface so that the system is user friendly, where image enhancement feature is of paramount significance. Traditional approaches such as Histogram equalization (HE) and contrast limited adaptive histogram equalization (CLAHE) are found to exhibit unnatural results. To improve upon, authors have suggested a new method, known as contrast limited adaptive histogram equalization with variable enhancement degree (CLAHEwVED) that gives better result as compared to HE and CLAHE. Chen et. al., [2004] presents significance of image enhancement in a fingerprint verification system as a mobile phone and embedded system application and suggests a special Gabor filter parameter selection constraint to reduce the computing complexity of the kernel generation step. Suggested approach exhibits to achieve significant speed improvement and is almost as effective as the traditional floating-point based implementation. Deborah et. al., [2010] in their paper present approaches to enhance the performance of character readability from old typewritten documents using GA based optical character recognition by tuning the fitness function. The paper presents results of comparative study of several fitness functions and shows that background variance (BV) is likely to be the best fitness function as it gives

the highest correlation. The PSO based image enhancement approach presented in this paper is targeted mostly for embedded system application.

## **BACKGROUND CONCEPTS**

For image enhancement task, a transformation function is required which will take the intensity value of each pixel from the input image and generate a new intensity value for the corresponding pixel to produce the enhanced image. To evaluate the quality of the enhanced image automatically, an evaluation function is needed which will tell us about the quality of the enhanced image. In this section we describe the function used for the proposed work.

#### **Transformation Function**

Image enhancement done on spatial domain uses a transform function which generates a new intensity value for each pixel of the  $M \times N$  original image to generate the enhanced image, where M denotes the number of columns and N denotes the number of rows. The enhancement process can be denoted by:

$$g(i,j) = T[g(i,j)]$$
(1)

where f(i,j) is the gray value of the  $f(i,j)^{th}$  pixel of the input image and g(i,j) is the gray value of the (i,j)<sup>th</sup> pixel of the enhanced image, T is the transformation function. Local enhancement method apply transformation on a pixel considering intensity distribution among its neighboring pixels. Adaptive histogram equalization (AHE) is one such local enhancement method which gives good result. However the expensive computational overhead makes AHE a prohibitive choice. The method used in this paper is less time consuming and is similar to statistical scaling [1]. The function used here is designed in such a way that it takes both global as well as local information to produce the enhanced image. Local information is extracted from a user defined window of size  $n \times n$ . The transformation T is defined as:

$$g(i,j) = K(i,j)[f(i,j) - c \times m(i,j)] + m(i,j)^{a}$$
(2)

In eq. (2), a and c are two parameters, m(i,j) is the local mean of the  $(i,j)^{th}$  pixel of the input image over a  $n \times n$  window and K(i,j) is enhancement function which takes both local and global information into account, Expression for local mean and enhancement function are defined as:

$$m(i,j) = \frac{1}{n \times n} \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} f(x,y)$$
(3)

One form of the enhancement function  $\mathbf{K}(i,j)$ , used in this work is:

$$\boldsymbol{K}(i,j) = \frac{k.D}{\sigma(i,j) + b} \tag{4}$$

where k, and b are two parameters, D is the global mean and  $\sigma(i,j)$  is the local standard deviation of  $(i,j)^{th}$  pixel of the input image over a  $n \times n$  window, which are defined as:

$$D = \frac{1}{M \times N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(x, y)$$
(5)

$$\sigma(i,j) = \sqrt{\frac{1}{n \times n} \sum_{x=0}^{n} \sum_{y=0}^{n} (f(x,y) - m(i,j))^2}$$
(6)

The final form of the transformation function appears as given below:

$$g(i,j) = \frac{k.D}{\sigma(i,j) + b} [f(i,j) - c \times m(i,j)] + m(i,j)^a$$
(7)

Using this transformation eq. (7), contrast of the image is stretched considering local mean as the center of stretch. Four parameters introduced in the transformation function are: a, b, c and k trigger large variations in the processed image.

#### **Evaluation** Criteria

To evaluate the quality of an enhanced image without human intervention, we need an objective function which will say all about the image quality. Many objective functions have been suggested in [Gorai and Ghosh, 2009 & Behera et. al., 2015]. In this paper, the objective function is formed by combining three performance measures, namely entropy value, sum of edge intensities and number of edgels (edge pixels). It is observed that compared to the original image good contrast enhanced image has more number of edgels [Gorai and Ghosh, 2009 & Behera et. al., 2015] and enhanced version should have a higher intensity of the edges [Braik et. al., 2007]. But these two are not sufficient to test an enhanced image; therefore one more measure has been taken into consideration, i.e. entropy value of the image that reveals the information content in the image. If the distribution of the intensities is uniform, then we can say that histogram is equalized and the entropy of the image will be more. The objective function considered here is:

$$F(I_e) = \log(\log(E(I_s))) \times \frac{n_e dgels(I_s)}{M \times N} \times H(I_e)$$
(8)

