PAPR reduction using DCT based sub carrier grouping with Companding Technique in OFDM system

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ABSTRACT:

Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique against the multipath fading channel for wireless communication. However, OFDM faces the Peak-to-Average Power Ratio (PAPR) problem that is a major drawback of multi-carrier transmission system, which leads to power inefficiency in RF section of the transmitter. In the proposed scheme, a joint reduction in PAPR of the OFDM system by grouping the sub carrier based on combining the Discrete Cosine Transform (DCT) with companding technique. The performance of the proposed PAPR is evaluated using a computer simulation. The simulation results show that the proposed scheme can yield good tradeoff between PAPR reduction performance and computational complexity.

Keywords:

Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Discrete Cosine Transform (DCT), Complementary Cumulative Distribution Function (CCDF).

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most attractive standard technology for fourth generation (4G) wireless communication. The basic principle of OFDM is to split a high rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of sub carriers. Usually, OFDM transmit signals has the problem of high peak-to-average power ratio(PAPR), that means when the signals of all sub-carriers are added constructively, the peak power can be the number of sub-carriers times the average power. The power consumption of a power amplifier depends largely on the peak power than average power. Thus, handling occasional large peaks leads to low power efficiency [3]. When the OFDM signals with high PAPR that is large variation in signal amplitudes, pass through HPA at the transmitter, signal distortion is caused, resulting in performance degradation. Large PAPR also demands Analog –to- Digital Converters (ADC's) with large dynamic range [1,2].

Recently, researchers have proposed many technologies and algorithms to tackle the PAPR problem including clipping companding [1], selected mapping (SLM) [3], non linear companding transforms [4] and DCT [5].Clipping reduces the signal power but degrading bit error rate (BER) performance and causing non linear phenomena such as spectral spreading. Spectral spread causes degradation of spectral efficiency. As an among those PAPR reduction methods DCT may reduce the PAPR of an OFDM signal, but does not increase the BER of the system. We proposed an efficient PAPR reducing scheme, DCT based sub carrier grouping with companding technique. This scheme compared with the original system for the reduction of PAPR.

The rest of this paper is organized as follows. In section II, some basics about PAPR problem in OFDM system is given . Section III represents about sub-carriers grouping. DCT are introduced in section IV. Proposed scheme discussed in section V. Measurement and simulation results are given in section VI, followed by the conclusions in section VII.

II. PAPR IN OFDM SYSTEMS

An OFDM signal consists of N data symbols (OFDM symbols) $X = \{X_n, n = 0,1,2,.........N-1\}$ will be transmitted in parallel such that each symbol is modulated by one of a set of sub carriers $\{f_n, n = 0,1,2........N-1\}$, where N is the number of sub carriers. The 'N' sub carriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where $\Delta f = \frac{1}{NT_s}$ (Hz) and T_s (sec) is the original symbol period. The complex envelope of the transmitted OFDM signal $x(t)$ can be represented as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \quad 0 \le t \le NT_s \qquad \ldots (1)$$

where $j = \sqrt{-1}$

Then the PAPR of the transmitted OFDM signal x(t) can be defined as the ratio between the maximum instantaneous power and its average power during its OFDM symbol.

$$PAPR[x(t)] = \frac{\max_{0 \le t \le NT_s} |x(t)|^2}{P_{avg}} \qquad \dots (2)$$

where , P_{avg} is the average power of x(t) and is expressed as

$$P_{avg} = \frac{1}{NT_s} \int_0^{NT_s} |x(t)|^2 \, dt \qquad \dots (3)$$

PAPR of continuous-time OFDM is generally defined as,

$$PAPR[x(t)] = \frac{\max_{0 \le t \le NT_s} |x(t)|^2}{\frac{1}{NT_s} \int_0^{NT_s} |x(t)|^2 \, dt} \qquad \dots (4)$$

In equation (4), PAPR reduction of OFDM signals is mainly achieved by minimizing the maximum instantaneous signal power $\max_{0 \le t \le NT_s} |x(t)|^2$.

If the bandwidth of the OFDM signal is $B = N \times \Delta f$ and the signal x(t) is sampled by the sampling time of $\Delta t = \frac{1}{B} = \frac{1}{N\Delta f}$, then the OFDM signal is in discrete time form and can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \, e^{j\frac{2\pi nk}{N}}, \quad k = 0,1,2,\dots\dots\dots N-1 \qquad \dots (5)$$

where, n denotes the index in frequency domain and X_n is the complex symbol in frequency domain. The PAPR computed from OFDM signal sample can be defined as

$$PAPR\{x[n]\} = \frac{Max_{0 \le n \le N-1} |x(n)|^2}{E\left[|x(n)|^2\right]} \qquad \dots (6)$$

E[.] – denotes the expectation operator and it will be taken over all OFDM symbols.

