COUPLING OF REPRODUCE KERNEL PARTICLE METHOD AND FINITE ELEMENT METHOD FOR IMPACT DYNAMICS SIMULATION

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ABSTRACT

In this paper, the reproduce kernel particle method (RKPM) is employed for 2D nonlinear analysis with deformation Elastic-Plastic. Using weak form of the problem, the discrete momentum equation and approximate function RKPM are substituted in the proposed method. Using explicit method and Gauss quadrature role and imposing the boundary conditions, the location and velocity equations of each particle are obtained. The penalty method is used for nonlinear contact algorithm. The discrete form of Johnson-Cook’s equation is utilized to describe the behavior of the material. Also, the Grunizen pressure equation is implemented for the given state equation. The computations associated with the experiments discussed in the present paper were performed in Fortran 90. To illustrate the validity and efficiency of the method comparisons are made with numerical and experimental results of other investigators and a good agreement is obtained.

KEYWORDS : RKPM, Penetration, Coupling, Metal Targets, 2d Simulation, Fortran 90

The mesh less methods are very suitable instruments for solving various type of problems. The main advantage of mesh less methods over Finite Element Methods (FEM) is that it can be applied to especially in large deformation impact and influence debate. The RKPM is one of most commonly used mesh less methods was introduced for the first time by Liu (1995). They added a correction function to kernel integral to reproduce the created conditions. Adding the corrected function dramatically increases the accuracy in comparison with the SPH method that is the lack of this function. Also, the main disadvantage of SPH method in restricted area is the optimal answer excluded in this way has been removed. Since this method make a general formula, this method provides a general formula for constructing shape function using special discrete in RKPM, the relations of the methods of EFG and SPH are achieved again. Recently, the numerical computation techniques without mesh scheme for simulating impact phenomenon associated with large deformation are found in many fields. Shaofan Li et al. [2] analyzed the technique of contact algorithm with mesh less RKPM method, Ming (2006) Analyzed impact formulation with mesh less RKPM method, the purpose of RKPM is to simulate the influence in the rigid and flexible deformation of projectile. Liu (1995). simulated plaque phenomenon with high speed. He used the algorithm grow of crack and got help from RKPM mesh less method for numerical simulation of projectile penetration in the steel target. In 2011, Zhang and his group (2011) simulated the impact phenomenon to metal target with couple of SPH and FEM in three dimensions. The aim of this paper is to simulate the impact phenomena and rigid projectile penetration numerically in metal target with couple of SPH and FEM is commutated in two dimensions. In this paper, for simplicity and matching the appropriate computation, it was assumed the hole of the projectile has been eliminated with FEM method and the the target is obtained by combining FEM and RKPM method.

GOVERNING RELATIONS IN THE FORMULATION RKPM

Methods of SPH and RKPM are relying on emulation function for the reproduction of the desired function. Liu and et al (1995, 2010). suggested a function for correct reproduction function in adjacent border areas and even in other parts of the range that lead to improvement the solutions even in the limited areas that they were considered weaknesses of SPH method. Thus Liu proposed a model so called RKPM can be expressed as follows

\[
{u^R}(x_j) = \int_{\Omega} u(x)\Psi(x-x_j,h)dx \quad (1)
\]

Where \(\Omega\) is computational domain and \(h\) is smoothing length, and \(\Psi\) is the Kernel function or simulator which is defined as follow

\[
\Psi(x-x_j,h) = c(x-x_j,h)\phi(x-x_j,h) \quad (2)
\]

Where \(c\) is correction function and \(\phi\) is the window function which is defined as follows
\[ \phi_a(x - x_j, h) = \frac{1}{a} \phi(\frac{x - x_j}{a}) \]  

(3) 

\[ c(x - x_j, h) = \beta_a(x)p(x - x_j) \]  

(4) 

Where \( a \) is the dilatation parameter and \( p(x - x_j) \) is the fundamental function. In this paper the two-dimensional and linear form are used the following form, where \( \beta_a(\xi) \) is unknown coefficients which is computed as follows

\[ p^T = (1, x, y)in \ 2D \]  

(5) 

\[ m(a, \xi)\beta_a(\xi) = P(0) \]  

(6) 

At first, in process of simulating the physical properties adapted with RKPM formulation approximate value of the central particle with neighbor’s particles is calculated as follows where the parameter \( I \) represents the central particle and calculated with neighbor’s particles \( J \).

\[ < u(r^J) > = \sum_{j}^{np} u(r^J) \Psi \Delta v^j \]  

