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DETECTION OF OBJECTS IN BINARY IMAGES USING MOMS

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Abstract — In this paper, object identification is done using MOMs (moments) in the binary images which are stored in the computer. The photograph is taken, converted from colored image to gray scale image & then further converted in binary image and stored in the memory of the computer. These images are further used for object identification.

Keywords- Moments, Invariance, Centroid, Moment of Inertia, Principal angle, Orientation, Binary image, Gray scale image.

I. Introduction

SHAPEanalysis is a method of finding the shape of irregular objects using two types of descriptors called as the line descriptors and the area descriptors and is used when the objects are not polyhedral objects. For example, circles, spheres, ellipses, boundaries, curves, arcs, objects of irregular shapes. There are two methods of performing the shape analysis of objects. viz., line descriptors & the area descriptors. Line descriptors are used to find out the length of the irregular boundary or curvature of an irregular object in a digital image in terms of pixels.

Area descriptors are used to find out the shape of the irregular object and its characteristic properties such as area, moments, central moments, centre of gravity (centroid), moment of inertia and the orientation of the object w.r.t. (x , y) axis of the image. The area descriptors are defined as the descriptors which are based on the analysis of the points enclosed by the boundary and are used to characterize the shape of the foreground region R in a image. In this paper, we discuss about the shape analysis concept of the captured objects in a 2 D image using moments [1].

The theory of moments provides an interesting method of describing the properties of an object in terms of its area, position and orientation parameters. The idea of moments was borrowed from the science of physics. In this paper, we discuss about a method of computing the features of objects in a 2D image.

The paper is organized as follows. A brief introduction about the shape analysis of objects was presented in the previous paragraphs. In section 2, a brief introduction about the foreground & background region in an image is dealt with. Shape analysis using moments is described briefly in section 3. Section 4 deals with the analytical / mathematical treatment of a simulation example. Section 5 shows the simulation results. Conclusions are presented in section 6 followed by the references.

II. Foreground Region and Background Region in a Binary Image

Consider a binary image B (k , j) as shown in Fig. 1 which is obtained by the segmentation / thresholding / binarization of a digital image or a GS image I (k , j). Foreground is represented by 1's & background is represented by 0's. There was a circular object with a hole inside [2]. First, it was captured by a camera, digitized &thenbinarized. B (k , j) is the binary representation of the object with the hole inside & is as shown in the Fig. 1.

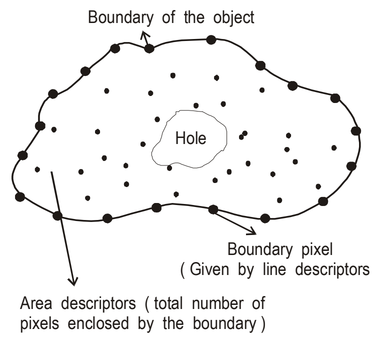

Fig. 1 : 2D representation of a elliptical object in a image

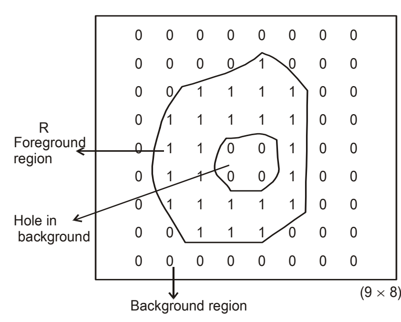

Fig. 2 : A foreground region R in an binary image ;

White - 1 (Foreground), Black - 0 (Background pixel)

A region R in a binary image is defined as a set of connected pixels as shown in Fig. 2, which are having the same gray level attribute. R is a connected set, i.e., for each pair of pixels in R, there is at least one path, which connects the pair. Foreground region is one connected set

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and background region forms another connected set. This makes sure that there are no breaks in the part. Since R is a connected set, R is a single part (with no breaks). Connected in the sense, there is a neighboring pixel, which is having the same gray scale value as the pixel considered. R can also have a hole in it. Now, we have to compute the shape of the alphabet O. In order to compute the shape of the alphabet O, we consider only the foreground region and the background region is neglected [3].

