NUMERICAL AND ANALYTICAL SOLUTION OF PENETRATION EQUATIONS BY USING DYNAMIC SPHERICAL EXPANSION METHOD AND REPRODUCING KERNEL PARTICLE METHOD

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ABSTRACT

The objective of this paper is to present the dynamic spherical cavity expansion method (DSCEM) and reproducing kernel particle method (RKPM) for solving penetration equation in the metal target. At first, weak form of the problem is used to discrete the linear momentum equations in spherical coordinate and Lagrangian environment. Then the approximate function RKPM is substituted in the method. Therefore, using explicit method and Gauss quadrature rule and imposing the boundary conditions, the velocity and location equations are obtained. To show the validity of proposed method, some comparisons are made between the experimental and analytical results of other investigators and a good agreement is obtained.

KEYWORDS: RKPM, Dynamic Spherical Expansion Penetration, Metal Targets.

Methods without mesh scheme, such as numerical methods that solve partial differential equations and ordinary differential equations are used. The range of these methods in the past ten years has developed a lot because it has no problem related to mesh and alsoanwer of this method is better than the other numerical methods. Perhaps these methods started in 1934 by Slater and Barta so called Collocation and in 1977 SPH method has been created by Monaghan and Lucy and their colleague, developed this method has been called RKPM in 1995 by Liu and his colleagues. Adding the corrected function dramatically increases the accuracy in comparison with the SPH method that is the lack of this function. In addition, the main disadvantage of SPH method in restricted area is the optimal answer excluded in this way has been removed. Since, this method make a general formula, therefore provides a general formula for constructing shape function using special discrete in RKPM, the relations of the methods of SPH are achieved again. Method was used in this paper is RKPM. According to the lack of mesh and the particles are capable of easily to the stumble numerical simulation of fluid with high deformation is possible. In this field, many article of numerical simulation of penetrationphenomena collision with high deformation and behavior like fluid during the phenomenon influence are presented. Recently, the numerical computation techniques without mesh scheme for simulating impact phenomenon associated with large deformation are found in many fields. Shaofan Li et al (2008) analyzed the technique of contact algorithm with mesh less RKPM method, Jau et al. (1996) analyzed impact formulation with mesh less RKPM method, and the purpose of RKPM is to simulate the influence in the rigid and flexible deformation of projectile. Shaofan Li (2001) simulated plaque phenomenon with high speed. He used the algorithm grow of crack and got help from RKPM mesh less method for numerical simulation of projectile penetration in the steel target. In 2011, Zhang and his group (Ls-Dyna manual theory) simulated the impact phenomenon to metal target with couple of SPH and FEM in three dimensions. In all these articles completely numerical simulation carried out, usually in term of stress momentum equation with using the stress of Pylakryshuf placement but in this article effort to combine many techniques that use two types of switching networks Euler and Lagrangian and are carried out in wich computation data exchange between these two types of networks so as to achieve the intended solution. The proposed method is a combination of these two networks as linear momentum balance equations was discrete in Lagrangian environment and also normal stress of Cauchy Euler in Euler environment is obtained numerical analytical solution using dynamical expansion of spherical cavity.

GOVERNING RELATIONS IN THE FORMULATION RKPM

Methods of SPH and RKPM are relying on emulation function for the reproduction of the desired function. Liu et al (1995, 1996) suggested a function for correct reproduction function in adjacent border areas and even in other parts of the range that lead to improvement the solutions even in the limited areas that they were considered weaknesses of SPH method. Thus, Liu proposed a model so called RKPM can be expressed as follows
\[ u^h(\xi) = \int_{\Omega} u(x)\Psi(\xi;\xi - x)dx \]  

(1)

Where \( \Omega \) is computational domain and \( h \) is smoothing length, and \( \Psi \) is the Kernel function or simulator, which is defined as, follows

\[ \Psi(\xi;\xi - x) = c(\xi;\xi - x)\phi_a(\xi - x) \]  

(2)

Where \( c \) is correction function and \( \phi_a \) is the window function which is defined as follows

\[ \phi_a(\xi - x) = \frac{1}{a}\phi(\frac{\xi - x}{a}) \]  

(3)

\[ c(\xi;\xi - x) = \beta_a (\xi)P(\xi - x) \]  

(4)

Where \( \alpha \) is the dilatation parameter and \( P(\xi - x) \) is the fundamental function. In this paper the two-dimensional and linear forms are used the following form, where \( \beta_a (\xi) \) is unknown coefficients which is computed as follows

\[ p^T = (1,x,y) \text{ in } 2D \]  

(5)

\[ m(a,\xi)\beta_a (\xi) = P(0) \]  

(6)

At first, in process of simulating the physical properties adapted with RKPM formulation approximate value of the central particle with neighbor's particles is calculated as follows where the parameter I represents the central particle and calculated with neighbor's particles \( J \).

