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LIMITS OF AMPLITUDE n^{th}-POWER SQUEEZING IN KERR EFFECT

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ABSTRACT

We study amplitude n^{th}-power squeezing of the Hermitian operator, $Y_{\theta}^{(n)} \equiv Y_1^{(n)} \cos\theta + Y_2^{(n)} \sin\theta$, of light initially in coherent state interacting with a non-absorbing non-linear Kerr medium, modelled as an anharmonic oscillator, described by well-known interaction Hamiltonian, $H = \frac{1}{2}\lambda a^{+^2} a^2$. Here, the parameter λ is proportional to cubic non-linearity $\chi^{(3)}$ of the nonlinear medium, a and a^+ are, respectively, the annihilation and creation operators for the interacting field, $Y_1^{(n)} + iY_2^{(n)} = a^n$, and θ is an arbitrary angle. We find almost complete amplitude n^{th}-power squeezing in such interaction for very small interaction time and very large intensity of interacting light and optimize it by an analytic estimation assuming realistic values of Kerr non-linearity and intensity of interacting coherent light. We obtain a scaling law for optimal amplitude n^{th}-power squeezing in terms of a dimensionless interaction time $\tau \equiv \lambda t$, Kerr parameter r, which is product of τ and the average number of photons and power of squeezing n. The validity of the obtained scaling law has been checked numerically and analytically in the optical domain of realistic values of Kerr non-linearity and intensity of interacting light.

KEYWORDS: Non-classical states, Squeezing, Amplitude n^{th}-power squeezing, Anharmonic oscillator, Optical Kerr effect, Phase shifting operator.

In quantum optics, much attention is being paid to non-classical states (Walls, 1983; Dodonov, 2002) of light, due to their applications in quantum information theory such as communication, quantum teleportation, dense coding and quantum cryptography. Squeezing, a well-known non-classical effect, has been generalized (Hong *et. al.*, 1985; Hillery, 1987; Zhang *et. al.*, 1990; Prakash *et. al.,* 2002) to case of several variables. Zhang *et al* (Zhang *et. al.*, 1990) generalized amplitude-squared squeezing defined by Hillery (Hillery, 1987) to amplitude n^{th}-power squeezing. According to Zhang *et al* definition, a state $|\psi\rangle$ is said to be amplitude n^{th}-power squeezed for the operator,

$$Y_{\theta}^{(n)} \equiv Y_1^{(n)} \cos\theta + Y_2^{(n)} \sin\theta , \tag{1}$$

if the n^{th}-order moment of $Y_{\theta}^{(n)}$,

$$\langle\psi|(\Delta Y_{\theta}^{(n)})^2|\psi\rangle < \frac{1}{4}\left|\langle\psi|[Y_{\theta}^{(n)}, Y_{\theta+\frac{\pi}{2}}^{(n)}]|\psi\rangle\right| \tag{2}$$

Here $Y_1^{(n)} + iY_2^{(n)} = a^n$, $\Delta Y_{\theta}^{(n)} = Y_{\theta}^{(n)} - \langle\psi|Y_{\theta}^{(n)}|\psi\rangle$, $[Y_{\theta}^{(n)}, Y_{\theta+\frac{\pi}{2}}^{(n)}] = \sum_{r=1}^{n} ({}^nC_r)^2 r! a^{+(n-r)} a^{(n-r)}$ and θ is an arbitrary angle.

Many schemes like four-wave mixing, resonance fluorescence, the use of free electron laser, cavities, harmonic generation, parametric amplification and J C model have been proposed for generation of non-classical states. However, the interaction of coherent light with a non-absorbing non-linear Kerr medium modelled as anharmonic oscillator with well-known interacting Hamiltonian (Maker *et. al.*, 1964),

$$H = \frac{1}{2}\lambda a^{+^2} a^2 = \frac{1}{2}\lambda N(N-1) , \tag{3}$$

Has been paid to much attention because of the exactly solvable model for generation of non-classical states, which exhibits significant squeezing. Here $N = a^+a$, a and a^+ are, respectively, the annihilation and creation operators for the interacting field and the parameter λ is proportional to cubic non-linearity $\chi^{(3)}$ of the nonlinear medium. Several authors (Millburn, 1986; Gerry et. al., 1987; Buzek, 1989; Si-De Du *et. al.*, 1992) have studied the non-classical effects in such interaction and reported almost complete squeezing and amplitude n^{th}-power squeezing for very small interaction time and very large intensity of the interaction light. Recently Bajer *et.al*. (Bajer *et.al*., 2002) and Prakash *et.al.* (Prakash *et.al.*, 2008) respectively, have studied the problem of optimization of squeezing and amplitude-squared squeezing in such interaction and reported scaling

