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BALANCED BIPARTITE TREES IN CRYPTOGRAPHY

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ABSTRACT

Mathematical concepts of Graph Theory, particularly labelled graphs which can be used to increase security in data transfer serve a very useful purpose in Cryptography. The field of Graph Theory, specially Graph Labeling and labeled graphs can be used as important tools for encryption for greater security, as compared to many other commonly used methods used in Cryptography. In this paper, Bipartite Trees employing certain graph labelling schemesnamely harmonious, graceful, sequential and felicitous are being presented as the preferred method for encryption and decryption. A balanced Bipartite Tree has the same number of vertices in left and right partite sets.

KEYWORDS: Label Matrix, Harmonious Labelling, Felicitous Labelling, Graceful Labelling, Sequential Labelling

Application of graph labelling schemes, namely *inner magic* and *inner antimagic* in Cryptography have been made in (Krishnaa, 2019). These labelling were discovered in (Krishnaa and Dulawat, 2006). Tokareva (2014) has seen the relationship between concepts of Graph Theory and Cryptography. Chase and Kamara (2010) has also discussed the encryption of cryptography. Krishnaa (2021) has given the application of some particular labelled graphs in cryptography. Sudarsana *et al.*, (2020) has given the super mean and magic labelling schemes in cryptography. Baskar Babujee and Babitha (2012) has discussed both encryption and decryption regarding labelled graphs. Maheswari *et al.*, (2020) has used star graphs for encryption and secret coding of messages. Krishnaa (2018) has given directions of using labelled graphs in cryptography and other applications while (Krishnaa, 2004) gives the *harmonious, felicitous, sequential* and *graceful* labelling of Bipartite Trees.

In this paper, Method providing two schemes is being presented: Scheme 1: adding of vertex labels and Scheme 2: adding induced edge labels for two kinds of choices of drawing the labelled graphs. These methods have been developed for balanced Bipartite Trees with the four kinds of labelling schemes namely harmonious, felicitous, sequential and graceful. These labellings have been obtained as per (Krishnaa, 2004) and form the basis for developing the Cryptographic algorithms in this paper.

MATERIALS AND METHODS

Methods used are from Graph Theory and other mathematical methods applicable for Cryptography.

Fundamental Definitions

a) **Harmonious labelling:** For a graph G(p,q) with p number of vertices and q number of edges a harmonious labeling is a function $f: V(G) \rightarrow Z_q = \{0,1,2,\ldots,(q-1)\}$; so that the induced edge label is given by $(f(x)+f(y)) \pmod q$; repetition of one vertex label is allowed for *trees*.

b) **Felicitous labelling:** A *felicitous labeling* is given by $f: V(G) \rightarrow Z_{q+1}$ so that the induced edge label is given by $(f(x)+f(y)) \pmod q$.

c) **Sequential labelling:** The *sequential labeling* is given by $f: V(G) \rightarrow Z_q = \{0,1,2,\ldots,(q-1)\}$; so that the induced edge label is given by $(f(x)+f(y))$.

d) **Graceful labelling:** A *graceful labeling* in a graph with q edges is assigning the numbers to the vertices $0,1,2,\ldots,q$ such that the induced edge labels found by taking the absolute value (the positive value) are from $1,2,\ldots,q$.

e) **Bipartite Tree:** A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets U&V such that every edge connects one vertex in U to one vertex in V. If vertex sets U and V are having equal number of elements they are being termed here having Equal Partite Sets, otherwise having Unequal Partite Sets.

f) **Label Matrix:** Depending on the type of graph labelling used, these matrices have been newly developed such as Harmonious Label Matrix for the harmoniouslabelling used for the Bipartite Tree.The entries of this type of matrix are the *induced edge*

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labels. For instance, a vertex is labelled in the Left Partite Set with i and with j in the Right Partite Set, then the matrix entry $a_{ij}$is the induced edge label of the edge formed by the end vertices labeld with i and j in the left and right respectively. Figure 3 shows the Harmonious Label Matrix for the HarmoniousTree of Figure 1.

RESULTS AND DISCUSSION

- **Method: (Add vertex label and induced edge labels)**

l = number of vertices in left partite set.
r = number of vertices in right partite set.

