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SAME GENDER STABLE FRIENDSHIP PREFERENCE PROBLEM (SGSFPP)

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ABSTRACT

In this paper, we have characterized absolutely stable friendship problems having a non empty core. This characterization has allowed us to state that if the core of an absolutely stable friendship problem is not empty, it contains a unique matching/mating in which all top ranked agents are mutually matched to one another and all other agents remain unmatched.

KEYWORDS: The Friendship algorithm (TFA), Same Gender Stable Friendship Preference Problem (SGSFPP), Stability of True Friendship.

Matching/Mating problems (Irving, 1985) arise so many applications in real life. Dating services want to pair up compatible couples. Interns need to be matched to Same Gender Stable Friendship Preference Problem (SGSFPP). Other assignment problems involving resource allocation arise frequently, including balancing the traffic load among servers on the internet.

The stable roommate's problem are originally posed by Gale and Shapley in 1962 involves a single set of even cardinality 2n, each member of which ranks every other member in order of preference.

In 1984, Irving published an algorithm that determines in polynomial time, if a stable matching is possible on a given set, and if so, finds such a matching. However, others have made efforts to redefine the concept of a stable matching, or even reframe the problem altogether to give it new real-world significance.

Especially in mathematics, the fields of game theory and combinatorics are used to find out stable friendship using roommate's problem.

Background

In their 1962 paper College Admissions and the Stability of Marriage,(Irving et al., 2002) David Gale and Loyd Shapley proposes the stable marriage problem. The problem concerns n men and n women, each of whom is to marry one partner.

Each man and woman ranks each woman and man, respectively (Kirubaharan and Nirmala,2012) , from 1 o n in order of preference. A matching is defined as a set of n disjoint pairs containing one woman and one man each. For convenience sake, we may refer to the woman's partner in the matching as her "husband" and the man's partner in the matching his wife.

A matching is stable when no woman x prefers a man y to her husband in the matching where y also prefers x to his wife.

In this paper, Gale and Shapley furnished an algorithm that always provides a stable matching/mating in polynomial time, and we propose here the related friendship problem. This problem concerns 2n participants who each rank the other 2n - 1 member in order of preference.

A matching in this context is just a set of n disjoint pairs of participants. The two participants in a same gender pair will henceforth be referred to as "friends" in the matching. In this problem, a matching is stable when no participants x and y exist who prefer each other to their present friends.

Definition: 1

In a graph G = (V, E), a matching is a sub graph of G where every node has degree 1. In particular, the matching consists of edges that do not share nodes.

The Friendship Algorithm (TFA)

Here is a method (Kirubaharan and Nirmala,2012) for getting everyone paired up in same gender. The stable friendship ritual takes place over several days. The idea is that each of the boys seek best boy friend one by one, in order of friendship preference, crossing off bad boy from their list as they get rejected. Here is a more detailed specification.

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Initial Condition: Each of the N boys has an ordered list of the N other boys according to their friendship preferences. Each of the other boys has an ordered list of the boys according to their preferences.

Each Day:

Ist day:

- All boys are standing in the play ground
- Each boy gives a company of his favorite best friend from other group boy whom he has not yet crossed off his list and serenades. If there are no boys left on his list, he stays at home and does homework.

IInd day:

- Boys who has at least one friend says to their favorite from among the friends that day: Maybe, come back tomorrow.
- To the other: they say No, I will never meet or contact you!

IIIrd day:

- Any boy who hears No crosses that boy off his list.

Termination Condition: If there is a day when every boy has at most one best friend, we stop the processes.

Definition: 2

In a stable friendship matching/mating, each boy is assigned to at most one best boy, and each other boy is assigned to at most one best boy.

As per figure:1 a matching is perfect if each boy is assigned with best boy and each best boy is assigned with boy (clearly we need the number of boys to be the same as the number of best boyfriends).

A matching is stable if it is perfect and there is no unstable pair.