In the above mentioned eq. (8),  $I_e$  is the enhanced image of  $I_o$  (the original image) produced by

the transformation function defined in eq. (7). In the above equation eq. (8), the edges or edgels can be detected by many efficient edge detector algorithms such as Sobel, Laplacian, Canny, etc. In this paper work, Sobel is used as an automatic threshold detector. After using Sobel edge operator we produce a Sobel edge image I<sub>e</sub>. on the produced enhanced image I<sub>e</sub>. E(I<sub>s</sub>) is the sum of M × N pixel intensities of Sobel edge image I<sub>s</sub>. n\_edgels(I<sub>s</sub>) is the number of pixels in Sobel edge image I<sub>s</sub>, whose intensity value is above a threshold in the Sobel edge image. Based on the histogram, entropy value is calculated on the enhanced image I<sub>e</sub> as:

$$H(I_e) = -\sum_{i=0}^{255} e_i$$
(9)

where  $e_i = h_i log_2(h_i)$  if  $h_i \neq 0$  otherwise  $e_i = 0$ , and  $h_i$  is the probability of occurrence of  $i^{th}$  intensity value of  $I_e$  image.

#### PARTICLE SWARM OPTIMIZATION (PSO)

PSO originally proposed by Kennedy and Eberhart [1995] is a novel evolutionary algorithm modelled after the flocking behaviour of birds. In PSO scheme, a swarm of birds (called particles) represents the solution space where each particle represents a candidate solution to the problem, characterised by its position vector  $\mathbf{x}$  and velocity vector  $\mathbf{v}$ . The quality or fitness of the solution that each particle implies is denoted by a function of its current position: f(x). Particles in PSO follow a very simple mechanism to evolve their behaviour: by trying to match the success of neighbouring particles and their own success achieved. Considering each particle of dimension n and the swarm of size Np, the position and velocity of the i<sup>th</sup> particle in the swarm,  $i \in [1, Np]$ , at the t<sup>th</sup> generation can be represented as:  $\mathbf{x}_i^t = (x_{i1}^t, x_{i2}^t, ..., x_{in}^t)$ and  $\mathbf{v}_{i}^{t} = (v_{i1}^{t}, v_{i2}^{t}, ..., v_{in}^{t})$  respectively. Mathematically, the new position and velocity of each bird is represented as:

$$\mathbf{v}_{i}^{t+1} = \omega^{t} \mathbf{v}_{i}^{t} + c_{1} r_{1} (\mathbf{p}_{i}^{t} - \mathbf{x}_{i}^{t}) + c_{2} r_{2} (\mathbf{g}^{t} - \mathbf{x}_{i}^{t}), \qquad (10)$$
$$\mathbf{x}_{i}^{t+1} = \mathbf{x}_{i}^{t} + \mathbf{v}_{i}^{t+1} \qquad (11)$$

where  $r_1, r_2 \in [0,1]$  are uniform random numbers,  $\omega^t$  is the inertia weight at the t<sup>th</sup> iteration that controls the fluidity of the medium in which the bird moves,  $c_1$ and  $c_2$  are the positive acceleration coefficients that controls the convergence by driving the particle towards the local and global best solutions, respectively;  $\mathbf{v}^t$  and  $\mathbf{x}^t$  are the velocity and position of the particle at the t<sup>th</sup> iteration;  $\mathbf{p}$  and  $\mathbf{g}$  are respectively the particle's personal and global best position attended so far. The rule for updating  $\mathbf{p}$  of i<sup>th</sup> particle in t<sup>th</sup> iteration is as follows:

$$\mathbf{p}_{i}^{t} = \begin{cases} \mathbf{p}_{i}^{t-1} & iff(\mathbf{x}_{i}^{t}) > f(\mathbf{p}_{i}^{t}) \\ \mathbf{x}_{i}^{t}, & otherwise \end{cases}$$
(12)

The value of  $\mathbf{g}$  of the swarm in  $t^{th}$  iteration is obtained as:

$$\mathbf{g}^{t} = \begin{cases} \min f(\mathbf{p}_{i}^{t}) \Big|_{i=1}^{N^{p}} & \text{if } \min f(\mathbf{p}_{i}^{t}) \leq f(\mathbf{g}^{t-1}) \\ \mathbf{g}^{t-1}, & \text{otherwise} \end{cases}$$
(13)

REGIN
INITIALIZE parameters;
FOR each porticle
INITIALIZE particle position and particle velocity;
END
00
FOR each particle
CALCULATE fitness value;
IF (the fitness volge is better than
the best fitness volue (pRest) in history)
set current value as the new pBest;
END
SELECT the particle with the best fitness value
of all the porticles as the gBest;
FOR each particle
CALCULATE particle velocity;
UPDATE particle position;
END
WHILE (TERMINATION CONDITION not satisfied)
END

Figure 1: Pseudo Code Algorithm for the PSO

The pseudo code algorithm and the flowchart of the PSO as an aid to understanding functioning of the algorithm is given below in Fig. 1 and Fig. 2, respectively:



