

## CHAOS PROCESS TESTING (TIME-SERIES IN THE FREQUENCY DOMAIN) IN PREDICTING STOCK RETURNS IN TEHRAN STOCK EXCHANGE

**ABOLFAZL GHADIRI MOGHADAM<sup>a</sup>, MEHDI JABBARI NOOGHABI<sup>b</sup>, MOHAMMAD MAHDI  
ROUNAGHI<sup>c1</sup>, MOHAMMAD HASSAN HAFEZI<sup>d</sup>, MORTEZA AYYOUBI<sup>e</sup>, ALI DANAEIF AND  
MAHDI GHOLAMI<sup>g</sup>**

<sup>a</sup>Associate Professor, Department of Accounting, Faculty of Economics and Business Administration , Ferdowsi University of Mashhad (FUM) , Mashhad, Iran

<sup>b</sup>Assistant Professor, Department of Statistics, Ferdowsi University of Mashhad , Mashhad, Iran

<sup>cde</sup>Ms Student, Department of Accounting , Mashhad Branch, Islamic Azad University, Mashhad, Iran

### ABSTRACT

Nowadays, the benefits of predicting are undeniably accepted in decision and policy making from different dimensions. Recently, structural models which were relatively successful in explaining the current situation have not been paid much attention in the field of forecasting. Most statistical observations have applied the tests which brought about wrong findings, and made them draw this conclusion that the obtained data from chaos-basis tests are random. However, these data are derived from systematic regulations which are accompanied with trivial disorders. Therefore, some other tests such as time-series tests in the frequency domain have been proposed. This test was utilized to assess the existence of chaotic processes in daily time-series on Tehran Stock Exchange over a period from 2007 to 2013. The achieved findings clearly demonstrate such processes during the process of this indicator.

**KEYWORDS:** Chaos Analysis, Stock Exchange, Time-series in Frequency Domain, Stock Return, Predicting

In 1776, Laplace stated that if we know preliminary conditions of a phenomenon, we can accurately predict its future. This idea was well accepted by scholars in the field of science. But Poë Ankaire in 1903 said that small errors in the forecast of today will lead to large errors in the future, and due to the fact that mostly we cannot achieve an exact knowledge of the existing condition, it is impossible to predict the future. Although Ankaire's idea is dominant now, many scholars, in different fields, still predict the assessed variables and phenomena. One of the youngest fields which still takes advantage of prediction is economics.

Most of the economic variables could not be predicted so far, since they cannot be accurately known. Some researchers believe that this unpredictability is rooted in the existence of a random procedure in the time series of these variables. Recently, structural models, which were relatively used to clarify the current conditions, could not be considerably successful. Comparing these models, economists have shown increasing interest in univariate models of time series in the field of forecasting. However, the acceptance of theoretical bases of predictability is greatly dependent upon the rejection of efficient market hypothesis (EMH)

associated with markets and pricing regulations (ozer and ortokatly, 2010).

Various methods have been applied to assess the efficient market hypothesis on Tehran Stock Exchange. Some scholars such as Salimifar and Shirzour (2010) rejected this hypothesis. It is mainly believed that the analyzed variable has a linear procedure which is accompanied with a random process (White Noise). In sum, such models are called linear-random models. Some researchers applied non-linear models such as ARCH and GARCH which simply use a non-linear model in residual variances with a random component. These models are called non-linear-random models.

In mathematics, such non-linear random procedures are named chaos which cause chaotic events. By and large, a chaos 1) is non-linear, 2) dynamic, 3) causes complex procedures, and 4) is highly sensitive in a way that seems to be random. Based on chaos theory, new approaches such as Fractal market hypothesis were proposed. This hypothesis attempts to explain financial markets' phenomena against efficient markets hypothesis. Fractal market hypothesis can be confirmed if maximal and reverse Lyapunov exponential (MLE) tests are accomplished. Maximal Lyapunov exponential (MLE) test is applied to ensure the predictability of analyzed

series based on non-linear models, while reverse maximal Lyapunov exponential (MLE) test is used to determine the predictable time. Mention must be made that if analyzed series are confirmed to be chaotic, they cannot be modelled or predicted on the basis of linear models anymore. In other words, linear models do not enjoy good results.

