

SQUARE HOLE CREATED BY DRILLING DOWN BY USING UNIVERSAL NON-COAXIAL COUPLINGS

SHAHRAMHOSSEINI CHALESHTORI^{1a}, EHSAN ZAMANI^b, AND AHMADREZA ASHRAFI^c

^aUniversity of Kashan

^bShahrekord University

^cShahrekord University

ABSTRACT

Producing square holes in the industry, is very common and useful and at the same time along with problems such as high cost and complexity of manufacture. Using a drill with non-coaxial couplings, in addition to easier manufacturing, also reduces the costs of production. A mathematical model for drill mechanism has been proposed in the first part of this paper. The mechanism of the drill and cam is designed by using the mathematical model. The main limitation of the square hole drilling mechanism is cam center of rotation, where it is not fixed and moves in a non-circular path. There is thus a need to non-coaxial couplings, in order to transmit rotational motion, made by drill to cam and bit. Examples of non-coaxial couplings are Richard Schmidt, Oldham and Universal couplings, which this paper was to examine the universal couplings.

KEYWORDS: Reuleaux Polygon, Square Hole, Universal Couplings, Edm Machining, Curves Of Constant Width

Square holes are used widely in industry. Examples of these applications are used in some of the couplings. Since the square shape of the hole and shaft, will lead to full involvement of couplings, accordingly prevent freewheeling. There are a variety of methods for creating square holes. One of these methods is making a square hole by CNC machine. Using CNC machine, despite the high accuracy, costs too much. In this method, the hole plan on workpiece, designed by CAD softwares, and then after becoming to G-Code, it transferred to CNC machine (Alan Overby, 1986). Another method is to use an Electrical Discharge Machine (EDM). EDM is a process that uses electrical discharges from an electrode to erode an electrically conductive material (Ben Fleming, 2005), regardless of the mechanical properties such as strength and hardness. In this machining method, the electrode is constructed to the shape of desired hole and by creating electrical discharge between tool electrode and workpiece, the spark is generated and the shape of electrode on workpiece caused (K.Y. Song et al., 2013). It should be noticed that no physical contact between the tool electrode and workpiece is created in the EDM method.

Broaching machining method is another method that can be used to create square holes on workpieces. Broaching is commonly used for machining of internal or external complex profiles that are difficult to generate by other machining processes such as milling and turning. Originally broaching was developed for non-circular internal profiles, such as square hole and keyways (U. Kokturk and E. Budak, 2004). Square hole drilling mechanism, including a triangle-like cam that can have rotational motion within a square frame. To investigate the mathematical model for the cam, a discussion called Curves of Constant Width is proposed. Curves of constant width provide a great example of vector space and also allow us to gain access to a large amount of information. In particular, Reuleaux polygons, a special set of curves of constant width give unexpected results to practical questions; such as drilling a square hole (Peter Fumich et al, 1987). The existence of non-circular curves of constant width in the standard Euclidean plane has been known since the time of Euler; e.g., the Reuleaux triangle was presented by Reuleaux

to Hornblower, the inventor of the compound steam-engine (Horst Martini and ZokhrabMustafaev, 2008). In recent years, mathematical properties of the Reuleaux triangle have led to some very important applications. Since a curve of constant width can be freely rotated in a square always maintaining contact to all four sides of the square, a Reuleaux triangle can be used for drilling holes of maximum area into squares. Reuleaux polygons are formed by connections through non-uniform circular arcs to each other (Kupitz, Y.S. and Martini, H., 2000). Some Reuleaux curves are shown in fig. 1. The curves of constant width have a plane shape with a constant width in all directions. The proposed term, width, is a vertical distance between the two parallel lines tangent to curve.

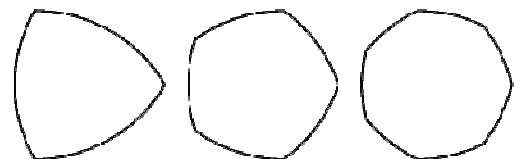


Figure 1. Curves of constant width

The square frame which the cam rotates inside it is fixed to the body of the drill. As long as the cam is rotated 360 degrees, the whole inner surface of square frame is covered by cam rotation inside the square frame. Accordingly, if a bit is designed on the cam, it could be able to cover entire square surface and do milling in a square shape. Off-axis couplings transmit torque between two off-axis shafts. For instances Richard Schmidt coupling (fig. 2), Oldham coupling (fig. 3), and Universal coupling or Hook coupling (fig. 4) are noted. The inventor of the coupling, Richard Schmidt of Madison, Alabama, said that a similar link arrangement had been known to some German engineers for years. But those engineers were discouraged from applying the theory because they erroneously assumed that the center disk had to be retained by its own bearing. Actually, Schmidt found that the center disk is free to assume its own center of rotation. In operation, all

three disks rotate with equal velocity (Neil Sclater and Nicholas P. Chironis, 2007).