(7) 

Each particle size is calculates as follows:

\[ \Delta v^J = \frac{m^J}{\rho^J} \]  

(8) 

Select a suitable weight function for RKPM shape function according to articles in the field of penetration or impact is being offered. In most cases the cubic B-spline weight function has been used that in this article is in a same manner.

\[
\phi(r, h) = \alpha_d \begin{cases} 
2/3 - r^2 + 1/2, & 0 \leq r \leq 1 \\
1/6(2 - r)^3, & 1 \leq r \leq 2 \\
0, & r \geq 2
\end{cases}
\]  

(9) 

\[ \alpha_d = \frac{1}{h}, \frac{15}{7\pi h^2}, \frac{3}{2\pi h^3} \]  

Respectively for 1,2 and 3 dimensional, \( h \) is smoothing length and usually considered a fixed number in the present calculations; the smoothing length is variable and obtained as follows:

\[ \frac{d \ln h}{dt} = -\frac{1}{v} \frac{d \ln \rho}{dt} \]  

(10) 

Where \( v \) represents the issue of dimension.

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**Figure 1. Two-dimensional shape function RKPM**

**FUNDAMENT FORMULA**

In plasticity theory, when a Elastic-Plastic material became plastic then the increasing strain that irrespective
of the effects of viscosity, this increasing is time-dependent and adding a term to the strain elastic is considered; of course the calculation of impact with high speed, the third term with the thermal strain will be appear.

\[ \varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^T \]  

\[ \sigma = D \varepsilon \]  

In this relation, D is matrix of material properties in elastic-plastic and viscoplastic states. Different criteria for the state of stress so called the yield functions, submission criteria can be expressed in terms of unknown variables are the stress and public face of the form is written as:

\[ R(\sigma) = K(k) \]  

K is a parameter for modeling the hard strain or the soft strain and it can be a function of plastic strain, temperature, void ratio, density of accumulated dislocation and etc. In this paper criteria yield Van Misses in relation to the Johnson Cook material model is hidden, has been used. Basically, for the use of plasticity theory in various problems, the following terms should be determined:

1- Yield surface function.

\[ x = \varphi(X,t) \]  

\[ \dot{F} = \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial X} \varphi(X,t) \right] = \frac{\partial}{\partial X} \left[ \frac{\partial}{\partial t} \varphi(X,t) \right] = \frac{\partial \dot{v}}{\partial X} \]  

CONVENTIONAL EQUATIONS IN RKPM FORM

\[ \frac{d\rho_j}{dt} = \rho_j \sum_{j=1}^{N} m_j \frac{v_j}{\rho_j} \nabla \psi_j \]  

\[ m_j \frac{dv_j}{dt} = -\sum_{j=1}^{N} m_j \frac{\sigma_j}{\rho_j^2} \nabla \psi_j \]  

\[ \frac{de_j}{dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left( \frac{\sigma_j}{\rho_j^2} + \frac{\sigma_j}{\rho_j^2} \right) \cdot (v_j \otimes \nabla \psi_j) \]  

RKPM–FEM COUPLING ALGORITHM

Approximate linear momentum balance Momentum equations — In the Lagrangian environment in discrete weak form is given as follows. In fact, this equation represents the virtual work system is \( \delta u \).

\[ \int_{\Omega} \delta u (\nabla \sigma + \rho \ddot{u} + b) dv = 0 \]  

Where \( \sigma \) is Cauchy stress, parameter, \( \rho \) is density and \( b \) is physical force. Eliminating the physical forces at the beginning we have:

\[ \int_{\Omega} \delta u (\nabla \sigma + \rho \ddot{u}) dv = 0 \]  

According to the definition of divergence, we remove the tension of gradient term from momentum equation as follows:

\[ \text{div}(\delta u, \sigma) = \nabla (\delta u, \sigma) \]  

\[ \Rightarrow \int_{\Omega} \delta u \nabla \sigma dv + \int_{\Omega} \sigma \nabla \delta u dv = \int_{\partial \Omega} t \delta u ds \]  

In above relation \( n \) is external normal vector on the surface \( r \) and \( t \) are the traction on the same level. With high relationship the momentum equation can be rewritten as following three separate integrals that the tension gradient is no separation:
\[ \int \delta u (\nabla \sigma + \rho \ddot{u}) dv = 0 \rightarrow \int \delta u \rho \ddot{u} dv + \int t \delta u ds - \int \sigma \nabla \cdot \delta u dv = 0 \quad (27) \]

