NUMERICAL SIMULATION AND PLAN STRESS ANALYTICAL SOLUTION OF ROTATING DISK IN HIGH SPEED

S. A. ZAMANI a, S. R. TAHMASBPOUR OM Ran b, B. ASADI c AND M. HOSSEINZADEH d

a Department of Mechanical Engineering Sama Technical and Vocational Training College, Islamic Azad University, Ayatollah Amoli Branch, Amoli.

b Department of Mechanical Engineering Sama Technical and Vocational Training College, Islamic Azad University, Ayatollah Amoli Branch, Amol, Iran.

c Department of Mechanical Engineering, Golpayegan University, Golpayegan, Iran.

d Department of Mechanical Engineering, Ayatollah Amoli Branch, Islamic Azad University

ABSTRACT

In this paper annular disk has been investigated as the most applied and simplest from affected by centrifugal force emanating from disk rotation. Firstly, elasticity theory has been presented by plane stress analytical solution of rotating annular disk with angular speed and fixed profile and boundary condition of non-movement in internal radius and without radial stress in external radius and effect of Poisson's ratio parameter on maximum displacements and stresses have been investigated. Also, comparison of numerical simulation of results of rotary annular disk by finite element method by ANSYS Workbench software with analytical results is indicative of coinciding of two results. According to extracted equations of elasticity solution and by assuming this fact that use of these equations are unable for predicting of precise value of maximum stress and maximum Displacement in two rotation speed and assumption of small deformation are applicable but are not applicable in high speed and by assumption of large deformation, maximum stress and maximum deformation of rotary annular disk in high speed have been predicted by preparation of computer program and revision of extracted equation emanating from elasticity solution. effect of rotation speed of disk has been resented on simulation results by computer program.

Keywords: Simulation, Analytical solution, Annular Disk, High Speed, Plane Stress.

In different mechanical systems, a component with characteristics of one annular disk is necessary as a primitive and fundamental element. These components are necessarily required in gas turbine, fly wheel, fan gear box, pumps, and big compressors (Serati and Alehossein, 2012). These rotating disks are usually rotating with high angular speed. High angular speed leads to formation of centrifugal force in disks and this phenomenon eventually causes deformation and radial displacement in disk. So far many researches have been done on rotary disk (Venkatarama Reddy, 2012; Hamid EkhteraeiToussi, 2012).

In this part, disk in the most applicable and the simplest condition, centrifugal force emanating from disk speed, will be investigated. Firstly, by elasticity theory with assumption of plane stress (Vullo and Vivio, 2008; Blazynski, 1983), analytical solution of rotating annular disk with angular speed, fixed profile, boundary condition of fixed support in internal radius and zero radial stress in external radius of disk, will be presented and effect of poisons ratio parameter on maximum displacement and maximum stresses will be investigate. Also, aforementioned issue will be simulated by finite element method (Sameguid and Kanth., 2002) by ANSYS Workbench software and elicited results will be compared with analytical results.

Theoretical Fundamentals and Extraction of Elasticity Equation of Issue

In practical, many engineering problems exist that stress distribution around an axil is axisymmetric. It must be wealthy mentioned that when this assumption is correct that geometry and boundary condition be axisymmetric. Analysis of this problem will be done under axisymmetric condition including rotating disk that is just rotating (Nie and Batra, 2010).

In this problem one thin and thick (t<<b) annular disk from isotropic, with internal radius (a), external radius (b), thickness (t) that is proved from internal radius region from Figure 1 is assumed. With assumption of without loading in direction of z, results to σz=0 and consequently plane stress is dominant to the
problem. Also, it must be kept in mind that due to axisymmetric condition of the problem, stress is symmetric around \( z \) direction and displacement is not of \( \theta \). In Figure 1, three-dimensional of assumed annular disk rotating with angular speed (\( w \)) of 18000 RPM with boundary condition, is illustrated.

![Image of three-dimensional profile of disk with its governed conditions](image)

Figure 1: Three-dimensional profile of disk with its governed conditions.

\[
F_r = \rho r \omega^2
\]  

(1)

In Table 1, mechanical properties and geometric characteristics of assumed disk has been presented.

