FINITE ELEMENT ANALYSIS OF MEMS STRUCTURE WITH FOUR CANTILEVER BEAMS AND A CENTRAL MASS IN SYMMETRIC AND ASYMMETRIC MODELS

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ABSTRACT
The basis of Micro-Electro mechanical systems is deal with electrical signal processing and their performance is based on mechanical principles that are mainly applied to the sensors and actuators. Today, since in the advanced industry, increasing speed and quent of dimensions are highly regarded. Therefore, study of mechanical parameters for designing, construction and optimization of these structures is inevitable. Since the performance of these structures are based on deformities and constant movement, are constantly exposed to vibrations. Also, other factors that are effect on the potential of these structures for effective vibrations are mass and elasticity in small scales. Therefore, the vibration behavior of these structures in order to design and subsequently removal or reduction of vibration is so important in engineering because vibrations have direct relationship with performance of these type of structures and determining their lifetimes. This research peruses study of Natural Frequency and Mode Shape on a typical MEMS structure having four cantilever beams and a central mass. Calculate the natural frequency and various modeshapes of vibration for both symmetric and asymmetric structures, based on finite element modal analysis using Ansys software was undertaken and the impact position of the center of mass vibrational behavior of structures has been investigated.

KEYWORDS: Vibration, Modal Analysis, Microelectromechanical Systems, Vibration Modes

Since the micro-electromechanical structural changes in their working environment movement forms and instruments are constantly updated, and they are completely normal vibrations. There is also a small-scale, mass and elasticity, another factor in the potential of these structures to vibrations. Therefore vibrational behavior of such structures aimed at eliminating or reducing vibrations in order to design more convenient, and the results can be important in determining the performance and life will work. In 1993, Chai & Low from Riley equation to find the natural frequency of a beam of uniform slender concentrated mass for two different boundary conditions and the accuracy of the proposed method used to perform practical tests have confirmed (Chai and Low, 1993; Chai and Low, 1993). Also in 1994, Low innovative methods such equivalent-midrange methodfrequencies to measure the mass of the vibrating beams are focused suggested that their results could be used for beams with different dimensions and boundary conditions (Low, 1994). In 2002, Sun et al study the vibrational behavior, to better assess the structural behavior and the environment provided by specifying the dimensions and domains this behavior improved manufacturing processes. He also showed that the natural frequency of the structure can be determined by studying the fatigue life of components should be taken (Sun et al., 2002). Skrinar possible to calculate the natural frequency of the beam structural parameters such as mass and stiffness have it tested (Skrinar, 2002). Hassanpour et al noted in 2007, As the exact solution methods for systems analysis mass and beam shooting proposed mass movement focused on the effect of axial force are considered (Hassanpour et al., 2007). This paper examines the various modes of vibration natural frequency micro-electromechanical a useful model for the center of mass and center of mass position of the four cantilever beam and addresses the impact on the vibrational behavior of structures has been investigated. Calculate the natural frequency of vibration for various modes in symmetric and asymmetric structures, based on finite element has been carried out using Ansys software.

MODELS & METHODS
The structure in this study were selected to study the vibration of a micro structure that has a central mass and four cantilever beam is attached to it. A schematic of this model is shown in Figure 1.

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Three models for the study of the structure selected. In the first model (a) as in Figure 1, the mass center of symmetrical elastic beams on the sides are the same length \( l_1 = l_2 \). In the second model (b) asymmetric structure and mass, is leaning to the left. So in this case the coupling beams over the left than the right pair of beams \( l_1 > l_2 \). In the third model (c) asymmetric structure relative to the mass model is more asymmetric than b. So in this case the coupling beams over the left than the right-hand side b is smaller than the coupling beams \( l_1 \gg l_2 \).

For vibration and modal analysis in this paper, the finite element software Ansys has been used. Linear modal analysis software to do, so any actions nonlinear properties such as plasticity properties and nonlinear contact elements defined ignored. Structures and industrial parts are constantly affected by fluctuations in loads and vibratory actuators. Analysis modal analysis is essential because they must be designed structures or pieces as possible, it is far from resonance frequency range. Fluctuations in the normal frequency range of the device, causing excessive amplitude and thus increases the risk of disintegration pieces. Therefore, the analysis of modal analysis to determine the natural frequencies and mode shape at the frequencies used. In addition, modeling is not enough simply to perform the analysis. Mechanical and physical properties of matter at the stage of pre-processing software should also be defined as a software analysis to consider them. The material used in the models discussed in this paper, single crystalline silicon to form silicon wafers that are available. Table 1 physical characteristics of the material included in this article will show.