\( J = \text{det}(F) \) is known as the rate of volume, \( F \) is the deformation gradient, \( P \) is first tension of Piola-Kirchhoff tension tensor and is defined as follows

\[ P = JF^{-1}\sigma = SF^T \]

(28)

In above relation, \( S \) is second tension of Piola-Kirchhoff and is represented as follows:

\[ S_{ik}^{pq} = \frac{\partial X_i}{\partial x_k} \frac{\partial X_j}{\partial x_l} \sigma_{kl} \quad i, j, k, l = 1, 2, 3 \]

(29)

where \( x \) and \( \Gamma \) respectively is coordinate initial reference systems and deformation of the findings and \( \sigma_{kl} \) is real tension or Euler rotation or the tensions of non-explorer rotation. In action, if the following equation be establish then Piola-Kirchhoff’s can be replaced with explorer rotation. Of course with updated geometry can be satisfy this requirement.

\[ \left| \frac{\partial X_i}{\partial x_k} - \delta_{ik} \right| < 1 \]

(30)

\( \delta_{ik} \) is the Kronecker delta, as at the beginning defined \( \Psi_1 \) is RKPM shape function.

\[ \int \sigma \nabla \delta u dv = \int \frac{\partial \delta u}{\partial X} F^{-1}\sigma dv = \int \frac{\partial \delta u}{\partial X} Pd\Omega \]

(31)

Integral of weight remaining is obtained as follows:

\[ R_u = \int \rho \dddot{u} \delta u dv + \int t \delta u ds - \int PV \delta u dv = 0 \]

(32)

Substituting RKPM function instead of \( \delta u \) in the following equation and differentiation of it yields:

\[ R_u = \int \rho \dddot{u} \sum \delta u \phi_a (r') \Delta v' dv + \int t \sum \delta u \phi_a (r') \Delta v' ds - \]

\[ \int P \sum \delta u \phi_a (r') \Delta v' dv = 0 \]

(33)

\[ \frac{\partial R_u}{\partial \delta u} = 0 \Rightarrow \int \rho \dddot{u} \phi_a (r') \Delta v' dv + \int t \phi_a (r') \Delta v' ds - \]

\[ \int PV \phi_a (r') \Delta v' dv = 0 \]

(34)

Equation of motion above includes three integral equations of motion which is separated. Each integral separately is converted to discrete form:

\[ f^{\text{ext}} = \int b \Psi_j (X) dv + \int t \Psi_j (X) ds \]

(35)

\[ f^{\text{int}} = \int P \Psi_j (X) dv \]

(36)

\[ \int \rho \dddot{u} \phi_a (r') \Delta v' dv = \sum_j \rho \dddot{u} \phi_a (r'^j) \Delta v' \Delta v^j \]

(37)

\[ \int PV \phi_a (r') \Delta v' dv = \sum_{jab} P \nabla \Psi_j \phi_a \Delta v' \Delta v^j \]

(38)

\[ \int t \phi_a (r') \Delta v' ds = \sum_{j} t \phi_a (r'^j) \Delta v' \Delta s^j \]

(39)

Relationship mass at momentum balance RKPM approximation is as follows
\[ M = m_j = \int_{\Omega} \rho^0 \psi_j(X) \sum_{j=1}^{NP} \psi_j(X) d\Omega = \int_{\Omega} \rho^0 \psi_j(X) d\Omega \quad (40) \]

Lagrangian RKPM form of motion equation:
\[ \dot{M}u = f^{ext} - f^{int} \quad (41) \]

**RKPM-FEM ATTACHMENT ALGORITHM**

\[ u^b(x) = \sum_{j \in G^{RKPM}} \Psi_j u_y + \sum_{j \in G^{FEM}} N_j u_y \quad (42) \]

\[ G^{RKPM} \] is all node RKPM in a support and \( G^{FEM} \) is FEM node in the same support, \( \Psi_j \) is RKPM shape function and \( N_j \) is FEM shape function.

