

## OPTIMIZATION AND SYNTHESIS OF FIVE BAR MANIPULATOR: A REVIEW

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**Abstract**-In this paper an overview of the literature related to five bar manipulators has been presented. Both Geared and non- geared five bar mechanisms have been considered in this work and all essential information are presented in various aspects of a manipulator. Also a survey on circular and non-circular GFBM is carried out to present the optimization and synthesis of various path profiles. In this work various optimization techniques have been discussed for particular path generation and their effectiveness. An effort has been made to suggest the better optimization and synthesis technique to simulate the GFBM which can generate a pre-defined path. Some special applications like ornithopter are taken into account. The combination of various algorithms gives more accurate results and synthesis of the path profile provided that the combinations should be accordingly and appropriate to that mechanism and particular path profile problem. Numerical methods are found to fit in almost mechanisms, though they require more calculations, but it gives more accurate result if combined with specific techniques. Different graphical and analytical techniques have been used for the problems of dimensional synthesis. One-phase, two-phase, numerical methods, genetic algorithm, GA-DE hybrid evolutionary algorithm, differential evolution algorithm and much more optimization techniques has been surveyed to represent a huge variety of work done by various researchers.

**Keywords**-Geared Five Bar Mechanism (GFBM), Optimization, Synthesis.

### I. Introduction

The aim of path synthesis is to simulate such a mechanism whose coupler point can follow a pre-determined path or trajectory. A continuous trajectory can also be denoted by a combination of discrete points. A method is suggested to reduce the size of the design space is to find an optimum solution which satisfies the shape of desired trajectory. By translating, scaling and rotating the solution for the final mechanism we get the final solution. The behavior of the geared five-bar mechanism (with DOF is 1) and the five bar mechanism (with DOF is two) are the same, if the gear ratio for the former and the ratio of angular velocity for both the input links (link 1 to link 4) for the latter are the same and if the starting angular positions of input links 1 and 4 for the former and the latter are the same. The main difference is that the latter one needs two drivers whereas the former needs only one driver and a gear set.

Different graphical and analytical techniques have been used for the problems of dimensional synthesis. Numerical optimization methods are used to overcome the shortcomings of analytical and graphical methods. Any of the gradients based methods or direct search can be used as numerical optimization method. In direct search technique an increment is provided to the initial possible design and different

increments are tested to get an improved solution [11]. In order to solve a synthesis problem effectively an objective function is optimized under some constraints formulated. The constraints include the recognition of singular positions and the full-rotability conditions [5]. Traditional synthesis techniques consider only a finite number of precision points whereas optimization-based mechanism synthesis (OBMS) provides the best fit to any number of precision points. OBMS allows the synthesis of mechanisms for complex tasks also where traditional techniques may fail [9]. Based on a symmetric five-bar manipulator a new topology for the amplification of displacement is proposed [13], and a compliant mechanism is implemented. The new mechanical amplifier is known as compliant mechanical amplifier (CMA). The analysis with FEM shows that a double symmetric beam five-bar manipulator using corner-filletted hinges provides good performance [13] compared with the previous one. This is based on four-bar topology. That two design parameters, thickness and the initial height of the flexural hinge, have good effects on natural frequency and the amplification ratio of the system. It has been taken care excellently to cover the researches from 1984 to 2017 for better review, but it can't be guaranteed to view all the papers. Even some articles has been covered from 1963, 64 also.