\[ < u(r^j) > = \sum_{j} u(r^j)\Psi\Delta v^j \]  

(7)

Each particle size is calculates as follows:

\[ \Delta v^j = m^j \rho^j \]  

(8)

Select a suitable weight function for RKPM shape function according to articles in the field of penetration or impact is being offered. In most cases, the cubic B-spline weight function has been used that in this article is in a same manner.

\[ \phi(r,h) = \alpha_J \begin{cases} \frac{2}{3} - r^2 + \frac{1}{2}r^3 & 0 \leq r \leq 1 \\ \frac{1}{6}(2 - r)^3 & 1 \leq r \leq 2 \\ 0 & r \geq 2 \end{cases} \]  

(9)

dimensional, \( h \) is smooth length and usually considered a fixed number in the present calculations; the smooth length is without dimension and be considered 1.2.
FORRESTAL AND LUK MODEL WITH DYNAMIC SPHERICAL CAVITY EXPANSION.

Dynamic spherical cavity expansion problem is considered, symmetrically. Using similar change in mass and momentum conservation equations, equations with nonlinear partial differential equations becomes normal. Radial tensions at the cavity surface as a function of cavity expansion velocity, strain rate and strain of hard are taken Warren and Forrestal. Using the least squares fit on the numerical data of radial tension to matches the chart.

\[
\frac{\sigma_r(a)}{Y} = A + B(\sqrt{\frac{\rho_0}{Y} a}) + c \left( \sqrt{\frac{\rho_0}{Y} a} \right)^3
\]

\(a, \dot{a}\) are the radial velocity considered to fixed and cavity radius, respectively. A, B and C are experimentally dimensionless coefficient. Since, the analytical model presented in this paper is based on spherical cavity, this method and relationship offered by Forrestal and Luk is given fully described in brief.

\[\rho = k\eta = k(1 - \frac{\rho_0}{\rho}) = \frac{1}{3}(\sigma_r + \sigma_\theta + \sigma_\phi) \quad (11)\]

In this method, a cavity expansion with radial velocity \(V\) of zero radius was studied with considering \(r\) as radial coordinate and \(t\) as time, the material can be divided in to two area plastic limited of \((r=vt, r=ct)\) and elastic limited of \((r= c dt, r=ct)\) C is speed of elastic and plastic area boundary and \(cd\) is rate of expansion elastic.

![Figure 1: Response regions for the cavity expansion problem.](image)

Material behavior in plastic area is linear relationship pressure-strain with criterion Trskawill be assume the following

\[\sigma = \frac{2}{3}Y [1 + \ln(\frac{2E}{3Y})] + \frac{3}{2} \rho V^2 \quad (12)\]

Luk, Okajimo and Forrestal to calculate the normal stress on the cavity expanding surface with speed \(V\) for incompressible material provided the following relationship:

\[\sigma_n = AY + B \rho V^2 \quad (13)\]
where $A = \frac{2}{3}[1 + \ln\left(\frac{E}{3(1-\nu)Y}\right)]$ and $B$ is an experimental coefficient which is obtained with machine result of practical experiment; and $V$ is Poisson coefficient.

Stress calculated based on theory of dynamic expansion of spherical cavity placement in the linear momentum balanced discrete with RKPM approximation function. Momentum equation in one dimension and direction of influence can be solved as follows.

**Approximate Linear Momentum Balance Momentum Equations.**

In the Lagrangian environment, the discrete weak form is given as follows. In fact, this equation represents the virtual work system is $\delta u$.

$$\int_{\Omega} \delta u(\nabla \sigma_n + \rho \ddot{u} + b) d\Omega = 0 \quad (14)$$

where $\sigma_n$ is Cauchy stress, $\rho$ is density and $b$ is physical force. Eliminating the physical forces at the beginning, we have:

$$\int_{\Omega} \delta u(\nabla \sigma_n + \rho \ddot{u}) d\Omega = 0 \quad (15)$$

According to the definition of divergence, we remove the tension of gradient term from momentum equation as follows:

$$\text{div}(\delta u, \sigma_n) = \nabla (\delta u, \sigma_n) \quad (16)$$

$$t = \sigma_n n$$

$$\Rightarrow \int_{\Omega} \delta u \nabla \sigma_n d\Omega + \int_{\Omega} \sigma_n \nabla \delta u d\Omega = \int_{\Gamma} t \delta u d\Gamma \quad (17)$$