Above relationship can be open as follows and presents a new relationship for the function approximation:

\[ \sum_{j \in G^{RKPM}} H(x-x_j)H(x-x_j)\phi_a(x-x_j)b(x) + \]

\[ \sum_{j \in G^{FEM}} N_j H(x-x_j) = H(0) \quad (43) \]

\[ b(x) = m^{-1}(x)[H(0) - \sum_{j \in G^{FEM}} N_j H(x-x_j)] \quad (44) \]

\[ m(x) = \sum_{j \in G^{FEM}} \phi_a(x-x_j)H^T(x-x_j)H(x-x_j) \quad (45) \]

\[ \Psi_j(x) = H^T(x-x_j)m^{-1}(x)[H(0) - \sum_{j \in G^{FEM}} N_j H(x-x_j)]\phi_a(x-x_j) \quad (46) \]

\[ u^b(x) = \sum_{j \in G^{FEM}} H^T(x-x_j)m^{-1}(x)[H(0) - \sum_{j \in G^{FEM}} N_j H(x-x_j)]\phi_a(x-x_j)u_y + \sum_{j \in G^{FEM}} N_j u_y \quad (47) \]

**CONTACT ALGORITHM**

When projectile penetrate into the mental rigid deformation we are tackling with resistance forces that called contact force, these forces are separated to tangential and vertical direction.

\[ f^c = f^N + f^T \quad (48) \]

Where \( f^c \) is contact force, \( f^N \) is vertical contact force and \( f^T \) is tangential contact force. These forces are obtained by the following relations:

\[ f^N(s) = -\frac{m_s g_n}{\Delta t^2} n = f^N n \quad (49) \]

In this relation \( \Delta t \) is time of each step, \( m_s \) is mass of projectile in-which can be obtained as follows:

\[ g_n = (x_s - x_f) \cdot n \quad (50) \]

Where \( x_s \) represents the slave node coordinates and \( x_f \) represents the master node coordinates. The penalty parameter coefficients are also obtained as follows:

\[ \beta = \frac{m_s}{\Delta t^2} \quad (51) \]

Tangential forces are obtained from the following equation:

\[ f^T = -\frac{m_s}{\Delta t} v_f \quad (52) \]

Where

\[ v = v_s - v_f \quad (53) \]
where $v_s$ and $v_c$ are respectively slave node rate and master node rate, contact forces for all particle in RKPM-FEM coupling mode can be express as follows:

$$f^c_i = \sum_{j=1}^{n_c} f^c_i \psi_i(x_j) + \sum_{j=1}^{n_c} f^c_i N_i(x_j)$$

(55)

Since the projectile and target material in a number of experimental results in this article is steel. So they set the coefficient of friction between the relationships presented in (Liaghat and Malekzadeh., 1999), variable with change in the projectile velocity can be considered.

$$\mu = 0.51 - 0.135 \log v$$

(56)

Steel projectile when dealing with aluminum target the coefficient of friction are considered 0.1.

**MAKING DISCREET EQUATION OF MOTION**

Equation of motion is obtained as follow:

$$M\ddot{u}(t) + f^{\text{in}}(u(t)) + f^{\text{c}}(u(t)) = f^{\text{ext}}(t)$$

(57)

Where $M$ is mass matrix, and $t$ is step time, and $u$, $\dot{u}$, $\ddot{u}$ respectively are vectors of displacement, velocity and acceleration that with explicit method above equation rewrite as follow

$$\ddot{u}(t) = M^{-1}[f^{\text{ext}}(t) - f^{\text{in}}(u(t)) - f^{\text{c}}(u(t))]$$

(58)

$$\frac{d\dot{u}}{dt} = \frac{\ddot{u}_{t+1} - \ddot{u}_{t-1}}{2\Delta t} = M^{-1}[f^{\text{ext}}(t) - f^{\text{in}}(u(t)) - f^{\text{c}}(u(t))]$$

(59)

$$\frac{d^2u}{dt^2} = \frac{\ddot{u}_{t+1} - 2\ddot{u}_t + \ddot{u}_{t-1}}{\Delta t^2} = M^{-1}[f^{\text{ext}}(t) - f^{\text{in}}(u(t)) - f^{\text{c}}(u(t))]$$

(60)

Time index, index location, velocity and location of particle is obtained as follow:

$$\ddot{u}^{n+1} = \ddot{u}^{n} + \Delta t (\dddot{u}^{n} + \dddot{u}^{n+1})$$

(61)

$$\dddot{u}^{n+1} = \dddot{u}^{n} + \Delta t \dddot{u}^{n} + \frac{1}{2} \Delta t^2 \dddot{u}^{n}$$

(62)

After did the calculation for $\Delta t$ was observed that the sensitivity of results to change the parameters for smaller amounts of $\Delta t = 10^{-6}$ sec very small and is available regardless the calculations were done so in order.

