TIME SERIES FORECASTING MODEL FOR IRANIAN GOLD PRICE

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ABSTRACT

Gold demand in recent years due to rising prices and the economic crisis due to sanctions on Iran has increased. In this paper we have used AR model, AR-IGARCH model, SETAR and STAR models for forecasting and these methods are applied for modeling a monthly log return time series of gold price from August 2007 to November 2013 (price in Iranian Rial against 1 gram of gold). The result reviews that the time series is nonlinear and SETAR (2,1,3) model yields the best result.

KEYWORDS: Log return time series; ARCH/GARCH model; TAR model

Financial time series analysis is the fundamental tool of study in asset valuation over time. We know that one of the reasons for the change in the price of gold is the external factors such as economic policy, environmental, political and social issues. In this hypothesis the external effects are modeled as noise, and the phenomena one considered as accidental. When the price of a good is increased, usually a self-regulating force decreases the prices, and vice versa when a price decreased; this force caused the prices to go high. This feedback mechanism could be linear or nonlinear\cite{3}. A time series is stationary if its properties are statistically invariant over time and can be modeled by various parametric methods. ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant. Generalized Auto regressive Conditional Heteroscedastic (GARCH) model consider the moments of a time series as invariant and is one approach to modeling time series with heteroscedastic errors. Threshold Autoregressive (TAR) models are commonly referred to as piecewise linear models or regime-switching models. The Self-Exciting Threshold Autoregressive (SETAR) model, first introduced by Tong\cite{10}, is a special case of the TAR model. Here, the movements between the regimes are controlled or governed by a variable called threshold just as in the TAR model with the difference that the threshold of a SETAR model is Self-Exciting. This means that, unlike the TAR model, where the threshold is assumed to be an exogenous variable, the threshold variable of a SETAR model is a certain lagged value of the series itself, an endogenous variable. Smooth Transition Autoregressive (STAR) models are typically applied to time series data as an extension of autoregressive models, in order to allow for higher degree of flexibility in model parameters through a smooth transition. Assuming that behavior of the series changes depending on the value of the transition variable. The transition might depend on the past values of the data series (similar to the SETAR models), or exogenous variables.

Many financial studies are based on returns, rather than prices of assets. Campbell, Lo, and MacKinlay \cite{1997} give two main reasons for using returns. First, for average in vessels, return of an asset is a complete and scale-free summary of the investment to opportunity. Second, return series are easier to handle than price series because the former have more attractive statistical properties.

Let $P_t$ be the price of an asset at time index $t$. We introduce the definition of return that issued throughout the paper. The natural logarithm of the simple gross return of an asset is called the continuously compounded return or log return \cite{1}:

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}}$$

We organize the paper as below. Section 2 deals with ARCH/GARCH method and section 3 deals with Threshold Auto Regressive (TAR) model. In section 4 the analysis results of the monthly log return of gold price(price in Iranian Rial against 1 gram of gold) time series[August 2007 - November 2013] using these methods is presented and in section 5 the comparison of applied methods is discussed. Final conclusions are given in section 6.
ARCH/GARCH MODELS

Volatility is an important factor in financial applications and it means the conditional standard deviation of the underlying asset return. An Autoregressive Conditional Heteroscedastic model considers the variance of the current error term or innovation to be a function of the actual sizes of the previous time periods error terms. Often the variance is related to the squares of the previous innovations.

The univariate volatility models include the autoregressive conditional heteroscedastic (ARCH) model of Engle (1982) and the generalized ARCH(GARCH) model of Bollerslev (1986).