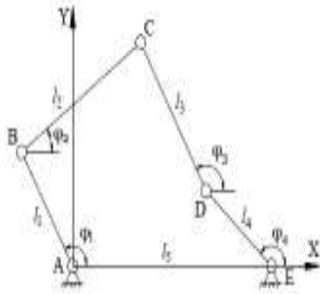


Fig. 1 The sketch of simple FBM [10]

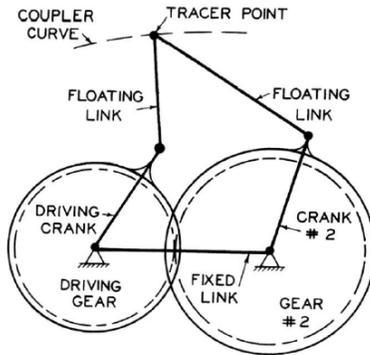


Fig. 2 Basic elements of GFBM [12]

## II. Simple Five-bar Mechanism

A Smaili and N Diab [7] proposed an ant-gradient optimization technique for dimensional synthesis of closed path generating 4 bar planar mechanisms through shape optimization. It is a two-phase synthesis process [7]. In first phase the coupler curve shape is optimized. In the second phase mathematical method is used to scale, rotate and translate linkage to its final configuration. They proposed cyclic angular deviation (CAD) vector for the shape optimization. The method uses less unknown variables (6), which converges the search in the design space [7]. They found that advantage of using CAD vector instead of Fourier descriptors is that this method is insensitive to the coupler curve profile and to any sharp edges it may have. This method can also be used in open-loop paths.

## III. Circular geared five bar Mechanism

Jacek Buskiewicz [5] proposed two functions which describes the closed curve and used it for the synthesis of planar geared five-bar linkage with 1-DOF for path generation. Mutual distribution of the curve points and the distance of the curve from its centroid are the two functions. He used normalized Fourier coefficients as objective functions in synthesis of planar geared five-bar mechanism with 1-DOF and evolutionary algorithm for the minimization. By means of the MDCP and the DCC functions a curve can be represented as a

sequence of NFCs [5]. Jacek found that MDCP and DCC functions are advantageous. When compared with the curvature-based method it was found that the MDCP function is less useful because it contains poor curve information, while the DCC method is better for synthesizing the sophisticated shapes [5]. Five continuous trajectories with up to combination of 40 discrete points for the GFBM are studied in [5] using the DCC, F-function, and curvature-based methods [25]. B. Roth and F. Freudenstein used a numerical method to solve the path generation problem of a GFBM. The approximate path-synthesis problem is solved [12] by specifying the precision points (points at which the desired curve and the exact path coincide); such techniques are called point-approximations to the path synthesis problem.

Sandhya R, et al, proposed a GFBM for the special application to generate the path like figure of eight (FO8). An ornithopter is a device that provides the flapping-wing motion of the birds. The mechanism studied in [17] is based on the Hummingbird morphology (birds float in the air without moving in any direction). Sensitivity studies have been carried out to determine the effect of various parameters of planar GFBM on the FO8 coupler curve. The most sensitive parameters and conditions for generating singly symmetric FO8 coupler curve have been also discussed in [17]. They found that the most effective parameters are the bounding box requirements, minimum transmission angle, symmetry of the coupler curve and rotatability criteria.

## IV. Non-circular geared five-bar Linkage

The limitations of cylindrical geared mechanism in a trajectory generation can be solved by using the non-circular gear pair can [10] and double crank hinge five-bar for closed combination. Inverse kinematics theory is used to solve thesequenceof rotation angle in a discrete manner for non-circular gear pair. J. Han has used reverse design method for non-circular GFBM as starting point and analyzed the design feasibility conditions to make a hinge pass over some pre-defined path. The discrete rotation angles of non-circular gear pair should meet progressive increasing relationship [10]. A five-bar linkage with non-circular gears is proposed [1] as a mechanism capable of precisely moving a coupler point along a desired path. The first step of the proposed methodology is the inverse kinematic analysis of the linkage, whose mobility, without geared bodies, is two.