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laws for optimal squeezing and amplitude-squared squeezing using an analytic estimation in the region of short interaction time and high optical power. In this paper we study amplitude nth-power squeezing in such interaction and generalize the results for arbitrary power n. We optimize amplitude nth-power squeezing by an analytic estimation assuming high intensity of interacting light and realistic values of Kerr non-linearity and obtain a scaling law for optimal amplitude nth-power squeezing. The validity of the obtained scaling law has been checked numerically and analytically in the optical domain of realistic values of Kerr non-linearity and intensity of interacting light.

AMPLITUDE nth -POWER SQUEEZING IN INTERACTION OF COHERENT STATE WITH A NON-ABSORBING NON-LINEAR KERR MEDIUM

It is useful to define amplitude nth-power squeezing defined (Eq. (2)) earlier in another form, by the parameter

$$S_{\theta,n}=\frac{\langle\psi|(\Delta Y_\theta^{(n)})^2|\psi\rangle-\frac{1}{4}\left|\langle\psi|[Y_\theta^{(n)},Y_{\theta+\frac{\pi}{2}}^{(n)}]|\psi\rangle\right|}{\frac{1}{4}\left|\langle\psi|[Y_\theta^{(n)},Y_{\theta+\frac{\pi}{2}}^{(n)}]|\psi\rangle\right|}, \quad (4)$$

with $-1\le S_{\theta,n}<0$ for amplitude nth-power squeezed state. For the analytic estimation of optimal amplitude nth-power squeezing, we minimize $S_{\theta,n}$ with respect to all possible phasesθ. If for minimum of $S_{\theta,n}$ with respect to all possible phasesθ, we have $\frac{dS_{\theta,n}}{d\theta}=0$, which gives

$$e^{4i\theta}=\frac{\langle\psi|a^{2n}|\psi\rangle-\langle\psi|a^n|\psi\rangle^2}{\langle\psi|a^{+2n}|\psi\rangle-\langle\psi|a^{+n}|\psi\rangle^2} \quad (5)$$

Using Eqs. (4) and (5), we finally get amplitude nth-power squeezing factor P_n, which is minimum of $S_{\theta,n}$ with respect to all possible phases θ as

$$P_n=(S_{\theta,n})_{min\theta}=\frac{\frac{1}{2}\left[\langle\psi|a^{+n}a^n|\psi\rangle-\left|\langle\psi|a^n|\psi\rangle\right|^2-\left|\langle\psi|a^{2n}|\psi\rangle-\langle\psi|a^n|\psi\rangle^2\right|\right]}{\frac{1}{4}\sum_{r=1}^{n}({}^nC_r)^2 r!a^{+(n-r)}a^{(n-r)}}$$

. (6)

It is further useful to use principal amplitude nth-power squeezing factor S_n that takes values between 0 and 1 for amplitude nth-power squeezing and for present case we can write $S_n=1+P_n$, with $0<S_n<1$ for amplitude nth-power squeezed state.

If we consider the interaction system with interaction Hamiltonian H defined by Eq. (3) and interaction light in coherent state defined [9] by

$$|\alpha\rangle=\exp(-\tfrac{1}{2}|\alpha|^2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle=D(\alpha)|0\rangle, \quad (7)$$

where $\alpha=Ae^{i\theta_\alpha}$, $|n\rangle$ is the occupation number and $D(\alpha)=\exp(\alpha a^+-\alpha^* a)$ is the displacement operator, then in interaction picture, we have the Kerr state at time t,

$$|\psi\rangle=U(t)|\alpha\rangle=e^{-i(N^2-N)\frac{\tau}{2}}|\alpha\rangle. \quad (8)$$