II. Shape Analysis Using Moments

To analyze and characterize the shape of the given foreground region R, we compute some numbers. These numbers are called as the moments of the foreground region R. Moments gives the characteristic features of the objects such as the shape, area, centroid, moment of inertia and the orientation of the object in the image. There are four types of moments, viz., lower order moments, central moments, normalized central moments & the principal angle [4].

Moments are defined as the sequence of numbers which are used to characterize the shape of any object OR the sum of products of the row value **x** raised to the power k and column value **y** raised to the power j in R.

$$m_{kj} \triangleq \sum_{row\,,\,column\ \in\ R} row^{k}\ column^{j} \quad (1)$$

$$\overset{\Delta}{=} \sum_{x,\,y \in R} x^{k} y^{j} \quad ; \quad k \geq 0\ ;\ j \geq 0 \quad (2)$$

x and y : Row and column values of the pixel in the foreground region R.

Order of the moment = Sum of the powers = (k + j).

In calculation of moments, consider only the foreground pixels and ignore the background pixels.

A. Lower Order Moments : Ordinary Moments, m_{kj}

Geometric meanings and physical significance of the LOM could be explained as follows. Let $\{m_{kj}\}$ be the ordinary moments of the foreground region R of a binary image B (k , j), A being the area of the foreground region R and $\{ x_c , y_c\}$ be the position of the centroid of the region R. Then [2], [5],

$$\begin{aligned} Area &= A = m_{00} \\ x_c &= \frac{m_{10}}{m_{00}} \\ y_c &= \frac{m_{01}}{m_{00}} \\ (x_c, y_c) &= Centroid = \left\{ \frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right\} \end{aligned} \quad (3)$$

- Zeroth order moment m_{00} gives the area of the foreground region R or the count of the total number of pixels in the region R or it is the measure of the size of the region R.
- m_{10} gives the first order moment (lower order) along x-axis.
- m_{01} gives the first order moment (lower order) along y-axis.
- m_{20} gives the second order moment (lower order) along x-axis.
- m_{02} gives the second order moment (lower order) along y-axis.
- m_{11} gives the product moment.
- m_{22} gives the product moment.
- Normalize the first order moments with the zeroth order moments, i.e., divide the first order moments with the zeroth order moments $\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}}$. This gives the centroid of the foreground region R.

The physical significance of lower order moments is, they just give the area and the centroid of the foreground region R.

B. Central Moments of a Foreground Region R, μ_{kj}

The geometric meanings and physical significance of central moments can be explained as follows. Central moments are called so because they are obtained using the centroid (x_c, y_c). The central moments of foreground region R are nothing but the ordinary moments m_{kj} of the foreground region R, but, translated by an amount equal to the centroid so that the centroid (x_c,y_c) now coincides with the origin, as a result of which the first order central moments are equal to zero and are invariant to translations of the foreground region R (i.e., $\mu_{10} = \mu_{01} = 0$). Let (x_c, y_c) be the centroid of a region R and let (x, y) be the row and column of a pixel p in R. The central moments μ_{kj} of R are given by

$$\mu_{kj} = \sum_{(x,y)\in R} (x - x_c)^k (y - y_c)^j \quad ; \ (k, j) \geq 0 \tag{4}$$

The physical significance of the central moments are, they just give the area and the moment of inertia and they are invariant to translations, but are variant to scale changes and rotations [2], [6].

- Lower order central moments (Zeroth), μ_{00} is the same as m_{00} ; i.e., gives the area of the region R or the size of the region, i.e., the number of pixel counts [16].
- Lower order central moments (First), μ_{10} : gives the first order central moment along x-axis = 0 (since centroid coincides with origin) &μ_{01} : gives the first order central moment along y-axis = 0 (since centroid coincides with origin).