ARCH Model

The first model that provides a systematic framework for volatility modeling is the ARCH model of Engle (1982). The basic idea of ARCH models is that

(a) The shock \( a_t \) of an asset return is serially uncorrelated, but dependent, and

(b) The dependence of \( a_t \) can be described by a simple quadratic function of its lagged values. Specifically, an ARCH(m) model assumes that

\[
 r_t = \mu_t + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2.
\]

Where \( \epsilon_t \sim \text{IID}(0,1) \), \( \sigma_t > 0 \), and \( \alpha_i > 0 \) for \( i > 0 \). The equation of \( \mu_t \) should be an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence. The coefficients \( \alpha_i \) must satisfy some regularity conditions to ensure that the unconditional variance of \( a_t \) is finite. In practice, \( \epsilon_t \) is often assumed to follow the standard normal or a generalized Student-t or a generalized error distribution [1].

GARCH Model

Bollerslev (1986) proposes a useful extension known as the generalized ARCH (GARCH) model. For a log return series \( r_t \), let \( a_t = \Delta r_t - \mu_t \) be the innovation at time \( t \). Then \( a_t \) follows a GARCH(m, s) model if

\[
a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,
\]

Where \( \epsilon_t \sim \text{IID}(0,1) \), \( \alpha_0 > 0 \), \( \alpha_i > 0 \), \( \beta_j > 0 \), and \( \alpha \) is the delay parameter. \( \epsilon_t \) is a Gaussian white noise sequence with mean zero and variance 1. TAR models are piecewise linear, \( p_1 \) is AR order of the lower regime and \( p_2 \) is AR order of the upper regime. And finally \( \alpha_1, \sigma_2 \) are respectively standardized deviations in the lower and upper regimes. Such a process makes the model nonlinear for at least two regimes, but remains locally linear (Tsay, 1989). A comparatively recent development is the Smooth Transition Autoregressive (STAR) model, developed by Terasvirta and Anderson (1992). STAR models, unlike standard TAR models, allow for a more gradual transition of the dependent variable between regimes. The regime indicator in these models is a continuous function rather than an on-off switch (Brooks, 2002). The 2-regime STAR model of order \( p \) is defined by

\[
r_t = c_0 + \sum_{i=1}^{p} \phi_{1i} r_{t-i} + F(r_{t-d})(c_1 + \sum_{i=1}^{p} \phi_{1i} r_{t-i}) + a_t
\]
Where \( d \) is the delay parameter, and \( F(.t) \) is a smooth transition function. In practice, \( F(.) \) often assumes one of three forms: namely, logistic, exponential, or a cumulative distribution function.

The logistic STAR (LSTAR) is an extension of the standard STAR model with the following transition function:

\[
F(r_{t-d}) = \frac{1}{1 + \exp\left(-\frac{r_{t-d} - th}{\gamma}\right)}
\]

Where \( r_{t-d} \) is the threshold variable, \( th \) is transition parameter and \( \gamma \) is standard deviation of the threshold variable.

**MODEL ESTIMATION AND PREDICTION OF LOG RETURN TIME SERIES**

In recent years gold demand due to rising prices and the economic crisis due to sanctions on Iran has increased. The gold price time series is one of the most important economic time series. This particular time series is involved socially because investors prefer to deposit in gold whenever a negative swing in the economy caused.

![Figure 1: Time plot of monthly log returns](image1)

![Figure 2: Sample ACF and PACF of monthly log](image2)

**Figure 2: Sample ACF and PACF of monthly log**

Also, the currency exchange rate is closely connected to the price of gold and the reserve of gold kept by the central bank of the government [11]. So this analysis is revealing in many different ways. Here we analyze the time series of monthly log return gold price from August 2007 to November 2013 using four different models. The plot of the time series is shown in figure 1. The auto correlation function of the time series is given in figure 2.

**Estimation of ARMA Model to Log Returns**

An AR(3) model is estimated for the monthly log returns of gold price. The fitted model is

\[
r_t = 0.024 + 0.428 r_{t-1} - 0.353 r_{t-2} + 0.25 r_{t-3} + a_t,
\]

\[
\sigma_a = 0.062, R^2 = 0.198, mse = 0.0038
\]

The plot of the given time series and its prediction using the estimated AR(3) model is given in figure 3.
The Ljung–Box statistics of standardized residuals $Q(12) = 6.0812$ with a value 0.91 based on its asymptotic chi-squared distribution with 12 degrees of freedom. Thus, the null hypothesis of no residual serial correlation in the first 12 lags is not rejected.