In Oldham coupling, it should be noticed that it can be proved all three disks rotate with equal rotational velocity, and thus the rotational velocity of driver shaft and driven shaft will be equal (Yutaka Nishiyama, 2009).

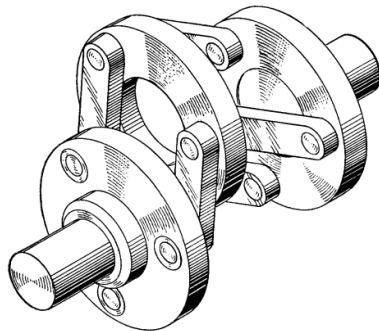


Figure 2. Richard Schmidt coupling

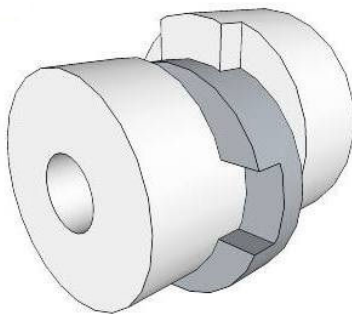


Figure 3. Oldham coupling



Figure 4. Universal coupling

A mathematical model used for cam mechanism is provided in the first part of this paper. In the second part, the universal coupling used for transmitting torque from drill to cam, has been studied.

MATHEMATICAL MODEL FOR THE CAM

Equations have been derived based on the simplest curves of constant width, the circle. According to fig. 5, width of curve calculated by the equation:

$$h(t) + h(t + \pi) = k \quad (1)$$

Where h shows perpendicular line to draw tangent to the curve, t is the angle between h line and horizontal axis and k is a constant. Family of tangents is obtained from equation:

$$F(x, y, t) = x \cdot \cos t + y \cdot \sin t - h(t) \quad (2)$$

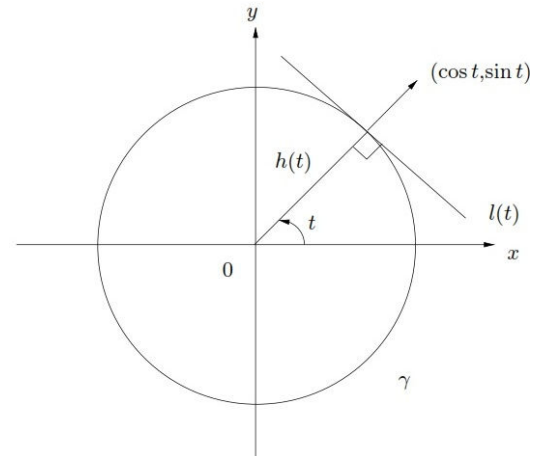


Figure 5. circle, simplest curve of constant width

To find envelope of this family, we use the equation (Andrew David Irving, 2006):

$$F = \frac{\partial F}{\partial t} = 0 \quad (3)$$

x and y are measured in terms of the parameter t , using equations 2 and 3:

$$\begin{aligned} x(t) &= h(t) \cdot \cos t - h'(t) \cdot \sin t \\ y(t) &= h(t) \cdot \sin t + h'(t) \cdot \cos t \end{aligned} \quad (4)$$

By choosing $h(t)$ to different forms, variety curves of constant width equations are obtained. It should be noted that $h(t)$, should satisfy equation 1. For example by choosing $h(t)=1$, equation of the simplest curves of constant width, the circle is obtained.

By choosing $h(t) = a \cdot \cos^2\left(\frac{3t}{2}\right) + b$, curve width would be equal $to a+2b$ (Andrew David Irving, 2006) (Stanley Rabinowitz, 1997). If $a=2$ and $b=8$, then:

$$\begin{aligned} x(t) &= (\cos 3t + 9) \cdot \cos t + 3 \sin 3t \cdot \sin t \\ y(t) &= (\cos 3t + 9) \cdot \sin t - 3 \sin 3t \cdot \cos t \end{aligned} \quad (5)$$

The diagram of equation 5 is plotted in fig. 6, using the software, *maple*, which has shown the profile of cam curve.

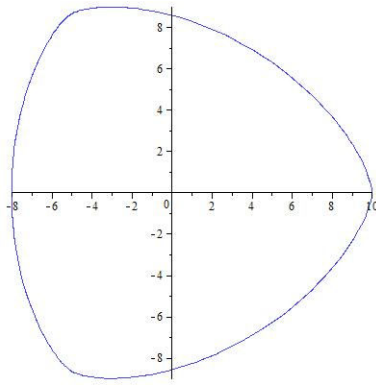


Figure 6. diagram of cam curve

1. Universal couplings for torque transmission

In square hole drilling mechanism, the cam center of rotation, is not fixed and is displayed in a non-circular path. Due to this, a non-coaxial coupling should be used for torque transmission. The universal coupling is one of the most widely used of non-coaxial couplings, which has the ability to transmit torque between parallel and non-parallel shafts and is composed of two universal joints. The joint was used on the clock mechanism for the first time at 1352 AD. Figure 7 shows the universal joint, which is designed in the 16th century and is maintained in Deutsches museum in munich (H.Chr. Seherr-Thoss et al., 2006).