III. GROUPED SUB-CARRIERS

The basic idea for the grouping the adjacent sub-carriers is to increase the frequency separation between the sub-carriers. So number of required sub-carriers are reduced to half. In conventional OFDM systems, each modulated symbol X_n is mapped to one of sub-carriers. However, in the grouping sub-carrier OFDM system, each pair of modulated symbol is jointly mapped into one sub-carrier[7]. The modified complex base band OFDM signal can be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{l=1}^{N/2} (X_{l1} e^{j2\pi(2l-1)\Delta f t} + X_{l2} e^{j2\pi(2l)\Delta f t}) \qquad \ldots (7)$$

where l is the total number of groups

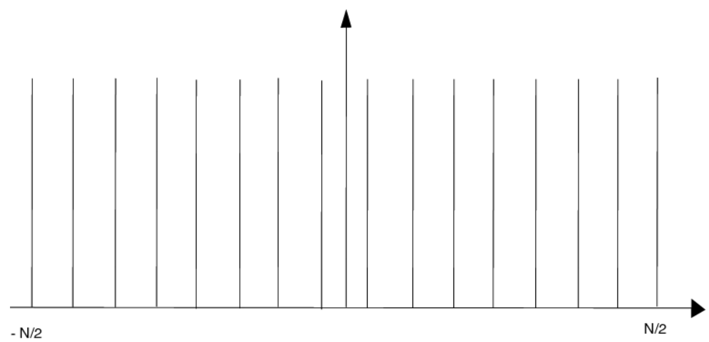

Fig.1. Conventional OFDM sub-carriers

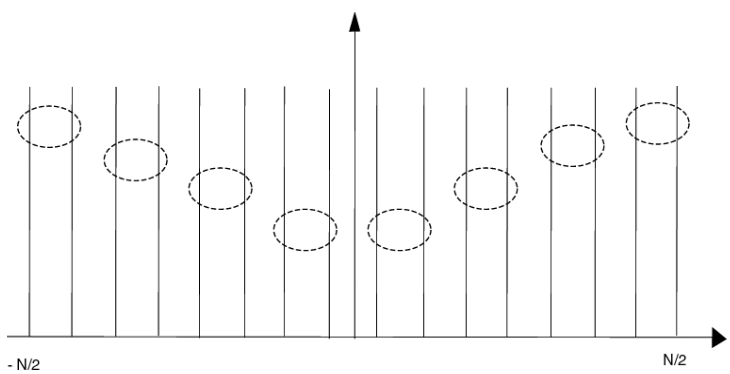

Fig.2.Grouped OFDM sub-carriers

IV. DISCRETE COSINE TRANSFORM

DCT decorrelates the data sequences, like other transforms such as the Hadamard transform. First we can define the basic relationship between the maximum of PAPR and auto correlation of IFFT input [13].The peak value of the auto correlation is the average power of input sequence. When the DCT is applied to the data sequence, the sequence structure will not be changed. At the same time, many zero elements will appear in the transformed sequence. That means, the lower order of components will be dominated in the transform domain signal after converted by DCT.

The one-dimensional DCT with length N is expressed as,

$$X[k] = \alpha(k)\sum_{n=0}^{N-1} x(n)\cos\left\{\frac{(2n+1)\pi k}{2N}\right\} \qquad 0 \le k \le N-1 \qquad \dots (8)$$

$$\alpha(k) = \sqrt{\frac{1}{N}} \qquad if \;\; k = 0$$

$$\alpha(k) = \sqrt{\frac{2}{N}} \qquad if \;\; k \neq 0$$

The IDCT is expressed as

$$x[n] = \alpha(k)\sum_{k=0}^{N-1} X[k]\cos\left[\frac{(2n+1)\pi k}{2N}\right] \;\; ,0 \le n \le N-1 \qquad \dots (9)$$

Let $\rho(i)$ be the aperiodic autocorrelation function (ACF) of an input vector X, then

$$\rho(i) = \sum_{n=0}^{N-1-i} X_{n+i} X_n^{*} \qquad \text{for i = 0,1, 2,……….N-1} \qquad \dots (10)$$

where the superscript * denotes the complex conjugate. Then, the relation between autocorrelation and PAPR of OFDM signals is bounded as

$$PAPR \quad \le 1 + \frac{2}{N}\sum_{i=0}^{N-1}|\rho(i)| \qquad \dots (11)$$

where $|p(i)|$ can be called the absolute aperiodic ACF of input vector X. From equations (10) and (11), we can find that the input vector with lower PAPR in OFDM systems.