In above relation $n$ is external normal vector on the surface rand $t$ is the traction on the same level. With high relationship, the momentum equation can be rewritten in following three separate integrals that the tension gradient has no separation:

$$\int_{\Omega} \delta u(\nabla \sigma + \rho \ddot{u}) d\nu = 0 \Rightarrow$$

$$\int_{\Omega} \rho \ddot{u} d\nu + \int_{\delta \Omega} t \delta u ds - \int_{\Omega} \sigma_n \nabla \delta u d\nu = 0 \quad (18)$$

$$R_u = \int_{\Omega} \rho \ddot{u} \sum \delta u_i \phi_a(r^i) \Delta V^i d\Gamma + \int_{\Gamma} t \sum \delta u_i \phi_a(r^i) \Delta V^i d\Gamma$$

$$- \int_{\Omega} \sigma_n \sum \delta u_i \nabla \phi_a(r^i) \Delta V^i d\Omega = 0 \quad (19)$$

With derivation from the residual value will have the following relation.
\[
\frac{\partial R_u}{\partial \delta u} = 0 \Rightarrow \int_\Omega \rho \ddot{u} \phi_a (r^I) \Delta V^I d\Omega \\
+ \int_\Gamma t \phi_a (r^I) \Delta V^I d\Gamma - \int_\Omega \sigma_n \nabla \phi_a (r^I) \Delta V^I d\Omega = 0
\]

Equation of motion in above includes three integral equations of motion which is separated. Each integral separately is converted to discrete form:

\[
\int_\Omega \rho \ddot{u} \phi_a (r^I) \Delta V^I d\Omega = \sum_j \rho \ddot{u} \phi_a (r^{I_j}) \Delta V^I \Delta V^{I_j} \quad (20)
\]
\[
\int_\Omega \sigma_n \nabla \phi_a (r^I) \Delta V^I d\Omega = \sum_{j \neq h} \sigma_n \nabla \Psi (r^I) \Delta V^I \Delta V^{I_j} \quad (21)
\]
\[
\int_\Gamma t \phi_a (r^I) \Delta V^I d\Gamma = \sum_j t \phi_a (r^{I_j}) \Delta V^I \Delta s^{I_j} \quad (22)
\]

Forces applied during the penetration process are as follows:

\[
f^{ext} = \int_\Omega b \Psi (X) d\Omega + \int_{\partial \Omega = \Gamma} t \Psi (X) d\Gamma \quad (23)
\]
\[
f^{int} = \int_\Omega \sigma_n \Psi (X) d\Omega \quad (24)
\]

**MAKING DISCREET EQUATION OF MOTION**

\[
M \ddot{u}(t) + f^{int}(u(t)) + f^c(u(t)) = f^{ext}(t)
\]

Where \(M\) is mass matrix, and \(t\) is step time, and \(u, \ddot{u}, \dddot{u}\) respectively are vectors of displacement, velocity and acceleration that with explicit method above equation rewrite as follow:

\[
\dddot{u}(t) = M^{-1} [f^{ext}(t) - f^{int}(u(t))] \quad (26)
\]
\[
\frac{d\dot{u}}{dt} = \frac{\dot{u}_{t+1} - \dot{u}_{t-1}}{2\Delta t} = M^{-1} [f^{ext}(t) - f^{int}(u(t))] \quad (27)
\]

Time index, index location, velocity and location of particle are obtained as follows:

\[
\dot{u}^{n+1} = \dot{u}^n + \Delta t (\ddot{u}^n + \dddot{u}^{n+1}) \quad (28)
\]
\[
\dddot{u}^{n+1} = \dddot{u}^n + \Delta t \dddot{u}^n + \frac{1}{2} \Delta t^2 \dddot{u}^n \quad (29)
\]

After computing the calculation for \(\Delta t\) was observed that the sensitivity of results to change the parameters for smaller amounts of \(\Delta t=10^{-9}\)sec very small and is available regardless, the calculations were done so in order.

Example: In this example, two kind of aluminum target has been used with this following information. Geometry; dimensions and gender of projectile are considered as (FORRESTA and WARREN, 2008).

| Table 1. Properties of material (target) |
Figure 2. Penetration depth (mm) versus striking velocity (m/s) for 6061T651 aluminum targets and 4340 Rc 38 steel projectiles.

Figure 3. Penetration depth (mm) versus striking velocity (m/s) for 6061T651 aluminum targets and both 4340 Rc 38 and Aer Met 100 Rc steel projectiles.

Figure 4. Penetration depth (mm) versus striking velocity (m/s) for 7075T651 aluminum targets and T-200 maraging steel projectiles.
CONCLUSION

The proposed method is a combination of Lagrangian and Euler environments, the main problem in this paper was discussed about the process of convergence of computing that minimizing the time, this problem was resolved. For numerical integration we have used Gauss Quadrate method, all relationship related to RKPM easily convert to SPH and EFG, thus the calculation done with substitution of shape function is convertible to another mesh less method. Time parameter was removed and the result based on speed-place have been offered, semi-infinite target was considered and also the result of analytical numerical solution was compared with Forrestal’s analytical result (FORRESTA and WARREN., 2008) and experimental result (Piekutowski et al., 1999).

REFERENCES


Ls-Dyna manual theory.