**Table 1.** Modelling the mechanical properties of the material used

<table>
<thead>
<tr>
<th>Young's modulus</th>
<th>(Gpa) 140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson Ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>Density</td>
<td>2.3(g cm(^{-3}))</td>
</tr>
</tbody>
</table>

After defining the material properties and the desired model drawn in the software Ansys, lattice and applying boundary conditions must be chosen analytical method for modal analysis. This problem is fixed with respect to the support beams and the beams are cantilever type. As a result, the boundary conditions should be applied to all the nodes on each end of the beam cross-section was chosen to be zero degrees of freedom in any be able to move extensions coordinate. In addition, the imposition of boundary conditions on a, b, and c is the same for all three models. Modes and to extract and calculate the eigenvalues, several numerical methods Ansys software to analyze the free vibration problems are considered, each of these methods in terms of practical, real and virtual memory to perform the analysis, the speed of precision of the calculations the size and number of frequencies requested models with different configurations are compared to each other.

**NUMERICAL RESULTS**

In the first analysis, modal analysis carried out for the first five vibration modes, and the range of frequencies ranging from 0Hz to 100000 Hz. The results indicate the presence of three vibration modes in the range of microstructural models discussed are given in Table 2.
As indicated in Table 2, the natural frequency rate is calculated for each of the models b and c differently than the symmetric model a. But the more accurately we can identify these differences, in tabular form, which will contain the percentage difference between the natural frequencies of the asymmetric models with the symmetric models (Table 3).

### Table 3. Percentagedifference between the natural frequencies of asymmetric models in comparison with the symmetric model.

<table>
<thead>
<tr>
<th>Asymmetric models</th>
<th>Percentasymmetric [%]</th>
<th>Percentagedifference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First vibration mode [%]</td>
</tr>
<tr>
<td>Model b</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Model c</td>
<td>30</td>
<td>9.9</td>
</tr>
</tbody>
</table>

This percentage indicates that a further increase in asymmetric structures investigated the natural frequency of the structure rises. It can be concluded that the modes of vibration studied for symmetric models (a) check and then the asymmetric model c, can be studied for representing asymmetric structures.

Figure 2 shows the first mode of symmetric structures (a). As is known, the transition of the deviation of the micro-beam swing on the x-y plane and along the y-axis has been done.

![Figure 2. First vibration mode of symmetric model a](image-url)
As the first natural frequency, Micro-beams deflection for the second natural frequency of the transition, with the difference that this time on a swing along the z-axis and they-z has occurred (Figure 3).

Figure 3. Second vibration mode of symmetric model a

The third natural frequency, deviation of the torsional and structureduring the x-axis (Figure 4).

Figure 4. Third vibration mode of symmetric model a

Similarly, Figure 5 an asymmetric structure (c) displays as is known, the transition of the deviation Micro-beamsswingonthex-y plane and along the y-axis has been done.
Figure 5. First vibration mode of asymmetric model c

As can be seen in Figure 6, in the second mode displacement, micro-beams swing on the x-y plane and along the y-axis has been done.

Figure 6. Second vibration mode of asymmetric model c

Finally, the third mode of vibration that torsional deviation of the x-axis has happened, can be seen in Figure 7.
THEORETICAL RESULTS

Structures and boundary conditions of symmetric structures can be divided into modular units. Mentioned structures, the elastic members (beams) to act as a shock absorber and a spring-mass as the department is attached to the other part together make up a larger system. In an idea these structures are like a spring-mass where the beams are combined into an equivalent spring. A schematic of this equivalent structure is shown in Figure 8.

Where \( m_i \) is the mass of elastic element, \( M_i \) is the central mass, \( K_i \) equivalent spring stiffness coefficient, \( M_{E1} \) equivalent mass, \( \omega \) is the regular speed, \( a \) the maximal displacement and \( t \) is time.