V. COMPANDING TECHNIQUES

The samples of OFDM signal x (n) are companded before it is converted into analog waveforms. A compression is used at the transmitter end after the IFFT block and a expansion is used at the receiver end before the FFT block. The companded signal S (n) can be expressed as

$$S(n) = C\{x(n)\} = \frac{v\ x(n)}{\ln(1+u)|x(n)|} \ln\left(1 + \frac{u}{v}|x(n)|\right) \quad \text{……..(12)}$$

where v is the average amplitude of the signal , u is the companding parameter, x(n) is the input of the compressor and S(n) is the output of the compressor. The companding transform should satisfy the following two conditions:

$$E\left(|S(n)|^2\right) \approx E\left(|x(n)|^2\right) \quad \text{……..(13)}$$

$$|S(n)| \geq |x(n)| \quad for \quad |x(n) \leq v| \quad \text{……..(14)}$$

$$|S(n)| \leq |x(n)| \quad for \quad |x(n) \geq v| \quad \text{……..(15)}$$

On the receiver end, the receiver signal must be expanded by the inverse companding transform before it can be sent to the FFT .The expanded signal at the receiver is

$$Y(n) = C^{-1}\{r(n)\} = \frac{v\ r(n)}{u|r(n)|}\left\{\exp\left[\frac{|r(n)|\ln(1+u)}{v}\right] - 1\right\} \quad \text{……..(16)}$$

V. PROPOSED SYSTEM

In the proposed system, the PAPR of an OFDM signal is reduced by using DCT based sub carrier grouping with companding technique In OFDM systems, the PAPR increases approximately linearly with the number of sub-carriers. Therefore, in order to reduce the PAPR, one of the best ways is to decrease the number of used sub-carriers. In this system, the adjacent two input data symbols are jointly mapped to a single sub-carrier, and then the frequency separation between the sub-carrier is increased. The largest instantaneous peak power will appear if all sub-carrier phases are accumulated to the same direction in the same phase. A block diagram of the proposed system is shown in figure.3.