## THEORETICAL BACKGROUND AND REVIEW OF LITERATURE

Chaos refers to a state of utter confusion or disorder. Philosophically, chaos is a total lack of organization in which accident determines the occurrence of events, but simply put, chaos is a dynamic system. Nowadays, assessing and analyzing dynamic systems is a branch of mathematics. As it was previously mentioned, chaos is derived from dynamic systems. A dynamic system or function is in fact a continuous static function, for example:

$$Y = \sqrt{X}$$

If in the above function, we put numbers instead of  $X$ , and consider the results as a new  $X$  and use it again in the function, we will have a dynamic system. As a matter of fact, the following equivalence can be obtained:

$$\begin{array}{c} \longleftarrow \downarrow \\ Y = \sqrt{X} \leftarrow Y = X \equiv X_{n+1} = \sqrt{X_n} \end{array}$$

Where  $n$  stands for the frequency. Repeating a function can bring about numbers or mathematically named 'circuits'. Thus an equivalent circuit is derived from the repetition of a function, as it has been mentioned. Circuits can differ based on the initial number. Sometimes, repeating a function can cause a specific homogenous number which is called 'fixed point'. For instance, the abovementioned function will be homogeneously moved towards number one, and the function  $Y = \cos(X)$  will finally reach 0.739085 and the fixed point will not change under more repetitions.

So, it can be written in this way:  $F(X) = X$

Some circuits are alternative, like the function  $F_N(X) = X$  in which  $N$  shows frequency. It means that starting from the initial point and repeating for  $N$  times can get us to the first point. For example  $F(X) = \frac{1}{X}$  possesses an alternative period of 2. If alternative period goes toward infinity ( $N \rightarrow \infty$ ), a chaotic system will be formed. Such condition will be impossible when the main

system or function is non-linear or simple enough. In theoretical discussions, chaos is considered as the order of seemingly disordered systems and derived from recognizing the existing secrets and regulations of the nature. Many interpretations have been utilized to conceptualize chaos theory. For instance, considering the smokes moving in ordered circles and gradually become disordered and fade, chaos theory can be understood. Chaotic systems are deterministic, meaning that their future behaviour is fully determined by their initial conditions, with no random elements involved. In other words, the deterministic nature of these systems does not make them predictable. But the analysts who are not aware of the nature of the system (or does not know it well enough), cannot distinguish between a chaotic and a random system. Unfortunately statistical tests cannot, also, distinguish between them. So, considering the measurement accuracy limit, the accuracy of forecasts based on usual statistical or econometric techniques, continuously at an exponential rate decreases.

Chaotic systems are nonlinear dynamic systems which (1) are highly sensitive to initial conditions; (2) have unusual complicated absorbents; (3) sudden structural breaks in their trajectory are distinguishable (Prokhorov, 2008). However, it should be noted that:

1. The behaviour of chaotic systems though seems random, in essence can be theoretically explained by deterministic rules and equations. Nevertheless, even though we accept the existence of equations explains the source of their chaotic behaviour, proximity and inaccuracy (though very small) are inevitable due to measurement limitations.
2. Even very small inaccuracy in initial conditions, because chaotic system is highly sensitive to them, leads to huge differences between expected and realized values in the long term. In other words, as time passes, forecasted series and measured values of realized series totally diverge so much that previous forecasts are no longer reliable; a fact called unpredictability in the long run (Williams, 2005).