During the calculation of the moments, each point (x , y) is shifted by an amount equal to the distance of the centroid from the origin ; so finally over the sum, the centroid coincides with the coordinate origin. Any translations of the region R are negated because of this shifting process. Since, central moments are invariant to translations, the first order central moments μ_{01} and μ_{10} are always = 0. Invariancy to translations means ; in the image, if the object is translated along x or y-axis the properties of the object remains the same. The second order central moments could be explained as follows :

- μ_{20} : Gives the second order central moment along x-axis. Gives the Moment of Inertia [M I] of the foreground region R about the x-axis.
- μ_{02} : Gives the second order central moment along y-axis. Gives the Moment of Inertia [M I] of the foreground region R about the y-axis.
- The x and y-axes pass through the centroid of the region R.
- μ_{11} is a product moment as it involves finding the product of (x – x_c) & (y – y_c) raised to a power after which, the products are summed to give μ_{11}.
- μ_{22} is another product moment which is not of any physical significance [15].

C. Normalized Central Moments, v_{kj}

The geometric meanings and physical significance can be explained as follows. Central moments are further normalized to produce another type of moments, called as the normalized central moments, which are invariant to scale changes of the foreground region R, in addition to translation invariance. If μ_{kj} are the central moments of region R and v_{kj}, then the normalized central moments v_{kj} of R are given by [2]

$$v_{kj} = \frac{\mu_{kj}}{\mu_{00}^{(k+j+2)/2}} \quad ; (k, j) \geq 0 \tag{5}$$

This property of invariancy to scale changes occurs because the area of the region R is scaled down by the factor of $\frac{(k+j+2)}{2}$. Invariancy to scale changes means ; in the image, if the object is zoomed in or zoomed out, the properties of the object remains the same.

- v_{00} :Zeroth order NCM = 1, from this, we can come to a conclusion that the NCM are invariant to scale changes of R.
- v_{10} and v_{01} : First order NCM along x and y-axis = 0, since they are invariant to translations.
- v_{20} and v_{02} : Second order NCM along x and y-axis.
- v_{11} and v_{22} : Product moments, not of any physical significance.

The physical significance of NCM's are they are invariant to scale changes, in addition to translation invariance, but variant to rotations [7].

D. Principal Angle : Orientation or Inclination of the Region, ϕ

Lower order moments m_{kj}, central moments $\mu_{kj}$and normalized central moments v_{kj}, characterize the region R and are invariant to translations and scaling of R, but are variant to rotations of the foreground region R.

Invariancy to rotations of R can also be obtained by finding the principal angle ϕ and is a measure of the orientation of the region R. The principal angle ϕ can be expressed in terms of the second order central moments and so does not involve additional calculations.

The physical significance of principal angle is ; it is invariant to rotations of the foreground region and gives the orientation of the foreground region R in the binary image. Invariancy to rotations means ; in the image, if the object is rotated by any amount, then, the properties of the object remains the same [8].

The physical interpretation of the principal angle ϕ is as follows. Draw a line L (β) through the centroid (x_c, y_c) of R at angle of β with the x-axis. The moment of inertia of R about the line L(β) depends on the angle β. Go on varying the angle β as shown in the Fig. 3.

At one point, the MI of the object will be minimum. The angle at which the moment of inertia I_R of region R is minimized is called the principal angle of R and is nothing but ϕ which is equal to β. It follows that the principal angle is well defined for an elongated object, but it becomes ambiguous when the object approaches a circular shape [9].

Principal angle ϕ of the foreground region R is defined as the angle at which the moment of inertia I_R of region R is minimized. Mathematically, it is given by the formula [2]

$$\phi = \frac{1}{2} a \tan 2\left(2\mu_{11}, \mu_{20} - \mu_{02}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right) \tag{6}$$

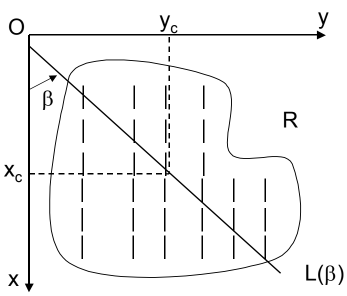

Fig. 3 : Principal angle of a region R

E. Invariant Moments

Now, we know the centroid of R and the principal angle of R, i.e., ϕ. If we translate R by an amount = ($-x_c$, $-y_c$), then the centroid coincides with the origin. Rotate R by an angle of $-\phi$(clockwise) ; then, the principal angle becomes zero. Now, if we take the moments of the region R, it will be seen that these moments are invariant to translations, rotations and scale changes [14]. The normalized moments of the resulting region will then be invariant to translations, rotations and scale changes of R. Such moments are called as invariant moments. Invariant moms are defined as the moments, which are invariant to translations, scale changes and rotations of the foreground region R [10].