Equal Partite Set ($l = r$): Plain text message to be sent is "**PLAINTXT**".

a. Harmonious Labelled Tree

Since the plain text message has 8 letters so take G(8,7) as shown in figure below. We have two choices for labeling the plain text message letters to the vertices of G(8,7) as given in Figure 1 and Figure 2.

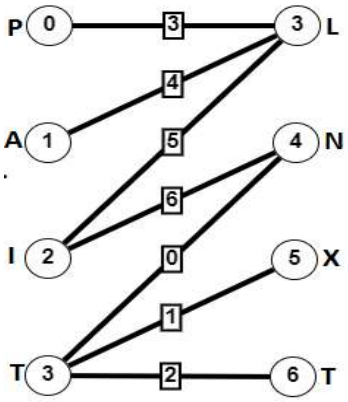

Figure 1: Harmonious Labelled Tree (Choice 1)

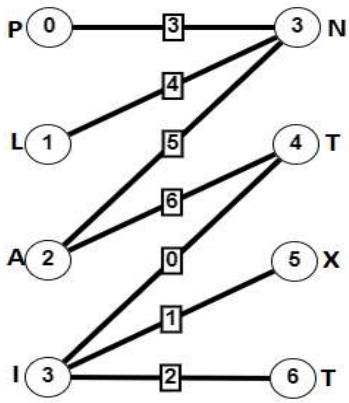

Figure 2: Harmonious Labelled Tree (Choice 2)

We will discuss here algorithm for choice 1 only and illustration will be discussed for both choices.

Algorithm for Scheme 1 of adding vertex labels (Choice 1):

m_l & m_r be the message array (Plain text for left and right partite sets respectively).

// Input vertex labels//

for$i$ = 1 to l // top to bottom//

Read $m_l[i]$ // input letters of left partite set.//

for$i$ = 1 to r //top to bottom//

Read $m_r[i]$// input letters of right partite set//

//Input vertex labels//

for$i$ =1 to l//(top to bottom)//

vertex_left_label [i] = i – 1. //This will input the vertex labels of the Left Partite Set in the array.//

for$i$ =1 to r//(top to bottom)//

vertex_right_label [i] = l + (i – 2). //This will input vertex labels of the Right Partite Set in the array ///

// Scheme 1 : Adding vertex labels //

//Computing Cipher Text (left partite set)//

for$i$ = 1 to l

$C_l[i]$ = vertex_left_label [i] + $m_l[i]$.

//Computing Cipher Text (right partite set)//

for$i$ = 1 to r

$C_r[i]$ = vertex_right_label [i] + $m_r[i]$.

For the Bipartite Tree in Figure 1, the newly developed Harmonious Label MatrixA will be as given below in Figure 3.

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \boxed{3} & 0 & 0 & 0 \\ \boxed{4} & 0 & 0 & 0 \\ \boxed{5} & \boxed{6} & 0 & 0 \\ 0 & \boxed{0} & \boxed{1} & \boxed{2} \end{bmatrix} \end{array}$$

Figure 3: Newly Developed Harmonious Label Matrix; boxes contain induced edge labels

Algorithm for Scheme 2 for adding induced edge label (Left Partite Set only):

for$i$ = 1 to l

// x : the number of non-zero entries in the i^{th} row ; col 1 = the column number of the 1^{st} non-zero entry in the i^{th}row when x = 1 ; col2 1 = the column number of the 1^{st} non-zero entry in the i^{th}row when x > 1//

If (x = 1) then Read col1

If (x > 1) then Read col2

If (x = 1) then

begin – if

j = col1

$C_l[i] = m_l[i] + A[i, j]$

end-if

if (x > 1)then

begin - if

$C_l[i] = 0$

j = col2 - 1

for$k$ = 1 to x

j = j + 1

$C_l[i] = C_l[i] + A[i, j]$

nextk

$C_l[i] = m_l[i] + C_l[i]$

end-if

next i

Encryption

Send the method or the name of the Algorithm which the Receiver knows, the Label Matrix, the Cipher text and the name of the labelling scheme to the Receiver. The labelled Bipartite Tree can be sent to the Receiver as an *adjacency matrix* also.