Schwarz (2010) applied a chaos test and examined various dimensions of financial markets and their effects on investing. Lento (2009) utilized Hurst chaos tests and investigated the relationship between long-term dependencies and interests and Canadian financial markets. Chaos and its application are of considerable importance for most of the current financial

markets. Financial markets can potentially provide long-term and transparent series which can be used in testing and estimating. An early example of a heterogeneous agent model can be found in Zeeman (1974). Many tests were proposed when chaos theory was discussed in the field of economics, before specific models of stock market were designed. Applying differential equations and assuming heterogeneity of market participants, Lux conducted some articles from 1995 to 1998 to clarify a model which could bring about chaotic results.

However, before that, Chiarella had shown the dynamics of this market. Muller et al. (1995) assessed heterogeneous business strategies in foreign exchange market. Sethi (1997) modeled the internal changes of speculative market. Brak and Fomes (1997, 1998) indicated that chaos is only effective in the market price when the agents' heterogeneous behaviours interfere in the assets pricing model. Using difference equations, Kaizoji (1998) suggested a model for speculative prices. Anderson (1999) proposed a continuous non-linear model to explain the fluctuation in global stock markets. He also described the collective behaviour of stock market changes by modeling by modeling and comparing them with individual behaviour. Joshi and Bedau (1999) defined financial market revolutions and classified different behaviours of current markets into four groups. These groups were defined on the basis of wealth and the type of strategies' complexity. And chaos can be assessed according to these four groups.

## TESTING METHODS

As it has been mentioned before, deterministic and random chaos are considerably different. This section deals with the fact that distinguishing between these two is so difficult. However, new studies have been accomplished in this field. Various tests have been discussed in the review of literature which make us capable of distinguishing a random system from a chaotic system. Some tests examine the random process, while others assess the chaotic characteristics of the process. Latter is called direct test, and the former is indirect. Indirect tests such as BDS examine the randomness of residual linear or non-linear regression; therefore, if this hypothesis that the process is random is rejected, it cannot be necessarily accepted that it is chaotic, since it can be rooted in the type of applied linear or non-linear model (Shiri, 2002).

The concept of Lyapunov exponential had been used before the emergence of chaos theory in order to recognize the consistency of linear and non-linear systems. Lyapunov exponential is calculated through assessing system skewness or flexure. Various methods exist to calculate Lyapunov exponential such as direct method or Jacobian Matrix method (Moeini et al., 2006). In fact, Lyapunov exponential, which provide a qualitative and quantitative characterization of dynamical behavior, are related to the exponentially fast divergence or convergence of nearby orbits in phase space. A system with one or more positive Lyapunov exponential is defined to be chaotic. Reverse Lyapunov exponential shows the difference between deterministic and random process (Wolf, 1985).

## PROCESS TESTING ON TEHRAN STOCK EXCHANGE

Since 1996, there has been a broad range of records on Tehran Stock Exchange which made it difficult for economists to investigate. Before that, some tests could not be conducted due to insufficient information. But now, considering the current status of Tehran Stock Exchange and existing data, it is possible to analyze and test the processes. To this end, variables of Tehran Stock Exchange (TEPIX) and industry have been assessed over a period from 2007 to 2013. Besides, needed data related to volume and number of stock transactions has been utilized.

### Assessing the Predictability of Different Models (Stock Returns)

Having estimated different models, model's predictability was examined out of the sample. Thus data was classified into two groups. First group included training or estimating set and the second was consisting of testing set. The model's coefficients were estimated using first set's data and then, second set was applied to test the model's predictability to see whether it can be generalized out of the data set or not. For this purpose, Mean Square Error (MSE) and Root Mean Square Error (RMSE) were used. By and large, among different methods which investigate goodness of fit, these two are of higher frequency. Many scholars consider RMSE as the best criterion for assessing goodness of fit, since it is the root of MSE, so errors are less probable (Suanson et al., 2011).

**Variance Ratio Test (VR)**

Variance ratio test is based on Lo and Mackinlay’s test and examines the hypothesis that a given time series follows a martingale difference square.