Table 1: Assumed parameters in numerical simulation and analytical solution of annular rotating disk.

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( t )</td>
<td>m</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>( b (R_{out}) )</td>
<td>m</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>( a (R_{in}) )</td>
<td>m</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>( \omega )</td>
<td>RPM</td>
<td>18000</td>
</tr>
<tr>
<td>5</td>
<td>( E )</td>
<td>Gpa</td>
<td>71</td>
</tr>
<tr>
<td>6</td>
<td>( \nu )</td>
<td></td>
<td>0-0.5</td>
</tr>
<tr>
<td>7</td>
<td>( \rho )</td>
<td>Kg/m(^3)</td>
<td>2770</td>
</tr>
</tbody>
</table>

Correlating centrifugal force, density of disk material and angular speed, created body force in radial direction has been obtained by Equation 1.

Also, by considering of problem condition, stress-strain equations in polar coordinates have been presented in Equations 2 and 3, where \( \sigma_r \), \( \epsilon_r \), \( \sigma_\theta \), \( \epsilon_\theta \), \( E \) and \( \nu \) are radial stress and strain, circumferential stress and strain, Young's modulus and Poisson's ratio, respectively.

\[
\sigma_r = \frac{E}{1-\nu^2} (\epsilon_r + \nu \epsilon_\theta)
\]  

(2)

\[
\sigma_\theta = \frac{E}{1-\nu^2} (\epsilon_\theta + \nu \epsilon_r)
\]  

(3)

By considering of problem, equilibrium equations will be simplified to Equation 4.

\[
\frac{d \sigma_r}{dr} = \frac{\sigma_r - \sigma_\theta}{r} + \rho r \omega^2 = 0
\]  

(4)

By substituting equation 2 and 3 into 4 gives:
\[ r \frac{d(\epsilon_r + v \epsilon_0)}{dr} + (1-v)(\epsilon_r - \epsilon_0) = - \frac{(1-v^2)}{r} \rho r^2 \omega^2 \]  

(5)

By applying of boundary condition (11), unknown coefficients \( c_1, c_2 \) will be calculated by Equations 12 and 13.

\[ \begin{align*}
\sigma_0 &= A \text{ at } r = a \\
\sigma_r &= B \text{ at } r = b
\end{align*} \]

(11)

\[ C_1 = \frac{B b^2 - A a^2 \left[ \frac{(3+v)}{8} \times \rho \omega^2 (a^2 - b^2) \right]}{(b^2 - a^2) \left[ \frac{E}{(v-1)} \right]} \]

(12)

\[ C_2 = \frac{(B-A) b^2 a^2 \left[ \frac{(3+v)}{8} \times \rho \omega^2 (a^2 - b^2) \right]}{(b^2 - a^2) \left[ \frac{E}{(v+1)} \right]} \]

(13)

By solving of differential equation, Equation 8 will be obtained.

\[ u_r(r) = \frac{(v^2-1) \rho r^3 \omega^2}{8 E} + \frac{C_1 r + C_2}{r} \]

(8)

(14)

\[ \begin{align*}
\sigma_r &= S \text{ at } r = a \\
\sigma_r &= B \text{ at } r = b
\end{align*} \]

By applying of boundary condition (14), unknown coefficients \( c_1, c_2 \) will be calculated by Equations 15 and 16.

\[ C_1 = \frac{B b^2 - A a^2 \left[ \frac{(3+v)}{8} \times \rho \omega^2 (a^2 - b^2) \right]}{(b^2 - a^2) \left[ \frac{E}{(v-1)} \right]} \]

(15)

\[ C_2 = \frac{(B-A) b^2 a^2 \left[ \frac{(3+v)}{8} \times \rho \omega^2 (a^2 - b^2) \right]}{(b^2 - a^2) \left[ \frac{E}{(v+1)} \right]} \]

(16)

\[ \sigma_r(r) = \frac{(3+v) \rho r^3 \omega^2}{8} + \frac{E \left[ C_1(1+v) - \frac{C_2}{r}(1-v) \right]}{(1-v^2)} \]