Decryption: The Receiver can decrypt the letters of the Cipher text by *subtraction*.

Similar Encryption and Decryption can be followed in similar other methods also.

Illustration (For Choice 1)

Encryption

For G(8,7) in Figure 1, Plain text is PLAINTXT.

Scheme 1 (add vertexlabel)

P+0 = P, A+1 = B, I+2 = K, T+3 = W

L+3 = O, N+4 = R, X+5 = C, T+6 = Z

Cipher text is: PBKWORCZ

Scheme 2 (Add induced edge label)

P+3 = S, A+4 = E, I+(5+6) = T, T+(0+1+2) = W

L+(3+4+5) = X, N+(6+0) = T, X+1 = Y, T+2 = V

Cipher text is: SETWXTYV

Decryption:

Scheme 1

P-0=P, B-1=A, K-2=I, W-3=T

O-3=L, R-4=N, C-5=X, Z-6=T

Scheme 2

S-3 = P, E-4 = A, T- (5+6) = I, W-(0+1+2) = T, X-(3+4+5) = L, T-(6+0)= N, Y-1 = X, V-2 = T

The original Plain text obtained is: PLAINTXT

Illustration (For Choice 2)

Encryption

For G(8,7) in Figure 2, Plain text is : PLAINTXT

Scheme 1 (Add vertex label)

P+0 = P, L+1 = M, A+2 = C, I+3 = L, N+3 = Q, T+4 = X, X +5 = C, T+6 = Z

Cipher text is: PMCLQXCZ

Scheme 2 (Add induced edge label)

P+3 = S, L+4 = P, A+(5+6) = L, I+(0+1+2) = L

N+(3+4+5) = Z, T+(6+0) = Z, X+1 = Y, T+2 = V

Cipher text is: SPLLZZYV

Decryption

Scheme 1

P-0=P, M-1=L, C-2=A, L-3=I

Q-3=N, X-4=T,C-5=X, Z-6=T

Scheme 2

S-3 = P, P -4 = L, L-(5+6) = A, L-(0+1+2) = I,

Z-(3+4+5) = N, Z- (0+6) = T, Y-1 = X, V-2 = T.

The original Plain text obtained is: PLAINTXT

b. Felicitous Labelled Tree

We have two choices for labelling the Plain text message letters of the vertices of G(8,7) as given in Figure 5 and Figure 6. The newly developed Felicitous Label Matrix is given in Figure 4 for the Bipartite Tree shown in Figure 5.

$$B = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} \boxed{4} & 0 & 0 & 0 \\ \boxed{5} & 0 & 0 & 0 \\ \boxed{6} & \boxed{0} & 0 & 0 \\ 0 & \boxed{1} & \boxed{2} & \boxed{3} \end{bmatrix} \end{array}$$

Figure 4: Newly Developed Felicitous Label Matrix; boxes contain induced edge labels

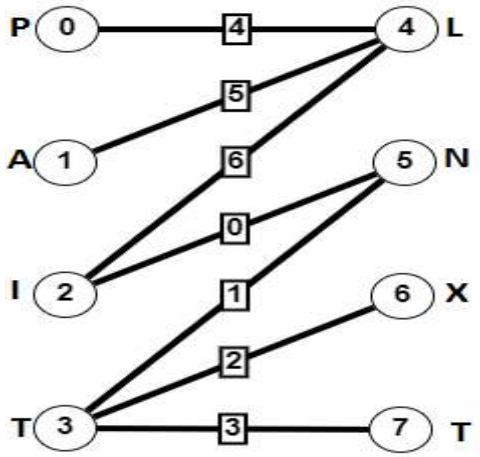

Figure 5: Felicitous Labelled Tree (Choice-1)

Illustration (For Choice 1)

For G(8, 7) in Figure 5 Plain Text is : PLAINTXT.

Scheme 1 (Add vertex label)

P+0 = P, A+1 = B, I+2 = K, T+3 =W,

L+4 = P, N+5 = S, X+6 = D, T+7 = A.

Cipher Text is: PBKWPSDA.

Scheme 2 (Add Induced edge label)

P+4 = T, A+5 = F, I+(6+0) = O, T+(1+2+3) = Z, L+(4+5+6) = A, N+(0+1) = O, X+2 = Z, T+3 = W.

