

NON LINEAR CONSTRAINED MODELS TO SOLVE THE VARIOUS MATHEMATICAL PROBLEMS

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ABSTRACT

In this work I will discuss constrained optimal models. Constrained nonlinear and non smooth models are much complicated to solve the unconstrained ones with a comparable number of independent variables and degree of non linearity because of the additional requirement that the solution must satisfy the constraints. The number of constrained non linear and non smooth optimization procedure is centered on one of three basic concepts.

KEY WORDS:Non linear optimization, optimal models, constrained models

Unconstrained optimization models arise directly in many practical applications. If there are natural constraints on the variables, it is sometime safe to disregard them and to assume that they have no effect on the optimal solution. Unconstrained models arise also as reformulations of constrained optimization models in which the constraints are replaced penalization terms in the objective function that have the effect of discouraging constraint violations. The general non linear programming model without constraint reduces to just.

$$\text{Minimize } f(x) \quad x \in E^n \quad \dots\dots\dots (1)$$

Where $f(x)$ is the objective function? Although most models arising in operations research have at least a few constraints with non negativity constraints being the type most often encountered. On the other hand constrained optimization model arise from which include explicit constraints on the variables.

The general non linear constrained model is stated as follows

$$\begin{aligned} \text{Min } f(x) \quad & x = (x_1, x_2, \dots, \dots, x_n, \\ \text{Subject to } g_i(x) & \leq \left. \begin{matrix} b_i, i = 1, 2, \dots, \dots, m \end{matrix} \right\} \end{aligned}$$

Where f and g_i are scalar valued functions and b_i are real numbers. The linearization of nonlinear models to meet the requirements for the iterative applications of linear programming method is one of the most obvious approaches to solve nonlinear programs.

Linear approximation of non linear functions is accomplished by replacing the non linear functions with their first order Taylor's series approximate expanded at the point of interest.

RESULTS AND DISCUSSION

Here I will use and explain the Quadratic Programming and some approximation methods. Various workers have developed algorithms to implement approximation of quadratic function instead of the linear functions Gill and Murray (1974) recognize that the second order methods in unconstrained optimization speed convergence extended the linear approximation to make a second order expansion of the general non linear model. First order conditions for a local stationary point were introduced. For an unconstrained model their algorithms stepped in the usual second order direction of descent with linear constraints and a quadratic objective function. Their computational procedure reduced to that of Rockasfellar(1973).

Generalized gradient Search (G.G.S.)

The generalized gradient search program was developed by Powell and Yuan (1990) which can accommodate both linear and non linear equality and inequality constraints. The program follows a steepest descent search in the interior of the feasible region where the numerical approximation of the partial derivatives of the objective function and the step size in a given direction at (K+1) stage are functions of the number of successful steps on kth stage. A gradient projection with linearized constraints is employed for the non trivial constraints. A projection technique is used to reach a feasible point from a non feasible starting point.

If a trivial constraint is violated, the stepsize is reduced by multiplying the current steplength by the ratio of distance between $x_j^{(k)}$ and its bounds if a projection is to be made onto bounding constraint as an inequality constraint a different procedure is used. The component $\frac{\partial f(x^{(k)})}{\partial x_j}$

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is set equal to zero and the variable x_j is removed from consideration. Thus the projection is carried out on a subspace in which x_j has been eliminated.

Let Δx represents the n -dimensional column vector and let Δb be a p -dimensional column vector.

To the first order $\Delta b = A\Delta x$

Where A is matrix of the first partial derivatives of the constraint with respect to x_j . The change in x , Δx is computed by multiplying A^T by a P -dimensional vector γ . $\Delta x = A^T\gamma$

So that $\Delta b = AA^T\gamma$. consequently since Δb and A are known.

$$\gamma = (AA^T)^{-1} \Delta b$$

The actual step taken is calculated from

$$x_j^{(k+1)} = x_j^{(k)} + (\Delta x)^T \left(\frac{\partial h}{\partial x_j} \right)$$

Where h include the matrix the active constraints.

Also the dot product of the gradient of the objective function and the unit normal n_i is

$$\nabla^T f(x^{(k)}) \cdot n_i = \nabla^T f(x^{(k)}) \frac{\nabla g_i(x^{(k)})}{\|\nabla g_i(x^{(k)})\|} \quad \parallel \quad \parallel$$

Extension of Davidon's Method To Accommodate Constraints

Davidon (1959) suggested that a method developed by him could be extended to non linear programs with linear equality and inequality constraints.

The discussion is based primarily on the work of Davidon (1959) who originally suggested the approach of Luksan (1994), who provided many of the details in the matrix manipulation. Murtagh and Sargent (1969) who modified the method and prepared a computer code to execute it.

ALGORITHM

Barnes (1967) derived the NLP algorithm for non linear programming method and extended the algorithms proposed by Di Bella and Stevens (1965), who designed algorithm only for equality constrained models. By using the linear approximates and near feasible points $x^{(k)}$. Dibella and Stevens (1965) suggested some changes to include the inequality constraints also. The NLP algorithm is divided conveniently into the steepest descent phase and the linear programming phase. The former is concerned only to improve the feasibility of the x -vector with respect to the equality and inequality constraints. Improvement of the objective function is

disregarded in this phase. After each linear programming move, a test is conducted to determine if the arbitrary constant. If the criterion is satisfied, the search is terminated, if not the search for an optimum is satisfied.

CONCLUSION

In this work I describe algorithms for the constrained optimization viz. linearization methods, penalty function methods technique which are currently use in optimization techniques. Constrained non linear and non smooth programming problems are much harder to solve than unconstrained problems with a comparable number of independent variables and degree of non linearly because of the additional Requirement that the solution must satisfy the constraints. Non linear programming models in which atleast non linear function tend to arise naturally in the physical sciences and engineering and are becoming more widely used in management and economic sciences having been discussed the unconstrained models. I focused concentration on constraints models. In this work after that I have discussed generalised gradients search method having being discussed several constraints models for non linear and non smooth optimization, I noticed that linear programming method have been successfully applied to large dimensional problems containing both equality and inequality constraints the linearisation of non linear programming problem to meet the requirement for the iteration application of linear programming method is one of the most obvious approach to solve the non linear programming methods.

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