Here $U(t)=\exp(-iHt)$ is the time evolution operator, and $\lambda t=\tau$, the dimensionless interaction time. Now we have for the Kerr state,

$$\langle\psi|a^n|\psi\rangle=\alpha^n e^{|\alpha|^2(e^{-in\tau}-1)-i(n^2-n)\frac{\tau}{2}}, \quad (9)$$

$$\langle\psi|a^{+^n}a^n|\psi\rangle=|\alpha|^{2n}, \quad (10)$$

and therefore finally we have amplitude nth-power squeezing parameter,

$$S_n=1+\frac{\frac{|\alpha|^{2n}}{2}\left[1-e^{2|\alpha|^2(\cos n\tau-1)}-\left|e^{|\alpha|^2(e^{-2in\tau}-1)-i(2n^2-n)\tau}-e^{2|\alpha|^2(e^{-in\tau}-1)-i(n^2-n)\tau}\right|\right]}{\frac{1}{4}\sum_{r=1}^{n}({}^nC_r)^2 r!|\alpha|^{2(n-r)}}. \quad (11)$$

We get finally amplitude nth-power squeezing parameter,

$$S_n = 1 + \frac{\frac{|\alpha|^{2n}}{2}\left[1 - e^{2|\alpha|^2(\cos n\tau - 1)} - \left\{e^{2|\alpha|^2(\cos 2n\tau - 1)} + e^{4|\alpha|^2(\cos n\tau - 1)} - 2e^{|\alpha|^2(\cos 2n\tau - 1)}e^{2|\alpha|^2(\cos n\tau - 1)}\cos\Phi\right\}\right]}{\frac{1}{4}\sum_{r=1}^{n}({}^nC_r)^2 r!|\alpha|^{2(n-r)}}, \quad (12)$$

where

$\Phi = (n^2\tau + |\alpha|^2 \sin 2n\tau - 2|\alpha|^2 \sin n\tau)$. Dependence of the amplitude nth-power squeezing parameter S_n on interaction time τ at the amplitude $|\alpha| = 5$ of interacting field for some values of power n is shown in the Fig.1.

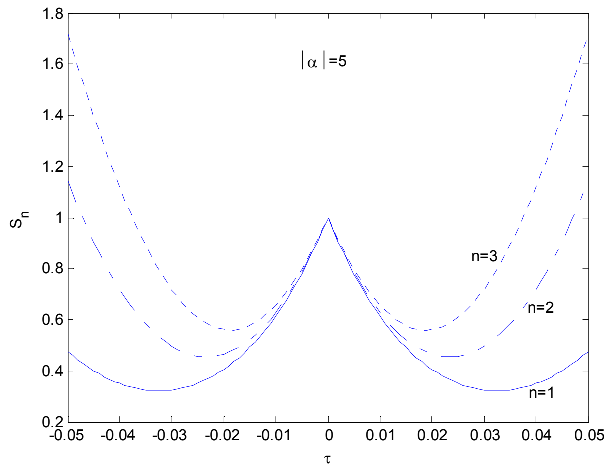

Figure 1: Variation of the squeezing parameter S_n with τ for some values of n at $|\alpha| = 5$.

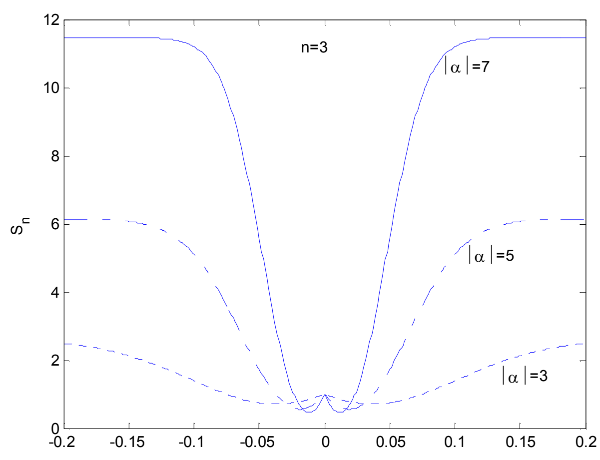

Figure 2: Variation of the squeezing parameter S_n with τ for n=3 at some values of $|\alpha|$.