Cipher Text is: TFOZAOZW.

Illustration (For Choice 2)

For G(8, 7) in Figure 6 Plain Text is : PLAINTXT.

Scheme 1 (Add vertex Label)

P+0 = P, L+1 = M, A+2 =C, I+3 =L,

N+4 = R, T+5 = Y, X+6 = D, T+7 = A

Cipher Text is: PMCLRYDA.

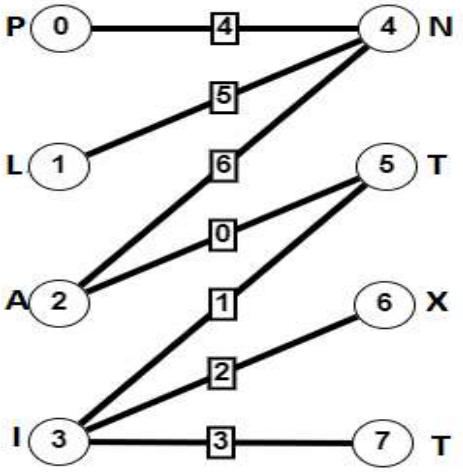

Figure 6: Felicitous Labelled Tree (Choice-2)

Scheme 2 (Add induced edge label)

P+4 = T, L+5 = Q, A+(6+0) = G, I+(1+2+3) = O, N+(4+5+6) = C, T+(0+1) = U, X+2 = Z, T+3 = W

Cipher Text is: TQGOCUZW.

c. Sequential Labelled Tree:

We have two choices for labeling the plain text message letters to the vertices of G(8,7) as given in Figure 8 and Figure 9. For the Bipartite Tree in Figure 8, the newly developed Sequential Label Matrix C will be as given below in Figure7.

$$C = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \boxed{3} & 0 & 0 & 0 \\ \boxed{4} & 0 & 0 & 0 \\ \boxed{5} & \boxed{6} & 0 & 0 \\ 0 & \boxed{7} & \boxed{8} & \boxed{9} \end{bmatrix} \end{array}$$

Figure 7: Newly Developed Sequential Label Matrix; boxes contain induced edge labels

Illustration (For Choice 1)

For G(8, 7) in Figure 8 Plain Text is : PLAINTXT.

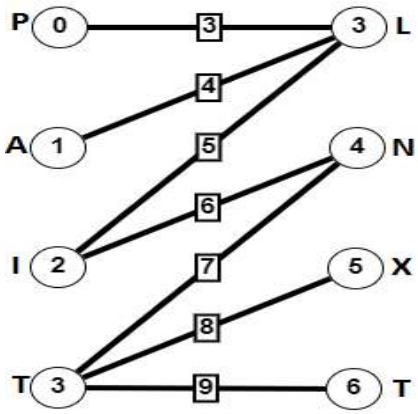

Figure 8: Sequential Labelled Tree (Choice-1)

Scheme 1 (Add vertex label)

P+0 = P, A+1 = B, I+2 = K, T+3 = W,

L+3 = O, N+4 = R, X+5 = C, T+6 = Z

Cipher Text is: PBKWORCZ.

Scheme 2 (Add induced edge label)

P+3 = S, A+4 = E, I+(5+6) = T, T+(7+8+9) = R, L+(3+4+5) = X, N+(6+7) = A, X+8 = F, T+9 = C.

Cipher Text is: SETRXAFC.

Illustration (For Choice 2)

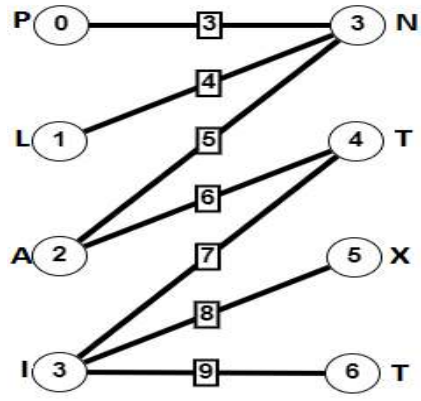

Figure 9: Sequential Labelled Tree (Choice-2)

For G(8, 7) in Figure 9 Plain Text is : PLAINTXT.