**Estimation of GARCH Model**

The Ljung-Box statistics of the $a_t^2$ series shows strong conditional heteroscedasticity or ARCH effects with $Q(12) = 44.7602$, the $p$ value of which is 1.133e-05. We can obtain the following AR(3)–IGARCH(1,1) model for the series:

\[
 r_t = 0.018 + 0.292 r_{t-1} - 0.2256 r_{t-2} + 0.205 r_{t-3} + a_t, \quad a_t = \sigma_t e_t, \quad \sigma_t^2 = 0.046 \sigma_{t-1}^2 + 0.954 a_{t-1}^2,
\]

\[
 R^2 = 0.17, \quad \text{mse} = 0.0038
\]

Where $r_t$ is the log returns. As a second method, the AR(3)–IGARCH(1,1) model is applied for the prediction of the log return time series and the plot of the given time series and its prediction using this method is given in figure 4.

**Prediction using SETAR Model**

The chosen SETAR model had a threshold delay of 1 and autoregressive order 1 in the lower and 3 in the upper regime. This gives:

\[
 r_t = 0.0234 + 0.4742 r_{t-1} + 0.0592 e_t, \quad \text{if } r_{t-1} < 0.04988
\]

\[
 R^2 = 0.21, \quad \text{mse} = 0.0034
\]

\[
 r_t = -0.0260 + 0.8399 r_{t-1} - 0.7969 r_{t-2} + 0.8170 r_{t-3} + 0.0506 e_t, \quad \text{if } r_{t-1} > 0.04988
\]

\[
 R^2 = 0.73, \quad \text{mse} = 0.0020
\]

All coefficients of the model are significant at level 0.05 except intercept in the upper regime -0.0260. As a third method, the SETAR(2,1,3) model is applied for the prediction of the given time series and its prediction is given in figure 5.

**Prediction using STAR Model**

Applying STAR models to the monthly log returns of gold prices, we obtain the following model:

\[
 r_t = 0.0279 + 0.5396 r_{t-1} - 0.0238 r_{t-2} - 0.0908 + \left(1 + \exp \left(\frac{r_{t-1} - 0.0625}{67}\right)^{-1} (-0.0908 + 0.5871 r_{t-1} - 0.8470 r_{t-2}) + a_t, \quad \text{if } r_{t-1} < 0.04988
\]

\[
 \text{AIC} = -421, \quad \text{mse} = 0.0032, \quad \text{MAPE} = 858.7%\]
All parameters in this model are not significant at level 0.05 except $\theta_1^{(2)} = -0.8470$.

**COMPARISON**

Comparison of the four methods considered for prediction is done by comparing their mean square error. The results of analysis are shown in table 2. Error comparison due to these four approaches given in table 2 shows that TAR(SETAR and STAR) models give smaller error. But it should be noted that the STAR model coefficients were not significant at the 0.05 level.

**Table 2: Error Comparison**

<table>
<thead>
<tr>
<th>M.S.E(AR)</th>
<th>M.S.E(ARxIGARCH)</th>
<th>M.S.E(SETAR)</th>
<th>M.S.E(STAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0037</td>
<td>0.0038</td>
<td>Lower regime: 0.0034</td>
<td>0.0032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper regime: 0.0020</td>
<td></td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

In this paper a AR model, AR-IGARCH model, SETAR and STAR models are applied for analyzing a monthly log return time series of gold price from August 2007 to November 2013 (price in Iranian Rial against 1 gram of gold). Error comparison due to these four approaches given in table 2 shows that SETAR and STAR models give smaller error. But most of the parameters of the STAR model are not significant at level 0.05. Also SETAR model gives larger R-Square than AR and AR-IGARCH models.

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