**Table 1: Results of VR Test in Stock Returns**

Test	p-value	Df	F
Criterion			
Variance	<0.0001	756	6.38

The achieved findings of this test show that there is no proof that time series or interruptions follow a martingale process, so data cannot be produced randomly. Thus this conclusion can be drawn that these series are predictable. Mention must be made though that this test cannot assess linearity or non-linearity of the process in stock returns.

**BDS Test**

BDS test was proposed by Brock, Dechert and Scheinkman in 1987 and uses times series to estimate randomness of a process based on integral correlation against general correlation. This test can efficiently assess the existence of non-linear processes such as chaotic processes in time series. The results of this test can be seen in the following table. Considering the above results, null hypothesis of this test cannot be confirmed. This hypothesis shows the deterministic characteristics of the model’s residuals. Thus the existence of a non-linear process in stock returns can be confirmed, which may be a chaotic one. It should be noticed that if the randomness of time series in dimension of two or more is rejected, the possibility of its non-linearity will be more. Due to the fact that alternative hypothesis is unknown, this test again proves the non-linearity of stock returns.

**Table 2: BDS Test’s Results in Stock Returns**

Dimension	BDS	SD	Z-Statistic	p-value
2	0.036784	0.003120	11.788	<0.0001
3	0.059570	0.004954	12.025	<0.0001
4	0.070707	0.005893	11.999	<0.0001
5	0.072014	0.006136	11.737	<0.0001

**Maximal Lyapunov exponential (MLE) test**

Maximal Lyapunov exponential examines the exponentially fast divergence or convergence of nearby orbits in phase space. In fact, Lyapunov exponential assess divergence or convergence through an exponential function to determine the local stability of linear or non-linear systems. Positive Lyapunov exponential shows the divergence of time series, high sensitivity and chaotic process, while negative Lyapunov exponent demonstrates the convergence of time series.

If Lyapunov exponential is zero, it can be concluded that the series has no convergence or divergence. In other words, it is fixed. So, the existence of one Lyapunov exponential can prove a chaotic process.

There are two methods of direct and Jacobian to calculate maximal Lyapunov exponential. Direct method is only used when equations of motion are clearly known. Equations of motion can be obtained through solving difference and differential equations. But due to the fact that economic system’s motions are not obvious, Jacobian method is utilized to calculate maximal Lyapunov exponential in such systems. Jacobian method was first discussed by Nichka et al. (1992).

They used Tiken theory to write equation associates with chaotic systems and estimated maximal Lyapunov exponential through the application of artificial neural networks.

**Table 3: Maximal Lyapunov Exponential Through The Application of Direct Method**

Maximum interruption	1	2	3	4	5
Lyapunov exponential	0	4.7X10 <sup>-19</sup>	0.03	0.04	0.02
Maximal Lyapunov exponential					0.02

**Table 4: Maximal Lyapunov Exponential Through The Application of Jacobian Method(based on artificial neural networks)**

Maximum interruption	1	2	3	4	5
Lyapunov exponential	4X10 <sup>-19</sup>	1.7X10 <sup>-21</sup>	0.001	0.09	0.2
Maximal Lyapunov exponential					0.2

Results of tables 3 and 4 show a chaotic system in stock returns, since maximal exponential are positive in both direct and Jacobian methods. Chaotic system can be also proved due to high sensitivity to the first condition or the first chosen point. the paths are divergent and no fixed point or cycle exist, so the process will be chaotic.

**Data Analysis (Time Series Test in Frequency Domain)**

The assumed model for the stock returns is consisting of random and deterministic sections which are separate from each other.

$$r_t = S_t + \varepsilon_t$$

Where;

$S_t$  represents signal, and  $\varepsilon_t$  is a know as white noise. random variables whose mean equals zero with finite variance. Furthermore, the regarded structure is a one-dimensional observation under DDS system in phase space.

$$S_t = g(X_t)$$

$$X_t = f(X_{t-1}) = \dots = f_{\theta}^t(X_0)$$

Where;

$g : X \rightarrow \mathbb{R}$  refers to the square integrable function; and  $f_{\theta} : X \rightarrow X$  stands for a measurable non-linear map which is dependent upon the parameters of vector  $\theta$ .

$$\theta \in \Theta \subset \mathbb{R}^l$$

$X_0$  refers to the preliminary conditions which are considered as random variables derived from a distribution with infinite second moment and fixed density. Thus a random process's signal is strongly fixed.