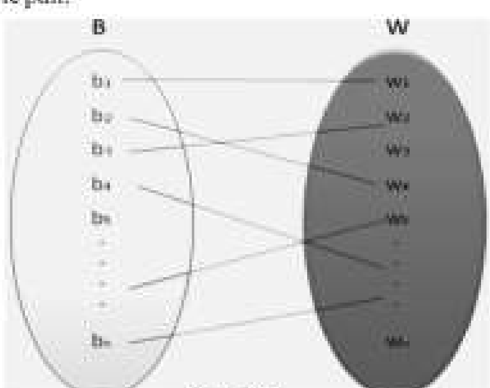

Figure: 1

Result: 1

There are n boys b_1, …. , b_n and other clever/best boys namely c_1, …. , c_n. For each boy, there is a preference list on the best boy (e.g. $c_1 > c2 > ... > c_n$), and for each best boy, there is a preference list on the boys. The TFA is to find a stable friendship for the two groups of same genders.

Result: 2

There are 2n people, each friendship can communicate with two people. Each person has a preference list on the other 2n - 1 people. The stable friendship problem is to find a stable matching/mating (i.e. no unstable pair and perfect) for the *2n* people.

Stable friendship problem (Irving,1985 and Thayer, 2007) is not like stable marriage problem but in general, have a solution. For a minimal counter example, consider 4 people A, B, C, and D where all prefer each other to D, and A prefers B over C, B prefers C over A, and C prefers A over B (so each of A,B,C is the most favorite of someone). In any solution, one of A,B,C, must be paired with D and the other 2 with each other, yet D's friend and the one for whom D's friend is most favorite would each prefer to be with each other.

The above algorithm consists of two phases, namely phase: 1 and phase: 2

Phase: 1

In this phase (Gusfield ,1988) participants propose their friendship to each other, in a manner similar to that of the Gale Shapley algorithm for the stable marriage problem. Here participants propose their friendship to each person on their preference list, in order, continuing to the next person if and when their current friendship proposal is rejected.

A participant rejects a friendship proposal if he already holds, or subsequently receives, a proposal from someone he prefers. That is, one participant might be rejected by all of the others, an indicator that no stable matching is possible.

Otherwise, phase 1 ends with each person holding a friendship proposal from one of the others this situation can be represented as a set S of ordered pairs of the form (p, q), where q holds a proposal from p - we say that q is p's current favorite. In this case that this set represents a matching, i.e., (q, p) is in S whenever (p, q) is, the algorithm terminates with this matching, which is bound to be stable.

Phase: 2

Otherwise the algorithm (Gusfield,1988) enters into Phase:2, in which the set S is repeatedly changed by the application of so-called rotations. Suppose that (p, q) is in the set S but (q, p) is not. For each such p we identify his current second favorite to be the first successor of q in p' s preference list who would reject the friendship proposal that he holds in favor of p.

A rotation (Kirubaharan and Nirmala, 2012) and(Harary,1969) relative to S is a sequence (p_0,q_0), (p_1,q_1) ,........, (p_{k-1},q_{k-1}) such that (p_i,q_i) is in S for each i, and q_{i+1} is p_i's current second favorite (where i + 1 is taken modulo k). If, such a rotation (p_0,q_0) ,......, (p_{2k},q_{2k}), of odd length, is found such that $p_i = q_{i+k+1}$ for all i (where i +k + 1 is taken modulo 2k + 1),this is what is referred to as an odd party, which is also an indicator that there is no stable matching. Otherwise, application of the rotation involves replacing the pairs (p_i,q_i), in S by the pairs (p_i,q_{i+1}), (where i + 1 is again taken modulo k).

Phase: 2 of the algorithm can now be summarized as follows

```
S = output of phase 1;
    while (true)
{
     identify a rotation r in s;
          if (r is an odd party)
return null ; (there is no stable friendship)
          else
          apply r to s ;
          if  (S is a good friendship)
          return S; (guaranteed to be stable)
    }
```

The following table 1 illustrates six friends (same gender) and preference list of their friendship.