## V. Synthesis

The synthesis of a mechanism typically involves synthesis about motion, path or function

generation. Synthesis of a mechanism involves a large number of design parameters. The discussed synthesis methods can be grouped as direct synthesis method [12, 23] and indirect synthesis method [19]. There are two further categories of direct synthesis method, i. e., One-phase [9, 12, 22, 23] and Two-phase synthesis method [5, 7, 25]. It is shown [20] that maximum number of precision points for the problem of approximate path-synthesis for unspecified crank rotation is generally equal to the number of mechanism-parameters. The number of mechanism-parameters is determined from the mechanism. For GFBM, Fig. 2, there will be nine parameters, i. e., length of all the five links, the phase angle, inclination angle of the fixed link and the coordinates of the fixed crank-pivot from origin. Primrose and Freudenstein [21] found that gear ratio  $N = \pm p/q$  provides the degree of generated locus equals to  $2p + 4q$ ; If gear ratio is positive, the circularity is  $p+2q$  and  $2q$  for negative gear ratio. Here  $p$  and  $q$  are co-prime integers and  $0 < p \leq q$ . Hence, gear ratio is decision parameter for the characteristics of the generated locus. A two-phase synthesis method generally cannot handle the path synthesis problems with prescribed timing [8].

Zhang et al. [19] used a nonlinear programming technique to an atlas containing 732 coupler curves for the symmetric GFBM. Continuation is a global and exhaustive numerical method [Alexander Morgan-1987] to solve the system of small polynomials. Continuation finds all solutions to the system of equations without prior knowledge of solutions of the system [24]. Continuation consists of two systems: the first system provides initial values for continuation path and the second is target system for which the solutions are desired. The genetic algorithm (GA) is an evolutionary algorithm and is an effective tool for solving the problems of synthesis and optimization [25]. This algorithm finds the better one solution out of the all possible alternative solution.

**A. One-Phase Method**

One phase synthesis method [8,9, 12, 15,19-23] simultaneously satisfies the shape, size, orientation and location details of the desired path. Wen-Yi Lin used [8] one-phase synthesis method, to solve the synthesis problems of the GFBM generating special trajectories. Approximately matched trajectories can be found if the objective function is evaluated typically several number of times [8]. W. Y. Lin validated the optimization techniques by generating the Triangle Curve with 22 Discrete Points, Asteroid Curve, ParamCurve Curve & Arc of Ellipse Curve with 41 Discrete Points. There is an optimization technique “exact gradient method” for optimal synthesis of mechanisms [16]. A unique feature of this method [J.

Mariappan and S. Krishnamurthy – 1996] is its applicability to many classes of mechanism synthesis problems, including different types of linkages and different kinds of objectives.

Position Equations for coupler point is given as [8]:

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} = r_1 e^{j\theta_0} + r_5 e^{j\theta_5} + r_4 e^{j\theta_4}$$

The objective function, which represents the error between the desired point and coupler point, may be expressed as:

$$f_{obj} = \sum_{i=1}^N [(C_{Xd}^i - C_X^i(X))^2 + (C_{Yd}^i - C_Y^i(X))^2]$$

Starns and Flugrad [23] and Subbian and Flugrad [22] applied the continuation method to a path synthesis task with seven precision points of a geared five-bar mechanism. In this method, the convergence of the synthesis equations doesn’t depend on the selection of initial values and, furthermore, it can be employed for a wide range of possible gear ratios. Roth and Freudenstein [12] proposed a numerical method for the path synthesis with nine target points of a geared five bar mechanism.

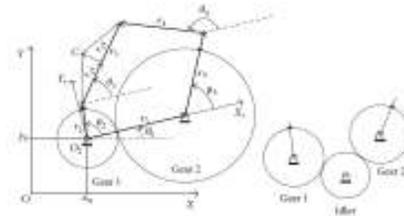


Fig. 3 Geared five bar mechanism in global coordinate system [8]

Path synthesis problems for special trajectories which are generated by GFBM have been studied in [15], where the square deviation of position was the objective function for the DE evolutionary algorithm. Although the proposed method needs a considerable number of evaluations of the objective function, it leads to accurate results. Comparing this method with the GA-DE evolutionary algorithm [5] shows a significant decrease in error. Nokleby and Podhorodeski [9] proposed a quasi-Newton optimization technique and Grashof criteria for the optimum path synthesis of a GFBM. They also demonstrated the effectiveness of the proposed synthesis routine at synthesizing Grashof GFBM.