**(0-1) Test for Chaos Analysis**

(0-1) test was used by Gotwald and Melborn to analyze chaotic system in 2005-2006. It was employed to analyze chaos in DDS  $\{X_t\}$  systems with the help of behavioural signs of equation's variables.

$$p_c(n) = \sum_{j=0}^{n-1} \cos(jc) g(X_j)$$

$$q_c(n) = \sum_{j=0}^{n-1} \sin(jc) g(X_j), n \in \mathbb{N}$$

where;

$c \in (0, \pi)$  is optional but accompanied with a fixed frequency.

If DDS systems do not show chaotic behaviour, both equation variables will remain in the domain of  $n \rightarrow \infty$ . Otherwise, variables show a chaotic behaviour whose results are as follows:

$$p_c(j+n) - p_c(j) \rightarrow 0$$

$$q_c(j+n) - q_c(j) \rightarrow n^{1/2} \quad n \rightarrow \infty, j \in \mathbb{N}$$

Applying behavioural signs and Pearson correlation coefficient, statistical results of time series tests in the frequency domain (0-1) are between  $n = (1, 2, \dots, n)^T$  and  $\Delta_n = (D_c(1), D_c(2), \dots, D_c(n))^T$ .

$$K_c = \lim_{n \rightarrow \infty} \frac{n^T \Delta_n - \frac{1}{n} 1_n^T n 1_n \Delta_n}{\sqrt{[n^T n - \frac{1}{n} (1_n^T n)^2] [ \Delta_n^T \Delta_n - \frac{1}{n} (1_n^T \Delta_n)^2 ]}}$$

$$1_n = (1, 1, \dots, 1)^T \in \mathbb{R}^n$$

$$D_c(n) = M_c(n) - \left( \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{j=1}^N g(X_j) \right)^2 \frac{1 - \cos(nc)}{1 - \cos c}$$

$$M_c(n) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{j=1}^N \{ [p_c(j+n) - p_c(j)]^2 + [q_c(j+n) - q_c(j)]^2 \}$$

In the aforementioned formulas,  $D_c$  stands for smoothed median square and  $M_c(n)$  refers to non-smoothed median square.

**Static Processes in Frequency Domain**

Spectral density function is a natural instrument to assess the properties of the frequency in a domain. Inferring on the basis of the spectral density function is called frequency domain analysis. Some statisticians accept the frequency method, thus the benefits of frequency method in the areas of electrical engineering, geophysics and meteorology can be widely observed. In order to introduce the spectral density or spectrum, first we have to examine the spectral distribution function. Although the chosen method which is going to be discussed here is not totally mathematical, it can hopefully help us gain a better understanding of the theoretical method. Suppose that a time series contains a periodic component with a known frequency, then the following normal pattern will be obtained:

4-2-2-1 pattern:

$$X_t = A \cos(\omega t + \theta) + Z_t$$

where;

$\omega$  stands for period changes frequency

A refers to the range of variations

$\theta$  refers to phase

$Z_t$  stands for the stationary random series

Figure 1 is an example of the period component. It should be noticed that  $(\omega t + \theta)$  is measured in radians. Since  $\omega$  equals the number of radians in time, angular frequency is sometimes called 'frequency'. Some scholars consider frequency as  $f = \omega / 2\pi$  (number of periods in time) which is mostly used to interpret the obtained data and shows  $2\pi / \omega$  (wavelength).