(9)

\[ \sigma_\theta(r) = \frac{(3+v) \rho r^3 \omega^2}{8} + \frac{E \left[ C_1(1+v) - \frac{C_2}{r}(1-v) \right]}{(1-v^2)} \]

(10)

**RESULTS AND DISCUSSION**

In this part results from numerical simulation and analytical solution will be presented. Figure 2 Shows change von-mises stress in different Poisson’s ratios by analytical and numerical method respectively.
Figure 2: Changes of equivalent von-mises stress by radius in different Poisson’s ratios by analytical solution and numerical simulation.

As it can be seen, from radial distance of 0.07 meter to radial distance of 0.09, change trend of equilibrium stress will be undependable of Poisson’s ratio.

Figure 3: Changes of radial displacement by radius in different Poisson’s ratios by analytical solution and numerical simulation.
Figure 4: Changes of radial strain by radius in different Poisson’s ratios by analytical solution and numerical simulation.

Figure 3 shows change trend of radial displacement by radial distance from center in different Poisson’s ratios (0-0.5) by analytical and numerical method, respectively. In a similar radial distance, radial displacement increases with decrease of Poisson’s ratio. Also, according to Figure 4 that shows change of radial strains by radius in different Poisson’s ratios with analytical and numerical method respectively, maximum displacement approach to internal radius by increase of Poisson’s ratio while only in zero Poisson’s ratio maximum displacement exists in external radius. In Figure 4, it can be concluded that except from zero Poisson’s ratio, in other states, circumferential strain will have a negative value in some radial distances. It can be concluded that in that distance, disk can go to internal direction of disk and can switch on the contrary of centrifugal force.

Edition of Equation for Obtaining of Maximum Stress and Maximum Deformation in High Speed

Extracted equation in previous section is acceptable by assumption of small deformation. But while the rotation speed of disk is high assumption of small deformation, will not be acceptable.

For editing of equation from elasticity solution boundary condition will be applied. On the other hand, after obtaining of displacement equation by radius (ut(r)), sum of external radius of disk without rotation in return of external radius and radial displacement value of rotating disk in external radius of disk will be inserted in extended computer program.

\[
\begin{cases}
  u = S & \text{at } r = a \\
  \sigma = B & \text{at } r = b' \\
  b' = b + u_{r=b}
\end{cases}
\]
**Figure 5:** Changes of radial displacement by radius.

**Figure 6:** Changes of radial stress by radius of disk.
Figure 7: Changes of circumferential stress by radius of disk.

Figure 8: Changes of radial strain by radius of disk.
As it can be seen in above results, a good accordance exists among assumption results of small deformation, assumption of large deformation and revised equation in low speed rotation.

**Figure 9:** Changes of circumferential strain by radius of disk.

**Figure 10:** Changes of radial displacement by radius of disk.
Figure 11: Changes of radial stress by radius of disk.

Figure 12: Changes of circumferential stress by radius of disk.
The difference between results from assumption of small deformation and large deformation is visible in high speed. Also, it can be seen that extended computer program can predict the maximum results value of assumption of large deformation very well.

In this section after validation of extended computer program that predicts maximum
displacement value, stress and strain very well, effect of angular speed will be investigated on maximum radial displacement, radial stress and circumferential stress and radial and circumferential strain.

**Figure 15:** Changes of maximum radial displacement by angular speed of disk.

**Figure 16:** Changes of maximum stress by angular speed of disk.
CONCLUSIONS

In this research, after analytical solution and coincidence of it with results of numerical simulation by limit element method and by ANSYS Workbench software, extracted equations from elasticity rotation of annular rotating disk have been edited for predicting of maximum von-mises stress and maximum radial displacement in high rotation speed and large deformation on the other hand, prediction of maximum von-mises stress and maximum radial displacement is possible in high speed and large deformation by elasticity solution. Effect of Poisson’s ration parameter has been investigated on place of maximum von-mises stress and maximum radial displacement and effect of speed on simulated results has been presented by computer program.


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