**B. Two-Phase Method**

Recently, a two-phase synthesis method has been proposed in [7, 25] for path synthesis problems of four-bar mechanisms. A Smali and N Diab [7] proposed an ant-gradient optimization technique for dimensional synthesis of closed path generating 4 bar planar mechanisms through shape optimization. It is a two-phase synthesis process [5, 7, 25]. In first phase the coupler curve shape is optimized. In the second phase mathematical method is used to scale, rotate and translate linkage to its final configuration. They proposed cyclic angular deviation (CAD) vector for the shape optimization. The method uses less unknown variables (6), which converges the search in the design space [7]. They found that advantage of using CAD vector instead of Fourier descriptors is that this method is insensitive to the coupler curve profile and to any sharp edges it may have. This method can also be used in open –loop paths. Jacek Bus'kiewicz discussed the effectiveness of the curvature based description method for optimal synthesis of a closed curve. The GA method is used for synthesis [25].

### VI. Optimization

There are various optimization algorithms, including genetic algorithm (GA) and modified GA [5, 25], exact gradient [16], ant-gradient [7], genetic algorithm fuzzy logic [27], differential evolution (DE) and modified DE [15, 28, 29], particle swarm optimization [30], GA-DE [18], and hybrid optimizer [26], which can solve the optimization problems of path synthesis of the mechanisms. A general objective function is ‘error function’ which is used in different fashion for different techniques. Recently, a modified distance error function [14] has been used for the problem of path synthesis with prescribed timing. Matekar and Gogate [14] found the lower transverse errors with respect to the higher longitudinal errors because their assumption was that the former is an indication of closeness between the generated and prescribed paths and the latter one is an indication of the error in the timing. The objective function proposed in [8] is the error function of the square deviation of positions and optimized by the GA-DE evolutionary algorithm. A new error estimator is used as objective function in [26] which is defined by means of the precision points. This new error estimator facilitates the evaluation of the fitness of the function. The main advantage is translation, rotation, and scaling effects are not influenced by this method.

W Y Lin proposed a modified idea by combining differential evolution (DE) with the real-valued genetic algorithm (RGA) which is termed as GA-DE hybrid algorithm [3]. Differential vector perturbation is used in the RGA instead of crossover, with the best individuals as the base vectors. An improved solution

for the kinematic synthesis of the four-bar mechanism with rolling contacts was studied by W Y Lin [3, 4]. He proposed the GA-DE hybrid evolutionary algorithm to solve the optimization and path synthesis for the four-link mechanism. Sheu et al. [2] studied about the motion and function syntheses of a four-bar mechanism with rolling contacts. Lee and Tsai [6] studied about the path synthesis of a four-link mechanism with rolling contacts. The demonstration has also been presented [3, 4] for the effectiveness of the proposed method with suitable examples. The number of design variables for the optimum path synthesis of four link mechanism with seven target points can be reduced by 10 [4] when compared with that used in [2, 6] formulation. Hence a seven-target point synthesis problem becomes simpler in [4]. It can be concluded that the proper combination of various evolutionary algorithms enable us to solve the optimization problem more easily. Optimization of the link-length and revolution ratio of gears has been carried out in [15] by DE algorithm considering the necessary constraints.

### VI. Summary

It has been found that sometimes as per the need the DE algorithm is better than the GA-DE but it need more evaluation. So we can conclude that DE technique is better for accuracy but GA-DE is better for calculation time. One researcher has found GA-DE algorithm is better for the GFBM. Many researchers have combined different techniques and some have given better results also. Numerical methods are found to fit almost mechanisms but they require more calculation and they are not better than the other path specific techniques.

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