4-2-2-1 pattern is so simple. Sometimes time-series changes may be the result of changes in different frequencies such as sales analysis, weekly, monthly and annual changes, or other periodic changes. In such cases, the above pattern can be generalized in the following manner:

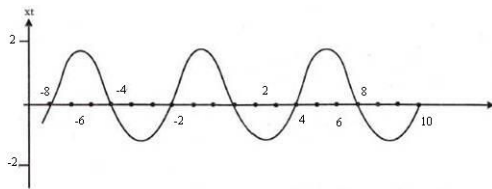


Figure 1

4-2-2-2 pattern:

$$X_t = \sum_{j=1}^k A_j \cos(\omega_j t + \theta_j) + Z_t$$

Where;

$A_j$  refers to the domain of  $\omega_j$  frequency.

In the pattern 4-2-2-1, if A and  $\theta$  are fixed, the pattern will not be stationary, and in the pattern 4-2-2-2, if  $A_j$  and  $\theta_j$  are fixed, the pattern cannot be stationary, since series average  $E(X_t)$  is dependent upon the time. So, if  $\theta$  is fixed in a case, but it changes in different cases, the pattern 4-2-2-1 will become stationary. 4-2-2-2 pattern can also change into a stationary pattern if a)  $A_j$  is a

discrete random variable with the mean of zero, and b)  $\theta_j$  is random variable with uniform distribution of  $(0, 2\pi)$ .

Pattern 4-2-2-3 can be also written in the following manner:

$$X_t = \sum (a_j \cos \omega_j t + b_j \sin \omega_j t) + Z_t$$

where;

$$a_j = A \cos \theta_j \text{ and } b_j = -A \sin \theta_j$$

Now if in the patterns 4-2-2-2 and 4-2-2-3, frequencies are infinite ( $k \rightarrow \infty$ ), it means that if:

$$X_t = \sum_{j=1}^{\infty} A_j \cos(\omega_j t + \theta_j) + Z_t$$

$$X_t = \sum_{j=1}^{\infty} (a_j \cos \omega_j t + b_j \sin \omega_j t) + Z_t$$

Each discrete stationary process which is measure in specific time period can be shown as follows:

4-2-2-4 pattern:

$$X_t = \int_0^{\pi} \cos \omega t dv(\omega) + \int_0^{\pi} \sin \omega t du(\omega)$$

Where;

$U(\omega)$  and  $V(\omega)$  are discrete processes with orthogonal data which are defined based on each  $\omega$  in the time period of  $(0, \pi)$ . It shows that the random variables of  $[U(\omega_2) - U(\omega_1)]$  and  $[U(\omega_4) - U(\omega_3)]$  are not correlated in the non-intersecting intervals of  $(\omega_1, \omega_2)$  and  $(\omega_3, \omega_4)$ . 4-2-2-4 pattern is called process spectral time series. In this pattern, it is important to show the significance of each frequency in the process changes of the interval  $(0, \pi)$ .

4-2-3- Wiener-Khinchin theorem (associated with the actual processes)

There is the increasing monotonic function of  $F(\omega)$  for each stationary random process with the autocovariance function of  $\gamma_k$ .

$$\gamma_k = \int_0^{\pi} \cos k\omega dF(\omega)$$

This equation is called auto covariance spectral function, and  $F(\omega)$  is called spectral distribution function which can be defined as follows:

For each  $\omega \in (0, \pi)$ ,  $F(\omega)$  shows total variance of harmonic components in the spectral function of  $X_t$  whose frequencies are less than or equal with  $\omega$ .  $\pi$  is the highest possible frequency for the discrete process which is measured at regular and identical intervals. Therefore, all changes will happen for frequencies less than  $\pi$ . So,

$$F(\pi) = \text{var}(X_t) = \sigma_x^2$$

If  $k=0$  in the spectral function,

$$\gamma_0 = \sigma_x^2 = \int_0^\pi dF(\omega)$$

$F(\omega)$  is an increasing monotonic function at the intervals of  $(0, \pi)$ . It can be understood that this function's behaviour is similar to the cumulative distribution functions'. The difference is that instead of 1, the higher limit in this function is  $\sigma_x^2$ . Due to the fact that  $F(\omega)$  is a monotonic function, it can fall into two separate functions of  $F_1(\omega)$  and  $F_2(\omega)$ .

$$F(\omega) = F_1(\omega) + F_2(\omega)$$

Where;  $F_1(\omega)$  is a non-decreasing continuous function which is related to the random component of the process, and  $F_2(\omega)$  is a step function which is related to the deterministic component..