F. Eccentricity

It is the maximum chord length along the principal axes or major axis of object divided by the minimum chord length, which is perpendicular to chord length. The maximum chord length or major diameter D of an object O is defined as

$$\text{Eccentricity} = \frac{\text{Maxmimum chord length}}{\text{Minimum chord length}} \tag{7}$$

$$D = \max_{i,j} \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} \tag{8}$$

where $p_i = (x_i, y_i)$ and $p_j = (x_j, y_j)$ are the pixels in the boundary of the object O.

$$\text{Thinness} = \frac{(\text{Perimeter})^2}{2} \quad 9)$$

$$\text{Roundness} = \frac{\left(x_c^{\,2} + y_c^{\,2}\right)^2}{A}$$

IV. Simulation example (analytical calculation)

In this section, we consider a zig-zag object was captured by the camera, digitized, segmented &binarized and stored in the memory of the computer as shown in Fig. 4. It is necessary for us to compute the shape of the objects using the methods discussed in the previous section. A brief analytical / mathematical treatment to the problem considered is dealt with in this section. The following computations can be seen as below [11].

Col j →

Row k ↓	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	1	1	0	1	0	0	0
4	0	0	1	1	1	1	0	0
5	0	0	1	1	1	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0

Fig. 4 : An image of size (8 × 8)

$$m_{00} = \sum_{x,\, y \in R} x^0 y^0 = \text{Area}$$

$$= 10$$

$$m_{01} = \sum_{x,\, y \in R} x^0 y^1$$

$$= 40$$

$$m_{10} = \sum_{x,\, y \in R} x^1 y^0$$

$$= 40$$

$$m_{02} = \sum_{x,\, y \in R} x^0 y^2 = \sum_{x,\, y \in R} y^2$$

$$= 174$$

$$m_{20} = \sum_{x,\,y \in R} x^2\, y^0 = \sum_{x,\,y \in R} x^2$$

$$= 166$$

$$m_{11} = \sum_{x,\,y \in R} x^1\, y^1$$

$$= 162$$

$$m_{22} = \sum_{x,\,y \in R} x^2\, y^2$$

$$= 2968$$

$$x_c = \frac{m_{10}}{m_{00}} = \frac{40}{10} = 4 \quad \text{and}$$

$$y_c = \frac{m_{01}}{m_{00}} = \frac{40}{10} = 4$$

Centroid , $(x_c y_c) = (4, 4)$

$$\mu_{00} = \sum_{x,\,y \in R} (x-4)^0\,(y-4)^0 = 10$$

$$\mu_{02} = \sum_{x,\,y \in R} (x-4)^0\,(y-4)^2 = 14$$

$$\mu_{20} = \sum_{x,\,y \in R} (x-4)^2\,(y-4)^0$$

$$= 6$$

$$\mu_{11} = \sum_{x,\,y \in R} (x-4)^1\,(y-4)^1$$

$$= 2$$

$$\mu_{22} = \sum_{x,\,y \in R} (x-4)^2\,(y-4)^2$$

$$= 8$$

Calculation of normalized central moments as follows

$$v_{kj} = \frac{\mu_{kj}}{\mu_{00}^{(k+j+2)/2}} \; ; \; k \ge 0 \;,\; j \ge 0$$

$$v_{00} = \frac{\mu_{00}}{\mu_{00}^{(0+0+2)/2}} = \frac{\mu_{00}}{\mu_{00}^{1}} = 1$$

$$v_{10} = \frac{\mu_{10}}{\mu_{00}^{(1+0+2)/2}} = \frac{\mu_{10}}{\mu_{00}^{1.5}} = 0$$

$$v_{01} = \frac{\mu_{01}}{\mu_{00}^{(0+1+2)/2}} = \frac{\mu_{01}}{\mu_{00}^{1.5}} = 0$$