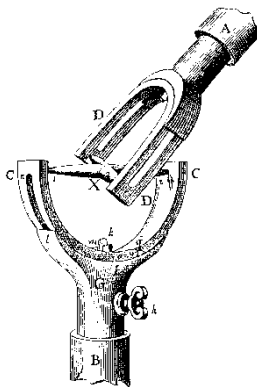


Figure 7. Initial figure of universal joint in the 16th century

According to fig. 8, the relation between the rotational angle of the driver shaft (ϕ_1) and the rotational angle of the driven shafts (ϕ_2), obtained as equation 6:

$$\tan \phi_2 = \frac{\tan \phi_1}{\cos \beta} \quad 6$$

From equation 6, the velocity ratio between the driver shaft and driven shaft is calculated into equation 7. The velocity ratio diagram has also been plotted in parameter ϕ_1 , in fig. 9.

$$\frac{\omega_2}{\omega_1} = \frac{\cos \beta}{1 - (\sin \beta \cdot \sin \phi_1)^2} \quad 7$$

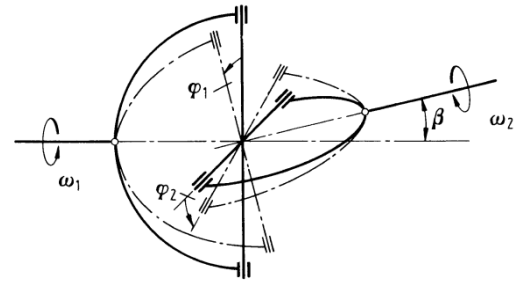


Figure 8. Universal joint angles in schematic form

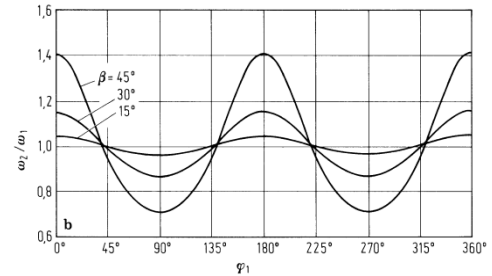


Figure 9. The ω_2/ω_1 ratio diagram in ϕ_1

The driven shaft acceleration is also calculated from equation:

$$\alpha_2 = \omega_1^2 \frac{\cos \beta \cdot \sin^2 \beta \cdot \sin 2\phi_1}{(1 - \sin^2 \beta \cdot \sin^2 \phi_1)^2} \quad 8$$

In the case shown in fig. 10, if two Universal joints are used in doubled mode, the Universal coupling is caused, which is capable to transmit torque between off-axis shafts.

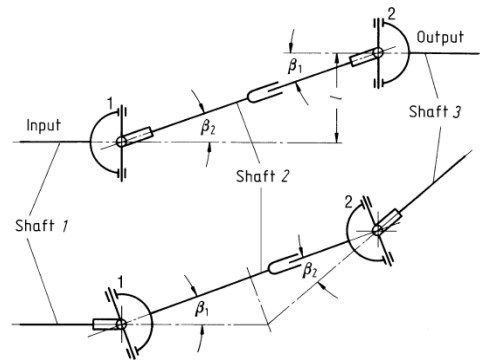


Figure 10. The universal coupling, made of successive connection of two universal joints

In Universal coupling, the relation between rotational angle of driver shaft, and rotational angle of driven shaft, is expressed in equation:

$$\tan \phi_3 = \frac{\cos \beta_1 \cdot \tan \phi_1}{\cos \beta_2} \quad 9$$

In case which driver and driven shafts are off-axis, then, and therefore the rotational velocity of driver and driven axes are equal.

CONCLUSION

Parametric equations of cam curve are obtained in t , as follows:

$$\begin{aligned}x(t) &= (\cos 3t + 9) \cdot \cos t + 3 \sin 3t \cdot \sin t \\y(t) &= (\cos 3t + 9) \cdot \sin t - 3 \sin 3t \cdot \cos t\end{aligned}\quad 10$$

Triangular cam profile is obtained from equation 10, which can be rotated within a square frame. So if the bit is mounted on the cam, it can make a square hole (fig. 11).

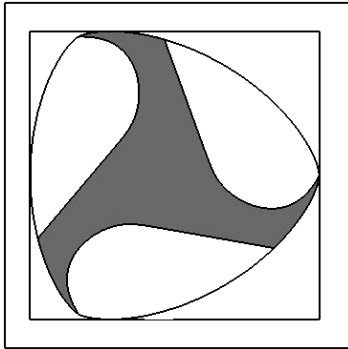


Figure 11. The cam and the bit mounted on it

Because the cam center of rotation is not fixed, universal couplings are used to transmit torque. Unlike the universal joints, which rotation velocity of driver and driven shafts are not equal, in universal couplings the driver and driven velocities are equal. And therefore these kinds of couplings are more appropriate to transmit torque from drill to cam.

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