It should be noted from the figure that amplitude nth-power squeezing in such interaction decreases with power n. Dependence of the amplitude nth-power squeezing parameter S_n (e.g. n=3) on interaction time τ for some values of the amplitude $|\alpha|$ of interacting field has been shown in Fig.2. It should be noted that amplitude nth-power squeezing appears significant when we decrease interaction time τ and increase the amplitude $|\alpha|$ of the interacting field.

AMPLITUDE nth-POWER SQUEEZING APPROXIMATION AND SCALING FORMULA

In the experiments, dimensionless interaction time τ is fixed by the length of fiber and hence in such interaction the amplitude nth-power squeezing can be controlled simply by adjusting the amplitude $|\alpha|$ of interacting field. For this reason the Kerr parameter r defined by $r = |\alpha|^2\tau$, is usually used to study the non-classical effects in such interaction. Hence we study amplitude nth-power squeezing parameter $S(r, \tau)$ in its dependence on the Kerr parameter r instead of its dependence on the amplitude $|\alpha|$ of the interacting field for the fixed interaction time τ. In general the value of Kerr non-linearity is usually very small and practically the dimensionless interaction time of the order of $\tau = 10^{-6}$ and $r \approx 1$ (for intense laser) can be reached in optical domain (Bachor: A Guide to Experiments in Quantum Optics, 1998). In Fig. 3 we show the dependence of amplitude nth-power squeezing parameter $S_n(r, \tau)$ on the Kerr parameter r for some fixed values of interaction time τ. The regular dependence of $(S_n)_{min}$ (the minimum value of amplitude nth-power squeezing parameter) on the interaction time τ may be noted form Fig. 3 indicates the existence of a scaling law of the form $(S_n)_{min} \approx r_{min}{}^{\gamma}$, for the optimal amplitude nth-power squeezing in such interaction, where r_{min} is the minimum value of r at which $S_n(r, \tau)$ is minimum for a fixed interaction time τ.

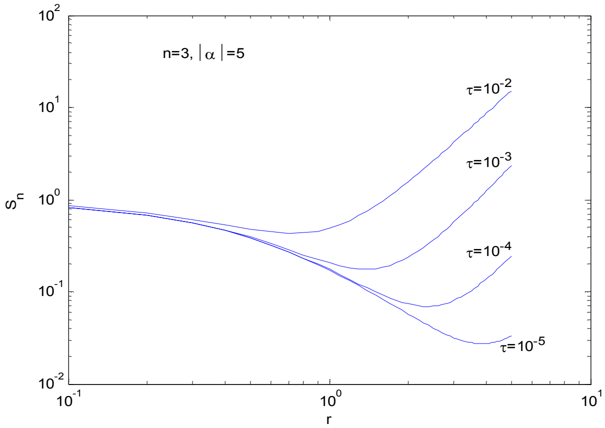

Figure 3: Variation of the squeezing parameter S_n with Kerr parameter r for some values τ for n=3 at $|\alpha| = 5$ (on log-log scale).

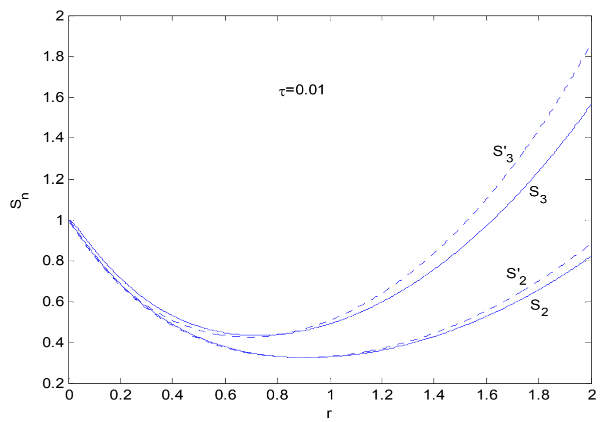

Figure 4: Comparison of the approximation S'_n with S_n as a function of r at $\tau = 10^{-2}$.