$$v_{02} = \frac{\mu_{02}}{\mu_{00}^{(0+2+2)/2}} = \frac{\mu_{02}}{\mu_{00}^{2}} = 0.140$$

$$v_{20} = \frac{\mu_{20}}{\mu_{00}^{(2+0+2)/2}} = \frac{\mu_{20}}{\mu_{00}^{2}} = 0.36$$

$$v_{11} = \frac{\mu_{11}}{\mu_{00}^{(1+1+2)/2}} = \frac{\mu_{11}}{\mu_{00}^{2}} = 0.02$$

$$v_{22} = \frac{\mu_{22}}{\mu_{00}^{(2+2+2)/2}} = \frac{\mu_{22}}{\mu_{00}^{3}} = 0.008$$

$$\text{Principal angle} = \phi = \frac{1}{2}\tan^{-1}\left(\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2\times 2}{6-14}\right)$$

$$= 76.72°$$

All the results are summarized as shown in the table 1. A graphical user interface program was developed in C / C++ language and the code was compiled and run [12].

On running the code, the following screens as shown below appeared, i.e., the inputs & the output screens, which are nothing but the simulation results as shown in the Figs. 5 - 8 respectively [13].

Type	Lower Order Moms m_{kj}	Central Moms μ_{kj}	Normalized Central Moms v_{kj}
Zeroth (0 , 0)	10	10	1
First (1 , 0)	40	0	0
First (0 , 1)	40	0	0
Second (2 , 0)	166	6	0.36
Second (0 , 2)	174	14	0.140
Product (1 , 1)	162	2	0.02
Product (2 , 2)	2968	8	0.008
Centroid (x_c , y_c)	(4 , 4)		
Principal angle ϕ	76. 72°		

Table 1

V. Simulation results

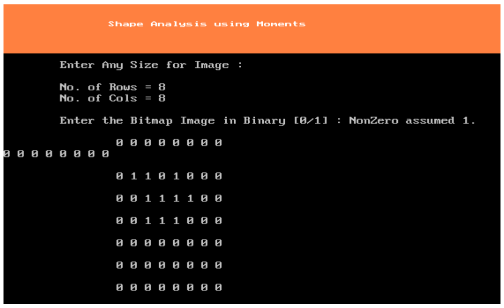

Fig.5 : Simulation result 1

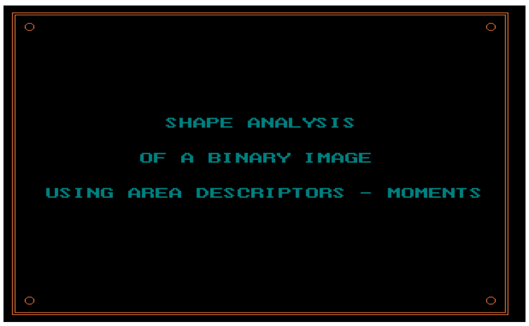

Fig.6 : Simulation result 2

Fig. 7 : Simulation result 3

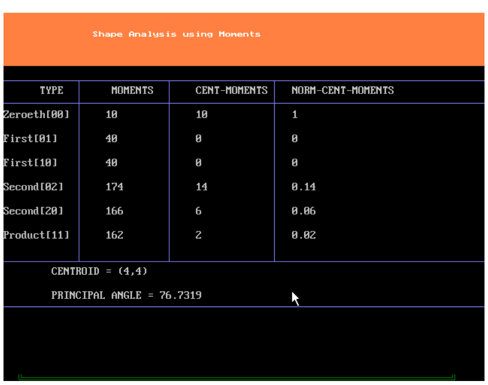

Fig.8 : Simulation result 4

VI. Conclusions

A method of computing the moments of a binary image was presented in this paper. The major disadvantage of moments in general is that they are global features rather than local. This makes them not suited for recognizing objects, which are partially obstructed. Moments are inherently location dependent, so some means must be adopted to insure location invariance (like the centroid). The mathematical results & the experimental results / simulated results shows the effectiveness of the developed method [2].

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