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HIGHER-ORDER SUB-POISSONIAN STATISTICS IN INTERACTION OF TWO TWO-LEVEL ATOMS WITH A SINGLE MODE COHERENT RADIATION

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ABSTRACT

In the present paper we study higher-order sub-Poissonian photon statistics in interaction of a single mode radiation initially in the coherent state $|\alpha\rangle$ with an assembly of two excited two-level atoms using the Hamiltonian, $H=\omega(a^{+}a + S_z) + g(aS_+ + a^{+}S_-)$, in the natural system of units. Here, $\alpha = |\alpha| e^{i\theta_\alpha}$; a is the annihilation operator for radiation, S_z and $S_\pm$ are the collective Dicke operators, g is the coupling constant, ω is the energy of the photons and the energy difference between the two atomic levels. We solve it exactly and conclude that higher-order sub-Poissonian photon statistics can be obtained for an arbitrary order by choosing suitably the square root of mean photon number |α| of the coherent radiation and coupling time gt. Variations of higher-order sub-Poissonian photon statistics with parameters |α| and gt have also been discussed.

KEYWORDS : Coherent State, Phase Shifting Operator, Higher-Order Sub-Poissonian Photon Statistics, Dicke Model.

The non-classical effects of a state (Loudon and Knight, 1987) can be manifested in different ways like squeezing, anti-bunching and sub-Poissonian photon statistics etc. Earlier study of such non-classical effects was largely in academic interest (Mollow and Glauber, 1967)), but now their applications in quantum information theory such as communication (Bennett et al., 1999), quantum teleportation (Braunstein, 2000), dense coding (Braunstein and Kimble, 2000) and quantum cryptography (Bennett et al., 1992) are well realized. It has been demonstrated that non-classicality is the necessary input for entangled state (Kim, 2002)

Squeezing, a well-known non-classical effect, is a phenomenon in which variance in one of the quadrature components become less than that in vacuum state or coherent state (Glauber, 1963) at the cost of increased fluctuations in the other quadrature component, has been generalized to case of several variables (Hong and Mandel, 1985; Hillery, 1986). According to Hong-Mandel's (1985) definition a state $|\psi\rangle$ is said to be $2n^{th}$-order squeezed for the operator,

$$X_\theta = X_1\cos\theta + X_2\sin\theta, \tag{1}$$

if the $2n^{th}$-order moment of X_θ,

$$\langle\psi|(\Delta X_\theta)^{2n}|\psi\rangle < \frac{(2n-1)!!}{2^{2n}} \tag{2}$$

Here Hermitian operators $X_{1,2}$ are defined by $X_1 + iX_2 = a$, a is the annihilation operator, θ is an arbitrary angle, $\Delta X_\theta = X_\theta - \langle\psi|X_\theta|\psi\rangle$ and (2n-1)!! is product of factors, starting with (2n-1) and decreasing in steps of 2 and ending at 1. Note that the right hand side in inequality (Eq. (2)) is the value of left hand side for coherent state. Such type of squeezing is quite distinct from ordinary squeezing because such squeezing does not require that the uncertainty product be a minimum and therefore both quadratures of the field can have higher-order squeezing. In other words, states exist for which product $\langle\psi|(\Delta X_1)^{2n}|\psi\rangle\langle\psi|(\Delta X_2)^{2n}|\psi\rangle$ takes a value less than that for a coherent state (Lynch, 1986). Recently simultaneous occurrence of higher-order squeezing of both quadrarture components in orthogonal even coherent state has been studied by Kumar and Kumar (2013).

In a similar analogy sub-Poissonian statistics (Kumar and Prakash, 2010) has also been generalized to higher-order sub-Poissonian photon statistics (Kim and Yoon, 2002). Kim et.al defined higher-order $Q^{(n)}$-parameter by

$$Q^{(n)} \equiv \frac{\langle\psi|(\Delta N)^{2n}|\psi\rangle - \ell^{(2n)}(\langle\psi|N|\psi\rangle)}{\ell^{(2n)}(\langle\psi|N|\psi\rangle)}, \tag{3}$$

to generalize the sub-Poissonian photon statistics into higher order similar as Q-parameter defined by Mandel (Mandel, 1982) for defining sub-Poissonian photon statistics. Here $N = a^{+}a$, $\Delta N = N - \langle\psi|N|\psi\rangle$ and $\ell^{(2n)}(\langle\psi|N|\psi\rangle)$ represents the higher moment for the coherent state, including the vacuum. The first four expressions for $\ell^{(2n)}(\langle\psi|N|\psi\rangle$ are

$$\ell^{(2)}(\langle\psi|N|\psi\rangle) = \langle\psi|N|\psi\rangle; \qquad 4(a)$$

$$\ell^{(4)}(\langle\psi|N|\psi\rangle) = 3(\langle\psi|N|\psi\rangle)^2 + \langle\psi|N|\psi\rangle; \qquad 4(b)$$