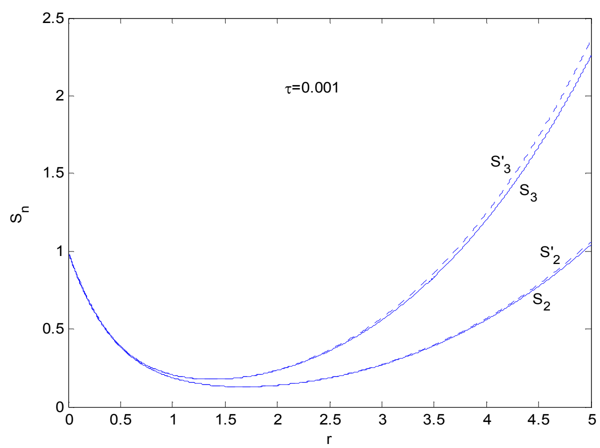

Figure 5: Comparison of the approximation S'_n with S_n as a function of r at $\tau = 10^{-3}$.

Now in the limit, $\tau \ll 1$, Eq. (12) gives,

$$S_n \cong S'_n = 1 + 2r^2 - n^2 r^3 \tau - 2r\sqrt{r^2+1} + \frac{nr^2\tau(3nr^2+3n+2)}{\sqrt{r^2+1}}, \quad (13)$$

to the first order in τ . Accuracy of the approximation (13) in comparison with exact values of S_n for two different values of τ can be seen in Fig. 4 and Fig. 5. The figures show that the approximation S' can be used to estimate the optimal amplitude n^{th}-power squeezing with a good precision. If we expand Eq. (13) in powers of r^{-1} and keep the smallest order terms in r^{-1} we obtain,

$$S'_n \cong S''_n = 2n^2 r^3 \tau + \frac{1}{4r^2}. \quad (14)$$

This gives

$$(S''_n)_{min} = \frac{5}{12}(12n^2\tau)^{2/5}, \quad (15)$$

at $r'_{min} = (12n^2\tau)^{-1/5}$. We see that in the limit $\tau \ll 1$, the minimum value $(S''_n)_{min}$ of S''_n is proportional to the $(2/5)^{th}$ power of the interaction time τ and it occurs at the Kerr parameter r'_{min} which is proportional to the $(-1/5)^{th}$ power of interaction time τ. It may be noted from Eq.(15) that the very same scaling laws but with different numerical coefficients have been obtained for normal squeezing by Bajer et.al..(Bajer *et.al.*, 2002) and amplitude-squared squeezing by Prakash *et. al.* (Prakash *et.al.*, 2008).

Table 1: Several numerical values of $(S_n)_{min}$ and the corresponding r_{min} obtained at the scaled interaction time and their estimations $(S''_n)_{min}$ and r'_{min} for n=3.

τ	$(S_n)_{min}$	$(S''_n)_{min}$	r_{min}	r'_{min}
10^{-1}	0.8732	1.0793	0.32	0.62
10^{-2}	0.4329	0.4297	0.72	0.98
10^{-3}	0.1761	0.1711	1.38	1.56
10^{-4}	0.0693	0.0681	2.36	2.47
10^{-5}	0.0273	0.0271	3.85	3.92
10^{-6}	0.0088	0.0108	6.60	6.21

Some numerical values of $(S_n)_{min}$ and $(S''_n)_{min}$ for different values of dimensionless interaction time τ are shown in Table 1. From the table it may be noted that in the region of realistic values of r and dimensionless interaction time τ, the approximation may be used with good precision.

CONCLUSION

We have analyzed amplitude n^{th}-power squeezing of light initially in coherent state interacting with a non-absorbing non-linear Kerr medium, modelled as an anharmonic oscillator modelled as an anharmonic oscillator, described by well-known interaction Hamiltonian, $H = \frac{1}{2}\lambda a^{+2} a^2$. Here, the parameter λ is proportional to cubic non-linearity $\chi^{(3)}$ of the nonlinear medium, a and a^+ are, respectively, the annihilation and creation operators for the interacting field. We found almost complete amplitude n^{th}-power squeezing in such interaction for very small interaction time and very large intensity of interacting light. We have optimized it by an analytic estimation assuming realistic values of Kerr non-linearity and intensity of interacting coherent light and obtained a scaling law for optimal amplitude n^{th}-power squeezing in terms of a dimensionless interaction time $\tau \equiv \lambda t$, Kerr parameter r, which is product of τ and the average number of photons and power of squeezing n. The validity of the obtained scaling law has been checked numerically and analytically in the optical domain of realistic values of Kerr non-linearity and intensity of interacting light.

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