![Figure 2. Flowchart of calculation.](image)
MATERIAL BEHAVIOR

Equation Of State
Change of the press, volume and energy of material through
the equation of state (EOS) to be considered. Micronesian
state equation for metallic materials is used as following

\[ P(\rho, e) = (1 - \frac{1}{2} L\eta) P_H(\rho) + L\rho e \]  \hspace{1cm} (63)

\[ \eta = \frac{P}{P_0} - 1 \]  \hspace{1cm} (64)

where \( H \) is Hugoniot curve, \( L \) is Micronisen and \( P_0 \) is the
initial density.

Model Material
Model material Johnson - Cook [1983], for soft materials such
as metals with high strain rate we are tackling to be used.
Ceylon tension or equivalent Van Misses stress is defined as
follow

\[ \sigma_f = (A + B\bar{\varepsilon}_p^m)(1 + c \ln \bar{\varepsilon}_p^*) (1 - T_m^n) \]  \hspace{1cm} (65)

Where \( \sigma_f \) is Ceylon tension, and \( A, B, c, m, n \) are fixed
parameters, and \( \bar{\varepsilon}_p \) is effective plastic strain, and \( \bar{\varepsilon}_p^* \)
is effective plastic strain rate for \( \dot{\varepsilon}_0 = 1 \text{s}^{-1} \) and \( T_m^* \) is defined as follow:

\[ T_m^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \]  \hspace{1cm} (65)

Defeated strain in the material model is defined as follow

\[ \varepsilon^f = [D_1 + D_2 \exp D_3 \sigma^*][1 + D_4 \ln \dot{\varepsilon}][1 + D_5 T^*] \]  \hspace{1cm} (66)

In the above equation, \( D_1 \) to \( D_5 \) are material constants and \( \sigma^* \)
rate of pressure is divided by effective tension.

\[ \frac{\sigma^*}{\sigma_{eff}} \]  \hspace{1cm} (67)

Defeated parameters are as follow in that for \( D=1 \) defeated
occurs or words \( \Delta \varepsilon^p \), total effective plastic strain will be
equal with failure strain.

\[ D = \sum \frac{\Delta \varepsilon^p}{\varepsilon^f} \]  \hspace{1cm} (68)

Example: To check the accuracy and precision of the new
numerical method presented in this paper with set of
experimental reference (Gupta and Madhu, 1992) and
(Piekutowski et al., 1999) has been compared. An objective
test is used in this references was a kind of Mild steel,
AL6061T651 & AL7075T651 and physical
properties and their coefficients are given in the Table in
below, projectile with nose Ogive accordance with considered
references.

| Table 1. Johnson-cook parameter for targets material |
|-----------------|---------|---------|-----------------|
| Material for target | Mild steel | AL6061 | AL7075 |
| A(MP) | 327 | 289 | 527 |
| B(MP) | 804 | 203 | 676 |
| \( c \) | 0.0114 | 0.011 | 0.017 |
| \( m \) | 0.94 | 1.34 | 1.61 |
| \( n \) | 0.73 | 0.35 | 0.71 |
| \( T_m^0 \) | 1526.85 | 652.22 | 652.22 |
| \( T_{room}^0 \) | 19.8 | 21.11 | 21.11 |
| B | 0.0005 | 0.001 | 0.001 |

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CONCLUSION

In this paper, using the RKPM- FEM coupling, process of penetration rigid projectile into metal target is simulated. Position of elements for total space target is background. Removing time parameter, remaining terms of velocity-target thickness and depth penetration-sticking velocity were obtained. Lagrangian FEM formulation was applied to the region without large deformation. The semi-Lagrangian RKPM formulation was employed in the region undergoing large deformation and material damage for desired regularity and resolution. Traditionally, the Gauss Quadrature (GQ) integration and nodal integration have been used in FEM and RKPM formulations, respectively. The proposed adaptively coupled FEM-RKPM formulation with frictional kernel contact algorithm has been applied to earth moving simulation and high velocity fragment contact-impact problems. The stable time step estimate has been introduced to update the time step in the numerical solution. Several simulations have been performed and have compared with experimental results.
Reasonable agreements between numerical method and experimental data have been observed.

REFERENCES


Nianfei G., L. Guangyao, L. Shuyao.; 2009. 3D adaptive RKPM method for contact problems with elastic-plastic dynamic large deformation, Eng Analysis with Boundary Elements **33**: 1211-1222.


Piekutowski A. J., M. M J. Forrestal, K. I. Poormon, T. L. Warren.; 1999. Penetration of 6061-T651 aluminum targets by ogive-nosed steel projectiles with striking velocities between 0.5 and 3.0 Km/s, Int J Impact Eng; **23**:723-34.