**RESULTS**

The present study assessed daily stock returns on Tehran Stock Exchange. Days in which no transaction has happened or whose return was zero were omitted from the model. Table 5 demonstrates the descriptive statistics of the returns on Tehran Stock Exchange in daily time-series based on annual periods.

**Table 5: Descriptive Statistics of The Stock Returns**

Year	Mean ± SD
2007	0.02879 ± 3.811676
2008	0.03564 ± 0.169521
2009	0.971213 ± 98.3987
2010	0.002781 ± 0.014956
2011	0.038487 ± 9.53924
2012	0.000537 ± 0.056608
2013	0.008495 ± 0.163849
Total	0.005621 ± 0.089242

**Table 6: Residuals**

Residuals				
Min	1Q	Median	3Q	Max
-0.024896	-0.005366	-0.001242	0.005434	0.071863
Coefficients				
	Estimate	Std.Error	V	p-value
Intercept	0.008249	0.001208	6.830	1.25e-09***
C <sub>1</sub>	-0.004152	0.001718	-2.417	0.01782*
S <sub>1</sub>	-0.004052	0.001698	-2.386	0.01928*
C <sub>2</sub>	0.001267	0.001718	0.738	0.46285
S <sub>2</sub>	0.005052	0.001698	2.975	0.00383**
Signif. Codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 '.', 0.1 '.', 1				
Residual standard error: 0.01139 on 84 degrees of Freedom      Multiple R-Squared: 0.2001				
Adjusted R-Squared: 0.162      F-Statistic: 5.253 on 4 and 84 Df      P-Value: 0.007957				

**Table 7: Residuals**

Residuals				
Min	1Q	Median	3Q	Max
-0.023752	-0.005307	-0.001124	0.005547	0.072924
Coefficients				
	Estimate	Std.Error	V	p-value
Intercept	0.008234	0.001204	6.837	1.16e-09***
C <sub>1</sub>	-0.004181	0.001713	-2.441	0.01672*
S <sub>1</sub>	-0.004052	0.001694	-2.392	0.01894*
S <sub>2</sub>	0.005052	0.001694	2.983	0.00373**
Signif. Codes: 0 '***', 0.001 '**', 0.01 '*', 0.05 ' ', 0.1 ' ', 1				
Residual standard error: 0.01136 on 85 degrees of Freedom				
Multiple R-Squared: 0.1949      Adjusted R-Squared: 0.1665 F-Statistic: 6.86 on 3 and 85 Df      P-Value: 0.000341				
F <sub>1</sub> = 0.0111 F <sub>2</sub> = 0.0222				

Time-series model was applied to test the returns in the frequency domain on Tehran Stock Exchange and the achieved results are shown in table 6. C<sub>2</sub> was omitted from this table since it is not significant at the error level of five percent in the model. The final results can be seen in table 7.

$$C_1 = \cos(2PF_1 \times t)$$

$$S_1 = \sin(2PF_1 \times t)$$

$$C_2 = \cos(2PF_2 \times t)$$

$$S_2 = \sin(2PF_2 \times t)$$

$$Ret = \alpha_0 + \sum_{i=1}^2 (\alpha_i \cos(2\pi f_i t) + \beta_i \sin(2\pi f_i t))$$

$\beta_1$  and  $\beta_2$  are coefficients of S<sub>1</sub> and S<sub>2</sub>, and  $\alpha_1$  and  $\alpha_2$  are coefficients of C<sub>1</sub> and C<sub>2</sub>.