Scheme 1 (Add vertex label)

P+0 = P, L+1 = M, A+2 =C, I+3 =L,

N+3 = Q, T+4 = X, X+5 = C, T+6 = Z

Cipher Text is: PMCLQXCZ.

Scheme 2 (Add induced edge label)

P+3 = S, L+4 = P, A+(5+6) = L, I+(7+8+9) = G, N+(3+4+5) = Z, T+(6+7) = G, X+8 = F, T+9 = C.

Cipher Text is: SPLGZGFC.

d. Graceful Labelled Tree:

We have two choices for labeling the plain text message letters to the vertices of G(8,7) as given in Figure 11 and Figure 12. For the Bipartite Tree in Figure 11, the newly developed GracefulLabel Matrix D will be as given below in Figure 10.

$$D = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 7 & 6 & 5 & 4 \end{array} \\ \begin{bmatrix} \boxed{7} & 0 & 0 & 0 \\ \boxed{6} & 0 & 0 & 0 \\ \boxed{5} & \boxed{4} & 0 & 0 \\ 0 & \boxed{3} & \boxed{2} & \boxed{1} \end{bmatrix} \end{array}$$

Figure 10: Newly Developed Graceful Label Matrix; boxes contain induced edge labels

Illustration (For Choice 1)

For G(8, 7) in Figure 11 Plain Text is : PLAINTXT.

Scheme 1 (Add vertex label)

P+0 = P, A+1 = B, I+2 = K, T+3 = W,

L+7 = S, N+6 = T, X+5 = C, T+4 = X.

Cipher Text is: PBKWSTCX.

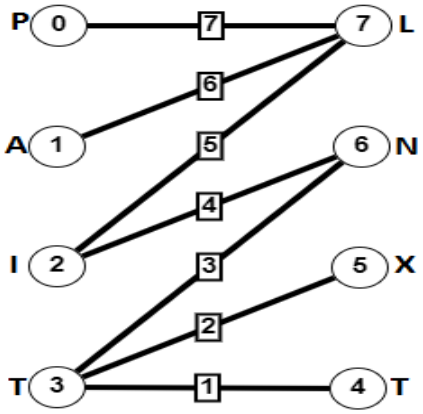

Figure 11: Sequential Labelled Tree (Choice-1)

Scheme 2 (Add induced edge label)

P+7 = W, A+6 = G, I+(5+4) = R, T+(3+2+1) = Z, L+(7+6+5) = D, N+(4+3) = U, X+2 = Z, T+1 = U.

Cipher Text is: WGRZDUZU.

Illustration (For Choice 2)

For G(8, 7) in Figure 12 Plain Text is : PLAINTXT.

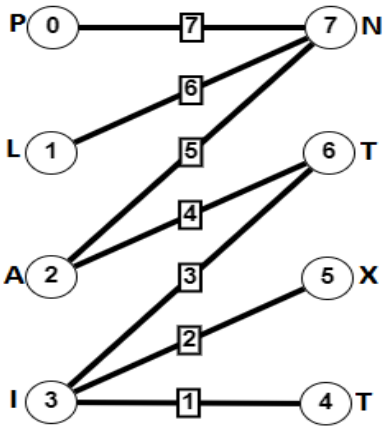

Figure 12: Graceful Labelled Tree (Choice-2)

Scheme 1 (Add vertex label)

P+0 = P, L+1 = M, A+2 =C, I+3 =L,

N+7 = U, T+6 = Z, X+5 = C, T+4 = X

Cipher Text is: PMCLUZCX.

Scheme 2 (Add Induced edge label)

P+7 = W, L+6 = R, A+(5+4) = J, I+(3+2+1) = O, N+(7+6+5) = F, T+(4+3) = A, X+2 = Z, T+1 = U.

Cipher Text is: WRJOFAZU.

CONCLUSION

Method presenting 2 schemes of adding vertex labels and induced edge labels for the labelling schemes of harmonious, graceful, felicitous and sequential have been presented for Balanced Bipartite Trees with equal number of vertices in left and right partite sets and for two kinds of choices of drawing the labelled graphs. These produce highly cryptic Cipher-texts thus providing for a safe data transfer.

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