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$$\ell^{(6)}(\langle\psi|N|\psi\rangle) = 15(\langle\psi|N|\psi\rangle)^3 + 25(\langle\psi|N|\psi\rangle)^2 + \langle\psi|N|\psi\rangle; \qquad 4(c)$$

and $$\ell^{(8)}(\langle\psi|N|\psi\rangle) = 105(\langle\psi|N|\psi\rangle)^4 + 490(\langle\psi|N|\psi\rangle)^3 + 119(\langle\psi|N|\psi\rangle)^2 + \langle\psi|N|\psi\rangle. \qquad 4(d)$$

According to Kim's definition [15], the photon statistics is called n^{th}-order sub-Poissonian if $-1 \le Q^{(n)} < 0$.

In this paper we study higher-order sub-Poissonian photon statistics in interaction of a single mode radiation initially in the coherent state $|\alpha\rangle$ with an assembly of two two-level atoms.

Higher Order Sub-Poissonian Photon Statistics in Interaction of Two Excited Two-Level Atoms With a Single Mode Radiation

Consider a system of two two-level atoms interacting with a single resonant mode of radiation with zero detuning. If the atoms are located in a region small in comparison with the wavelength of the field, but not so small so as to make them interact directly with each other, the Hamiltonian (Dicke, 1954) of the system in the dipole and rotating wave approximation is given in the natural system of units ($\hbar = 1$) by

$$H = H_0 + H_1;\ H_0 = H_F + H_A \qquad (5)$$

Here, $H_F = \omega_F N$, $H_A = \omega_A S_z$, $H_I = g(a S_+ + a^+ S_-)$, $N = a^+a$ and subscripts F, A, and I refer to field, atoms and interaction, N, a and a^+ are number, annihilation and creation operators respectively, g is coupling constant and are the Dicke's collective atom operators (Dicke, 1954). Since $[H_0, H_I] = 0$, the time evolution operator $U = e^{-iHt}$ can be written as $U = U_0 U_1$, where $U_0 = e^{-iH_0 t}$ and $U_I = e^{-iH_I t}$. The exact time evolution operator in interaction picture in the form (Prakash and Kumar, 2007),

$$U_I = e^{-iH_I t} =$$

$$\begin{pmatrix} 1+(N+1)\,C(N+1) & -i\,S(N+1)\,a & C(N+1)\,a^2 \\ -ia^+\,S(N+1) & \cos(gt\sqrt{4N+2}) & -i\,S(N)\,a \\ a^{+2}\,C(N+1) & -ia^+\,S(N) & 1+N\,C(N-1) \end{pmatrix} \qquad (6)$$

Here

$$C(N) \equiv \{\cos(gt\sqrt{4N+2}) - 1\}/(2N+1);$$

$$S(N) \equiv \{\sin(gt\sqrt{4N+2})\}/\sqrt{2N+1}.$$

If both atoms are excited and radiation is in the coherent state $|\alpha\rangle$ initially, the initial state is $|\alpha\rangle|1,1\rangle$ and the final state is then obtained using Eq. (6) in the form,

$$|\psi\rangle = [1+(N+1)\,C(N+1)]\,|\alpha\rangle|1,1\rangle - ia^+\,S(N+1)\,|\alpha\rangle|1,0\rangle + a^{+2}\,C(N+1)\,|\alpha\rangle|1,-1\rangle \qquad (7)$$

Straight forward calculations lead to

$$\langle\psi|a^{+k}a^k|\psi\rangle = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 \qquad (8)$$

Here,

$$A_1 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[|\alpha|^{2k}(1+(n+k+1)\,C(n+k+1))^2],$$

$$A_2 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[k^2|\alpha|^{2(k-1)}(S(n+k))^2],$$

$$A_3 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[(2k+1)|\alpha|^{2k}(S(n+k+1))^2],$$

$$A_4 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[|\alpha|^{2(k+1)}(S(n+k+2))^2],$$

$$A_5 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[k(k-1)|\alpha|^{2(k-2)}(C(n+k-1))^2],$$

$$A_6 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[4k^3|\alpha|^{2(k-1)}(C(n+k))^2],$$

$$A_7 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[(k^2+(k+1)^2 + (2k+1)^2)|\alpha|^{2k}(C(n+k+1))^2],$$

$$A_8 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[4(k+1)|\alpha|^{2(k+1)}(C(n+k+2))^2],$$

and

$$A_9 = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}[|\alpha|^{2(k+2)}(C(n+k+3))^2],$$