Table: 1(Phase - I pay off matrix)

Order/Nameof the Friends	Proposed Friendship Preference List (Same Gender)					
	I	II	III	IV	V	VI
f_1	-	3	1	2	5	4
f_2	4	-	5	3	2	1
f_3	4	1	-	2	3	5
f_4	5	2	3	-	1	4
f_5	2	3	1	4	-	5
f_6	2	5	3	4	1	-

Significant Symbols: denotes proposes and x denotes rejects

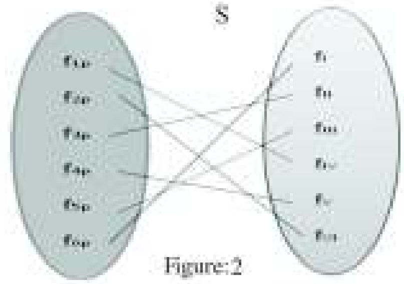
Figure:2

$f_{1p} \rightarrow f_{III}$

$f_{2p} \rightarrow f_{VI}$

$f_{3p} \rightarrow f_{II}$

$f_{4p} \rightarrow f_{V}$

$f_{5p} \rightarrow f_{III}$ (repeated) it is not possible in first preference and f_{5p} rejects theirs friendship with f_I.

(ie, f_{5p} x f_I) and again give second preference for f_{1p}.

$f_{1p} \rightarrow f_{III}$

Next, $f_{6p} \rightarrow f_V$, (repeated), this also already exists and give second preference for f_{6p}. (ie, $f_{5p} f_{VI}$). After some processor we get, $f_{6p} \rightarrow f_I$

Therefore, (Figure,2) phase 1 ends with the set S = {(f_{1p},f_{IV}), (f_{2p},f_{VI}), (f_{3p},f_{II}), (f_{4p},f_V), (f_{5p},f_{III}), (f_{6p},f_I) }

Next,

As per phase:1, S = {(f_{1p},f_{IV}), (f_{2p},f_{VI}), (f_{3p},f_{II}), (f_{4p},f_V), (f_{5p},f_{III}), (f_{6p},f_I) }and by rotation method in S we get figure,3 as follows;

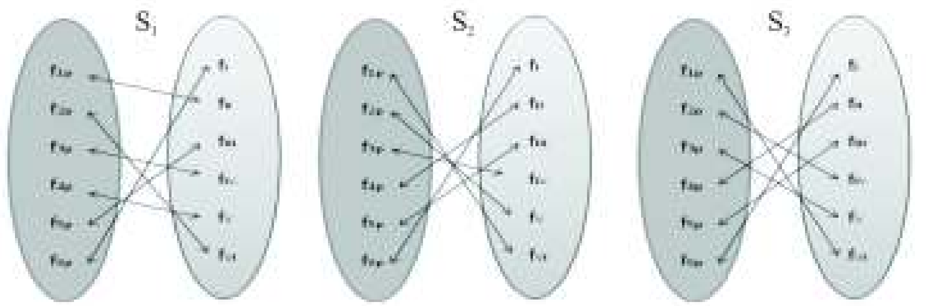
Figure : 3

r_1 = {(f_{1p},f_{IV}) (f_{3p},f_{II}) }, and so we get the first modified set S = {(f_{1p},f_{II}), (f_{2p},f_{VI}), (f_{3p},f_{IV}), (f_{4p},f_V), (f_{5p},f_{III}), (f_{6p},f_I) }

r_2 = {(f_{1p},f_{II}), (f_{2p},f_{VI}), (f_{4p},f_V) }and also the second modified set S = {(f_{1p},f_{VI},) (f_{2p},f_V), (f_{3p},f_{IV}), (f_{4p},f_{II}), (f_{5p},f_{III}), (f_{6p}, f_I) }

r_3 = {(f_{2p},f_V), (f_{3p},f_{IV}) }, Therefore the third modified set S = {(f_{1p},f_{VI}) (f_{2p},f_{IV}) (f_{3p},f_V) (f_{4p},f_{II}), (f_{5p},f_{III}) (f_{6p},f_I) }.

CONCLUSION

We have characterized absolutely stable true best friendship problems when preferences are strict. That is, we have obtained under which conditions on preference profiles indirect dominance implies direct dominance in roommate problems.

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