In this study, chaos process was analyzed and the performances of chaotic models or time-series frequencies were assessed to predict stock returns on Tehran Stock Exchange. The obtained results prove the existence of chaotic procedure at the error level of 5% on Tehran Stock Exchange. Furthermore, predictability, martingale process and non-linearity of the series were confirmed. Therefore, the efficient market hypothesis of the series cannot be confirmed. This conclusion could be also drawn

that the studied series are chaotic and Frektral market hypotheses about the stock returns are confirmed.

### CONCLUSION AND SUGGESTIONS

Significant advances in computational tools in recent decades enhanced the possibility of applying theories based on deterministic or chaotic non-linear models which seem to be random for a certain extent. Indeed, chaos theory provides more accurate studies of the complex behavioural characteristics among economic variables which cannot be assessed by means of the existing conventional instruments. In the reviewed literature of economics and econometrics, a random behaviour is considered for most economic variables. As a result, changes of these variables are not predictable. In fact, chaos theory provides this opportunity to discover the complex pattern and discipline which rule these variables, and apply them to predict during short periods of time. In section one and review of literature, chaos theory was discussed in the field of economics. There are many models which are based on the linear hypotheses or random procedures which are not valid anymore, and the previous estimations cannot be easily accepted, since the parameters suffered from a lack of sensitivity. Reviewing various tests in the third section, it was found that there are non-linear and chaotic procedures in economics. Deploying time-series tests in frequency domain at a high



confidence level confirmed the existence of a chaotic procedure in stock returns on Tehran Stock Exchange on a daily basis over a period from 2007 to 2013.

Recommendations derived from this study can be explained in terms of an efficient process for providing appropriate way of modeling and predictions in complex and volatile series. Applying time-series models in frequency domain can be considerably helpful in assessing the series which have been confirmed to be chaotic. Technically, models that are based on genetic algorithm, non-linear regression and Fractal models, which are the most useful non-linear models, can be employed to test chaotic series.

## REFERENCES

- Behdad Salami, A. (2003). Chaotic procedure testing in returns on Tehran Stock Exchange, *Journal of Economics*, pp. 35-73.
- Abbasi Nezhad, H. & Naderi, E. (2012). The analysis of chaos, wavelet and neural network in predicting Tehran Stock Exchange, *Journal of Economic Modeling Research*, pp. 119-140.
- Sheri, S. & Aghazarian, N. (2007). An experimental test of Fama and French model in explaining returns on Tehran Stock Exchange, *Experimental Studies of Financial Accounting*, issue 19, pp. 115-137.
- Moshiri, S. & Morovat, H. (2006). Predicting total index of Tehran stock returns applying linear and non-linear models, *Iranian Journal of Trade Studies*, issue 41, pp. 245-276.
- Akbari Moghaddam, B. & Rezaei, F. & Norouzi, A. (2009). Comparing predicting strength of Fama and French models, Beta value, and expected stock returns, *Journal of Economic Modeling*, 3<sup>rd</sup> year, pp. 55-76.
- Moshiri, S. (2002). A review of chaos theory and its applications in economics, *Iranian Journal of Economic Research*, issue 12, pp. 29-71.
- Haji Karimi, B. (2004). Chaos theory and its application in organizational decision making, pp. 31-46.
- Nobakht, M.B. & Gholami Nataj Amiri, S. & Majidzadeh, R. (2011). Evaluation of Iran development in the last century with emphasis on chaos theory and game theory, *Rahbord*, 20<sup>th</sup> year, issue 59, pp. 271-317.
- Ghaheri, A. & Ghorbani, M.A. & Del Afrouz, H. & Malekani, L. (2012). Evaluation of stream flow using chaos theory, *Iranian Water Research Journal*, 6<sup>th</sup> year, issue 10, pp. 117-137.
- Karsten, W., Chaos in german stock return-New evidence from the 0-1 test, *Journal of Economics Letters*.115 (2012) 487-489.