Therefore from Eq. (3) we finally get first-order Q-parameter (Mandel Q-parameter),

$$Q^{(1)} \equiv \frac{\langle\psi|(\Delta N)^2|\psi\rangle - (\langle\psi|N|\psi\rangle)}{(\langle\psi|N|\psi\rangle)} \qquad (9)$$

and second-order Q- parameter,

$$Q^{(2)} \equiv \frac{\langle\psi|(\Delta N)^4|\psi\rangle - (3\langle\psi|N^2|\psi\rangle + \langle\psi|N|\psi\rangle)}{(3\langle\psi|N^2|\psi\rangle + \langle\psi|N|\psi\rangle)} \qquad (10)$$

$$\langle a^{+}a\rangle = N\,;\quad \langle N^{2}\rangle = \langle a^{+}a\rangle + \langle a^{+2}a^{2}\rangle;\quad \langle N^{3}\rangle = \langle a^{+}a\rangle + 3\langle a^{+2}a^{2}\rangle + \langle a^{+3}a^{3}\rangle \tag{11}$$

$$\langle N^{4}\rangle = \langle a^{+}a\rangle + 7\langle a^{+2}a^{2}\rangle + 6\langle a^{+3}a^{3}\rangle + \langle a^{+4}a^{4}\rangle \tag{12}$$

Using Eqs. (8)-(12), we can easily calculate the value of $Q^{(1)}$ and $Q^{(2)}$.

RESULTS

We use C++ programming to find minimum $Q^{(1)}$ and $Q^{(2)}$ for studying maximum first-order and second-order sub-poissonian photon statistics. The Figures Figure 1, Figure 2, show that the variation of parameters $Q^{(1)}$ and $Q^{(2)}$ with coupling time gt for a fixed value of square root of mean photon number $|\alpha|$. We conclude that second order sub poissonian statistics i.e. the value of $Q^{(2)}$ is larger than the value of $Q^{(1)}$ (i.e Mandel Q-parameter) for the case when both atoms are excited. The second order sub poissoinian occurs at very small average photon number and small coupling time.

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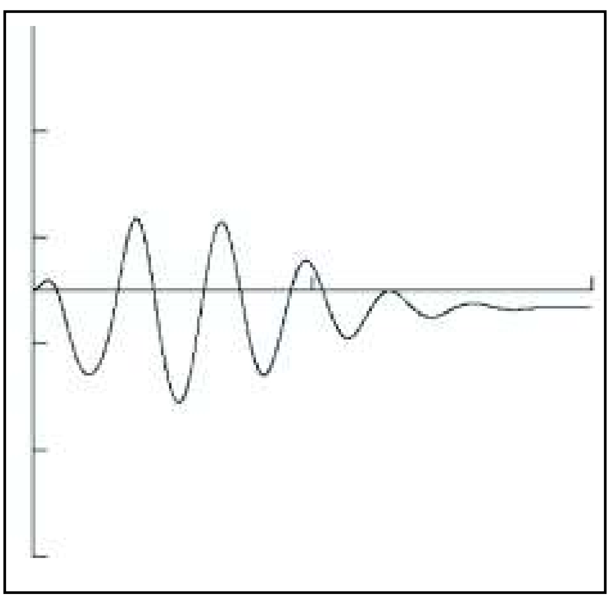

Figure 1 : Variation of $Q^{(1)}$ (0.5 to -0.5 on Y axis) with gt (0 to 4 on X axis) to for $|\alpha| = 5$

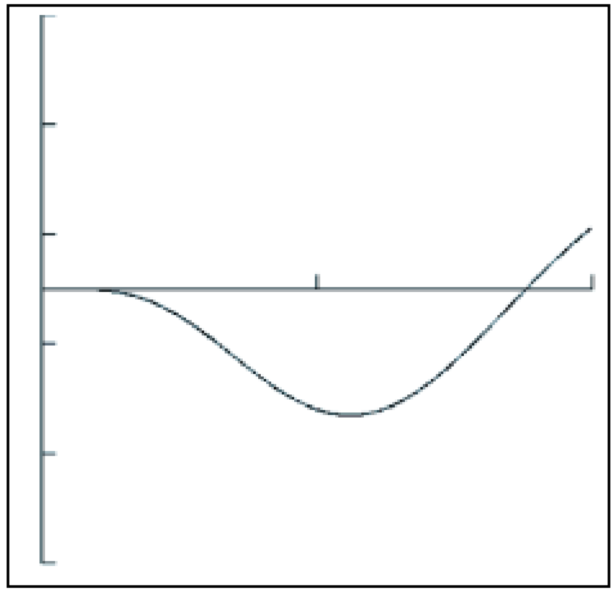

Figure 2 : Variation of $Q^{(2)}$ (0.5 to -0.5 on Y axis) with gt (0 to 1 on X axis) for $|\alpha| = 0.00001$

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