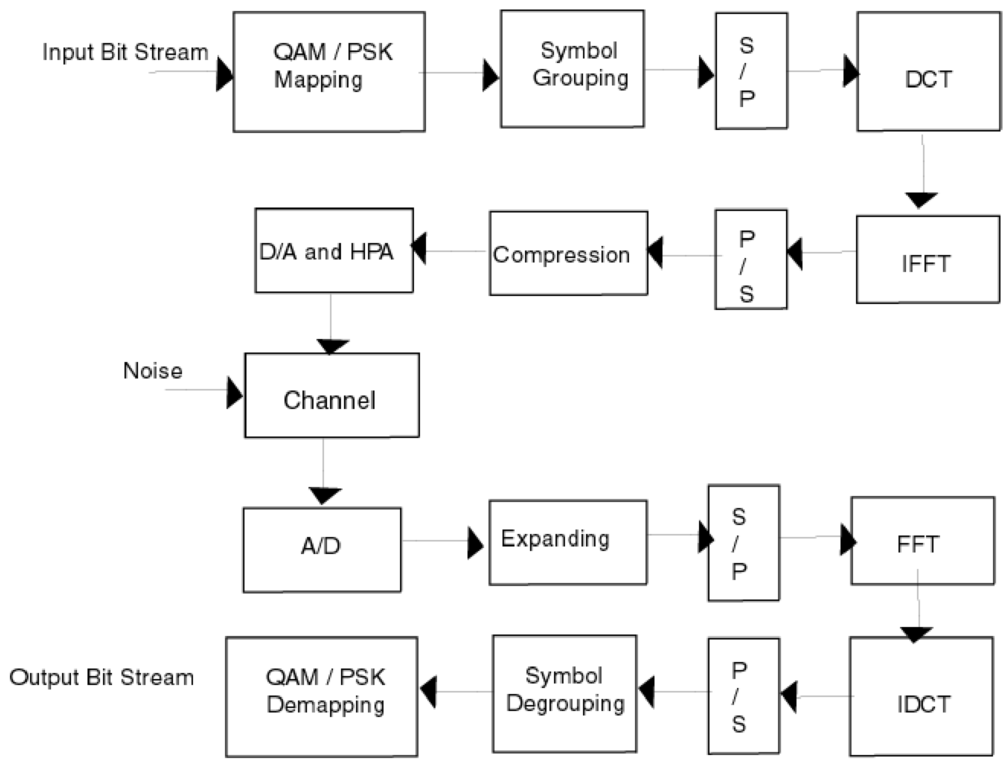

Fig.3.Proposed OFDM system

The PAPR of an OFDM signal can be defined as

$$PAPR[x(t)] = \frac{\max_{0 \le t \le NT_s} |x(t)|^2}{E\left[|x(t)|^2\right]} \qquad \ldots (17)$$

If the input data power is normalized, then $E\left[|x(t)|^2\right] = 1$, and we get

$$PAPR[x(t)] = \max_{0 \le t \le NT_s} |x(t)|^2$$

$$= \max_{0 \le t \le NT_s} \left| \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi k \Delta f t} \right|^2$$

$$\le \frac{1}{N} \left| \sum_{n=0}^{N-1} X_n e^{j2\pi k \Delta f t} \right|^2$$

$$\le \frac{1}{N} . N^2$$

$$\le N \qquad \ldots\ldots.(18)$$

It clearly shows that maximum PAPR is equal to the number of sub-carriers. Therefore, the maximum PAPR of an OFDM signal can be expressed as

$$\textbf{max}\ \text{PAPR}[x(t)] = N \qquad \dots (19)$$

S_{DCT} is the most of peak power of OFDM after DCT [4] and expressed as

$$\lim_{N\to\infty} |S_{DCT}| \leq 2.2825 + \frac{2}{\pi}\ln N \qquad \dots (20)$$

Therefore, DCT can improve the performance of OFDM, After DCT many data values are very small and even some zero elements appear in the transformed sequence.

VI. SIMULATION RESULTS

To show the overall effect of the PAPR reduction, randomly generated data are modulated into 16 QAM. The number of sub-carrier is depending upon symbol grouping and finally DCT and companding technique is used for grouped sub carriers. We can evaluate the performance of the PAPR reduction scheme using complementary cumulative distribution (CCDF) of the PAPR of the OFDM system.

CCDF Performance

Complementary cumulative distribution function (CCDF = 1-CDF) is used to evaluate the performance in PAPR reduction which denotes the probability that the PAPR exceeds a certain threshold. CCDF values are obtained by checking how often PAPR exceeds the threshold values.

$$\text{CCDF} = P\ (\text{PAPR} > \text{threshold}) = 1 - P(\text{PAPR} < \text{threshold})$$