Segmentation of structures into smaller parts so it can be analyzed continuously, the equivalent stiffness coefficient and mass equivalent to a complex structure, makes it easier. Therefore, the equivalent stiffness and mass accommodation coefficient of the equation (1), the natural frequency can be calculated. Another thing to be noted here is the combination of springs. If the spring-mass structure more than one spring is used, stiffness factor system in terms of its springs are connected in series or parallel, coefficient of tough times for all springs used in the system. A simple structural mass and stiffness factor beam is calculated using the following equation:

\[
K_e = \frac{P_e}{|y_B|} \quad (2)
\]

\( P_e \) The amount of force exerted on the end of the beam and \( y_B \) the rate displacement end of the beam in the direction of the applied force. As discussed above structure is symmetrical. Taking a part of a modular structure, the schematic force and its effect on the structure of the curve is shown in Figure 9.
The displacement $y_B$ will be a function of load $P_e$, length $l_e$, elasticity module $E_e$ and inertial moment of the transversal section $I_e$. When the force $P_e$ is applied, the energy potential of the beam deformation energy stored. According to the theory of elastic beam, the deflection curve will be equal to:

$$y = -\frac{P_e x^2}{6 E_e l_e} \left(\frac{3}{2} l_e x\right)$$

(3)

Consequently, taking into account the relations (2) and (3) it follows:

$$K_e = \frac{12E_e l_e}{l_e^3}$$

(4)

Obviously this is true for a cantilever beam attached to the central mass. In view of the structures in question have four cantilever beam are connected to the central mass. Using the proposed relationship for the equivalent stiffness coefficient of spring mix, hardness coefficient equal mass, equal to the natural frequency of the structure can be seen in Table 4.

Table 4. Equivalent mass, equivalent stiffness coefficient and natural frequency based on a microelectromechanical structures discussed

<table>
<thead>
<tr>
<th>$\frac{48E_e l_e}{l_e^3}$</th>
<th>$\frac{4Ehw^3}{I^3}$</th>
<th>$K_{eq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{13m_V}{35} + m_C$</td>
<td>$m_V = 4 m_V$</td>
<td>$M_{eq}$</td>
</tr>
<tr>
<td>$\frac{1}{2\pi} \sqrt{\frac{4Ehw^3}{I^3 \left(\frac{13m_V}{35} + m_C\right)}}$</td>
<td></td>
<td>$f_n$</td>
</tr>
</tbody>
</table>
Where the mass of each beam is \( m_i \); \( m_{bc} \) is total mass of all cantilever beams; \( m_c \) is the central mass and its value is equal to \( 5.0887 \times 10^{-11} \) kg. Theoretical and numerical values of the results are to be compared with the analysis by modeling and error rate data revealed. In Table 5, the accuracy of the results and percent errors are shown for the symmetric model.

### Table 5. Comparison of natural frequencies results and percentage error

<table>
<thead>
<tr>
<th>Typology</th>
<th>Natural frequencies [Hz]</th>
<th>Difference [Hz]</th>
<th>relative difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical Result [Hz]</td>
<td>Analytical Result [Hz]</td>
<td></td>
</tr>
<tr>
<td>Model a</td>
<td>23467</td>
<td>23532</td>
<td>65</td>
</tr>
</tbody>
</table>

According to the results of the numerical data and theory is observed that the relative error between the theory and the numerical values that are less than one percent error is reasonable and very good. Therefore, to calculate the natural frequency of the structure is more complicated and time consuming, which requires heavy computations, numerical method will increase the speed and accuracy of calculations.

### CONCLUSION

According to the results, it can be seen that by keep out the mass from central structure and increasing the asymmetric, such as first, second, and third frequency mode in the more asymmetric structure of c, had an impressive increasing as compared to their frequency in the symmetric structure a and asymmetric structure b. So when the more central mass's tendency to the backrests increases, the more natural frequency increases too. The relative error between the theory and the numerical values that are less than one percent error is reasonable and appropriate; Therefore, the calculated values for models of asymmetric structures with an approximately 0.3 percent considered valid. The results show the various modes of vibration of the first natural frequency deviation Microbeams for both symmetric and asymmetric structure of the transition, but the direction of oscillation is different for them. The symmetric one transition is along the y-axis and the asymmetric transfer along the z-axis.

### REFERENCES