Figure 2: Flowchart for the PSO

## PSO BASED PROPOSED IE METHODOLOGY

an То produce enhanced image а transformation function defined in eq. (7) is used, which incorporates both global and local information of the input image. The function also contains four parameters namely a, b, c, and k which are used to produce diverse result and help to find the optimal one according to the objective function. These four parameters have their defined range which is mentioned in the parameter setting section. Now our aim is to find the best set of values for these four parameters which can produce the optimal result and to perform this work PSO is used. P number of particles are initialized, each with four parameters a, b, c, and k by the random values within their range and corresponding random velocities. It means position vector of each particle X has four components a, b, c, and k. Now using these parameter values, each particle generates an enhanced image. Quality of the enhanced image is calculated by an objective function defined

#### Parameter Setting for PSO Algorithm

The result of PSO algorithm is very much parameter dependent. Fine tuning of the parameters can provide better result than other optimization algorithms. Parameter  $\omega$  used in eq. (10) is called the inertia weight. Maximum and minimum value for  $\omega$  is set to two and zero respectively, which is same for all particles. The process starts with maximum inertia value and gradually reduces it to minimum. Therefore initially inertia component is big and explore larger area in the solution space, but gradually inertia component becomes small and exploit better solutions in the solution space. Inertia value  $\omega$  is calculated as:

$$\omega^{t} = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}} \times t \tag{14}$$



Figure 3: PSO based Image Enhancement Algorithm

Parameters  $c_1$ , and  $c_2$  are positive acceleration constants, given a random number in the range [0,2]. Throughout the lifetime of a particle, these parameter values are fixed for each particle.  $r_1$  and  $r_2$  are random numbers in the range [0,1] and varies for each component of the particles in every generation (i.e., during each iteration).

In this work there are four problem specific parameters, a, b, c and k. The range for these parameters are the same for all, i.e., in the range [1,5].  $a \in [0.5, 1.5]$ ,  $b \in [0,0.5]$ ,  $c \in [0,1]$  and  $k \in [0.5, 1.5]$ . We have changed the range for parameter *b* as that range does not produce good result. It has been observed that small value for *b* stretch the intensity in a large amount. So, after normalizing those intensity values between [0,255] to display the image, it becomes very discritized and the originality of the image is lost. So, we increase the range to overcome this specific problem, and the range is fixed to [1,(D/2)], where D is the global mean of the original image.

#### **RESULTS ANALYSIS**

In this section, we present the experimental results for evaluating the efficiency of the proposed automatic image enhancement method. The implementation of the proposed algorithm is done in MATLAB 7.8 using a computer with Intel i3 Processor (2.20GHz) and 4 GB RAM. For getting the experimental result from the simulation study of the proposed method, we have considered three sample test images which include image taken in indoor as well as outdoor.

Figures below show variety of source (fig. 4, 6 and 8) and enhanced images (fig. 5, 7, and 9) chosen for conducting the simulation study. Plots presented in figures 10, 11 and 12 represents their corresponding improvement in fitness value.



Figure 4: Source Image-1 Figure 5: Output Image-1



Figure 6: Source Image-2 Figure 7: Output Image-2



Figure 8: Source Image-3 Figure 9: Output Image-3

Performance of the simulation was studied from the improvement in the objective function as well from the visual enhancement. For the parametric study of performance, variance in the intensity of neighbouring value pixel over an image block of  $n \times n$ window with n = 3 was taken in both input image and enhanced image. Fast rise and stabilization in the fitness value as shown in the fitness versus iteration plots (fig. 10, 11 and 12) indicates effectiveness of the proposed approach. From the visual analysis of the enhanced image against their corresponding input images appropriately matches with the performance plots.

In the execution of program we basically have taken in average 50 populations and 50 iterations to execute the operation. Results of simulation has established that a fast rise in fitness value (as low as 4 iterations) can make it suitable for implementation in low speed and low memory foot-print embedded processor environments.

The result of simulation shows the input image along with the enhanced image as well as the performance plots. The performance plot represents the number of iteration in x axis and the fitness values in y axis.



Figure 10: Image Enhancement Performance Plot-1



Figure 11: Image Enhancement Performance Plot-2



Figure 12: Image Enhancement Performance Plot-3 CONCLUSION

In this paper, a have introduced a new approach to achieve automatic image enhancement

using real-coded weight adjusted PSO, implemented by specifying a suitable fitness function proportional to the number and intensity values of the edgel pixels and to the entropic measure of the image. The objective of the algorithm was to maximize the total number of pixels in the edges thus being able to visualize more details in the images. The algorithm is tested on three selected images. The results obtained are tabulated and compared with the results obtained using other traditional and evolutionary approaches. It is clear from the obtained results that the proposed PSO based image enhancement is better than other image enhancement techniques implemented using other approaches in terms of both quality of the solution and computational efficiency. The proposed PSO based image enhancement scheme may be extended in several ways, such as: fine tuning of the PSO parameters in order to reduce the number of particles and reducing the maximum number of iterations. Another extension is to code local parameters of the method that applies to each neighbourhood.

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