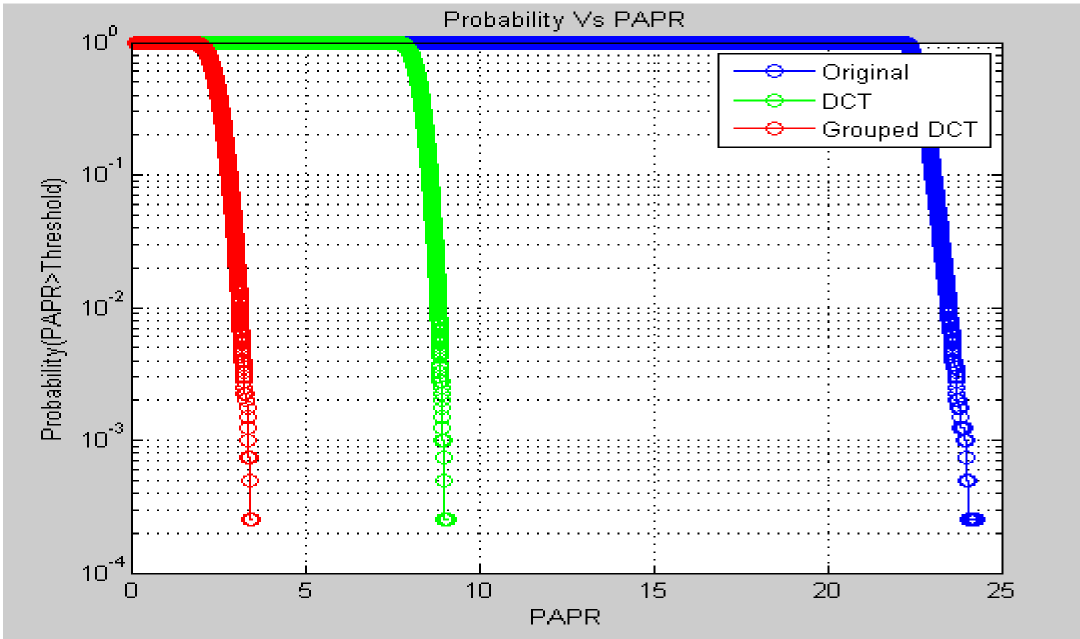

Fig.4. Comparisons of CCDF of different PAPR reduction techniques

In Fig.4, clearly shows that the DCT transformed signal have a much lower PAPR than the original signals. For example, when CCDF = 10^{-3}, the PAPR of the original signal is 24 dB, whereas that of the transformed signals is 8.5 dB and transformed grouped sub-carrier signal is 3.5 dB and, thus, 21 dB improvement in performance is achieved by the grouped DCT scheme.

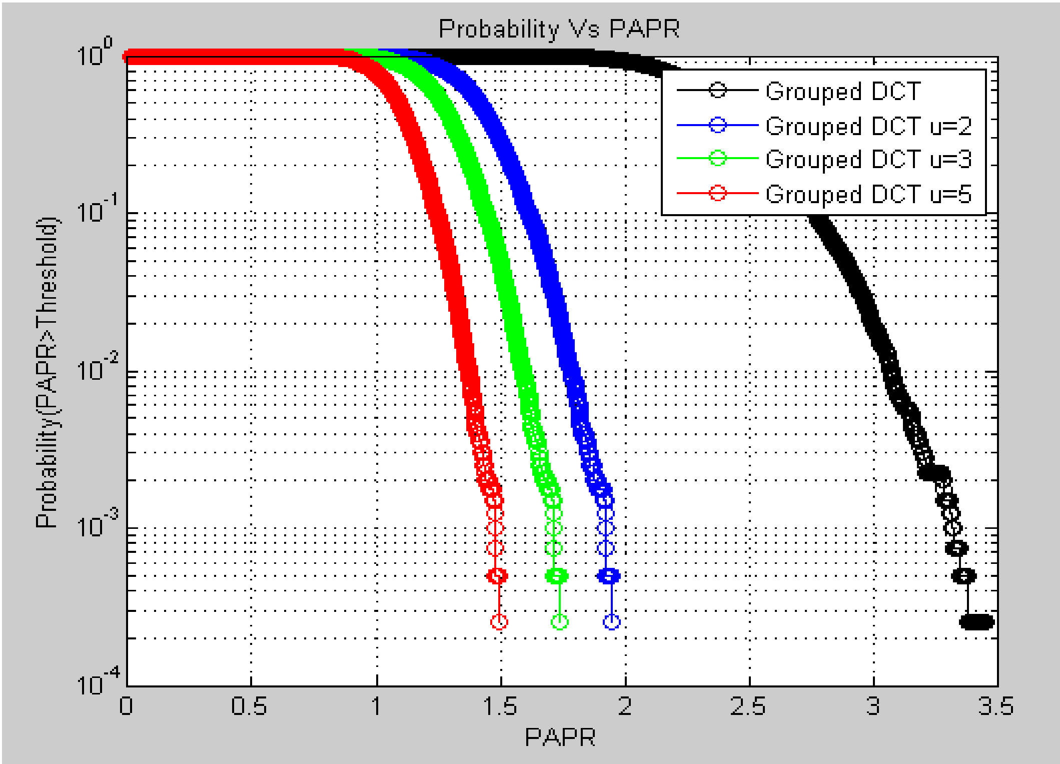

Fig.5. Comparisons of CCDF of proposed PAPR reduction scheme with different companding factor u.

In Fig.5, shows the CCDF performance of DCT transformed sub-carrier grouped signal of companding algorithm for PAPR reduction. The value of the companding factor u was fixed to 2,3, and 5. With this companding method, the peak power at CCDF = 10^{-3}, is reduced to 1.9 dB, 1.75 dB and 1.5 dB when compared with the original system.

VII. CONCLUSION

In this paper, a joint reduction in PAPR of the OFDM system by grouping the sub carrier based on combining the Discrete Cosine Transform (DCT) with companding technique has been proposed. The simulation results show that the proposed scheme can yield good tradeoff between PAPR reduction performance and computational complexity.

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